Analytical Modeling of Forced Convection in Slotted Plate Fin Heat Sinks

P. Teertstra, J. R. Culham & M. M. Yovanovich

Microelectronics Heat Transfer Laboratory
Department of Mechanical Engineering
University of Waterloo
Waterloo, Ontario, Canada

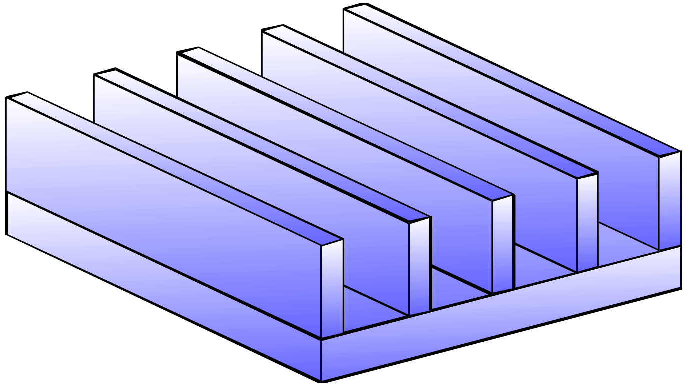
http://www.mhtl.uwaterloo.ca

Future Challenges in Electronic Packaging
International ME'99 Congress and Exposition

Nashville, TN
November 17, 1999

Introduction: Plate Fin Heat Sinks

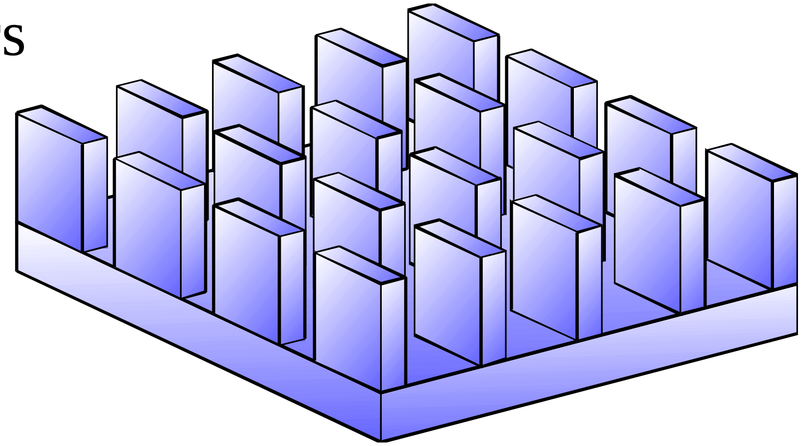
- Heat transfer enhancement for air cooled applications:
  - increase effective surface area
  - decrease thermal resistance
  - control operating temperatures



- Plate fin heat sinks:
  - most common configuration
  - convection in channels between fins

Introduction: Slotted Fin Heat Sinks

- Enhanced thermal performance:
  - new thermal boundary layers initiated at each fin section
  - increase in average heat transfer coefficient
  - decrease in surface area



- Performance of slotted fin heat sinks function of slot size and spacing
- Optimal slotted fin heat sink design balances enhancement of *h* with reduction of *A*

Introduction: Heat Sink Selection

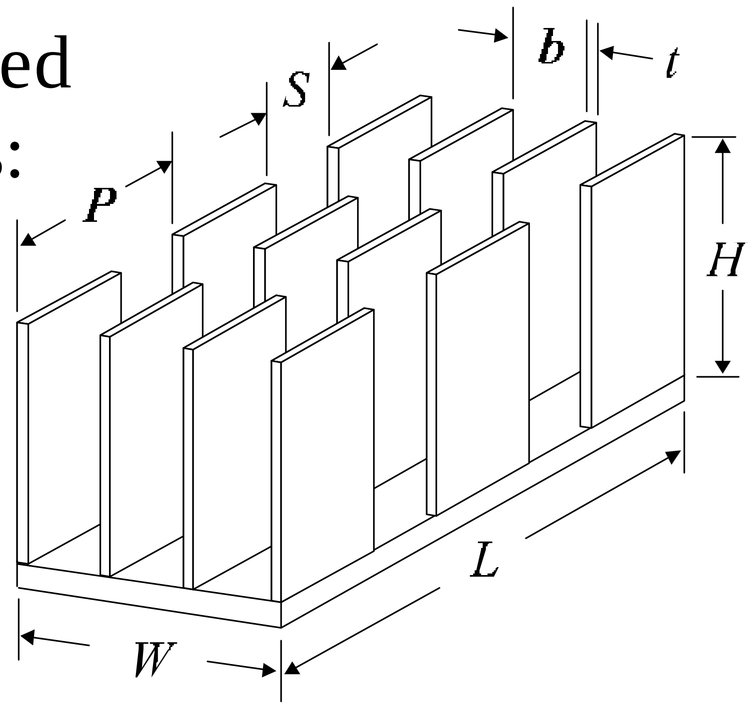
- Heat sink selection depends on many factors:
  - performance
  - dimensional constraints
  - available airflow
  - cost
- Quick and accurate design tools are required:
  - predict performance early in design
  - perform parametric studies
  - alternative to numerical simulations, experiments

Objectives

- Develop analytical models for average heat transfer rate for slotted fin heat sinks:
  - laminar, forced convection flow
  - full range of developing and fully-developed flow
  - non-isothermal fins
- Perform experimental measurements to validate proposed models:
  - range of slot sizes and spacing
  - inline and staggered slot arrangement

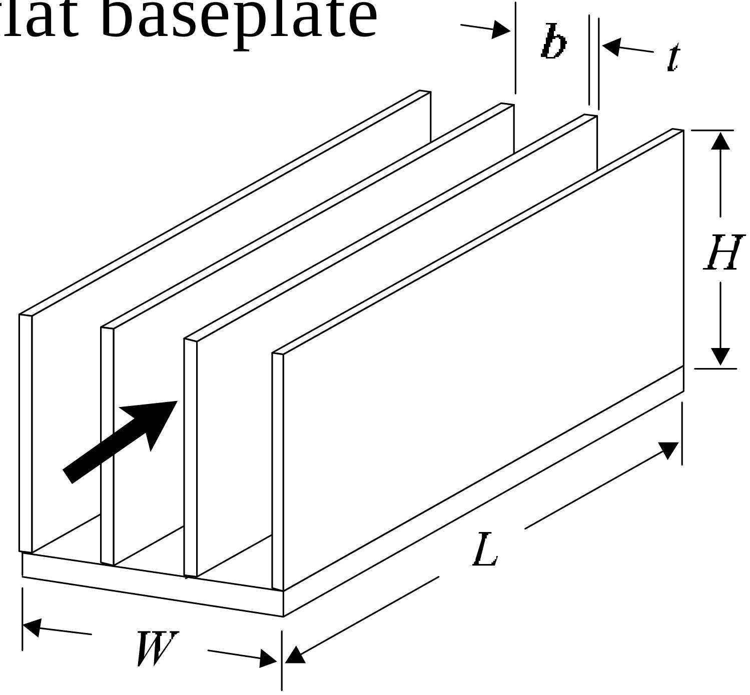
Problem Definition: Slotted Heat Sinks

- Uniformly sized and spaced slots in fins
  - fins slotted from tip to baseplate
  - fin sections connected only by baseplate
- Slot size and spacing described by dimensionless parameters:
  - pitch, $P/L \quad (0 < P/L \leq 1)$
  - width, $S/P \quad (0 \leq S/P < 1)$
- Slot arrangement:
  - inline
  - staggered

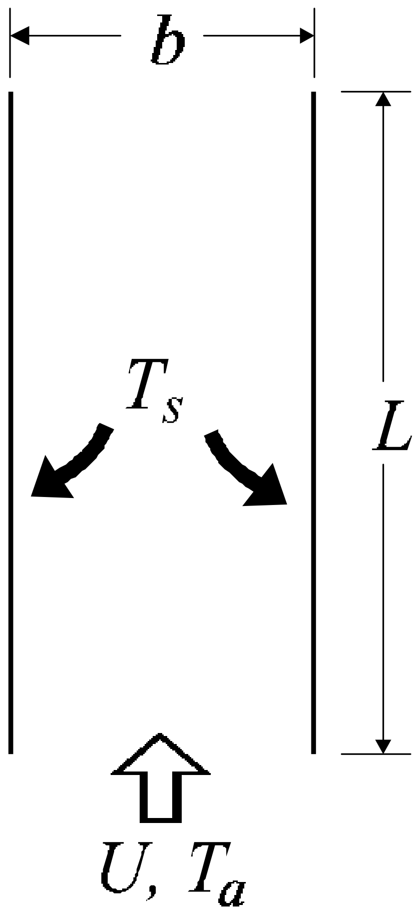


Problem Definition: Plate Fin Heat Sink

- Array of N plates on a single, flat baseplate
- Baseplate assumptions:
  - fins in perfect thermal contact
  - isothermal
  - adiabatic lower surface, edges
- Uniform velocity in all channels with no bypass:
  - shrouded heat sink
  - with flow bypass model for un-shrouded heat sinks
- Heat sink modeled as N-1 parallel plate channels



Problem Definition: Parallel Plate Channel



- Assume $b << H$
  - 2D channel flow
  - neglect baseplate, shroud effects
- Isothermal boundary conditions
- Reynolds number:

$$\mathrm{Re}_b = \frac{U\,b}{\nu}$$

- Nusselt number:

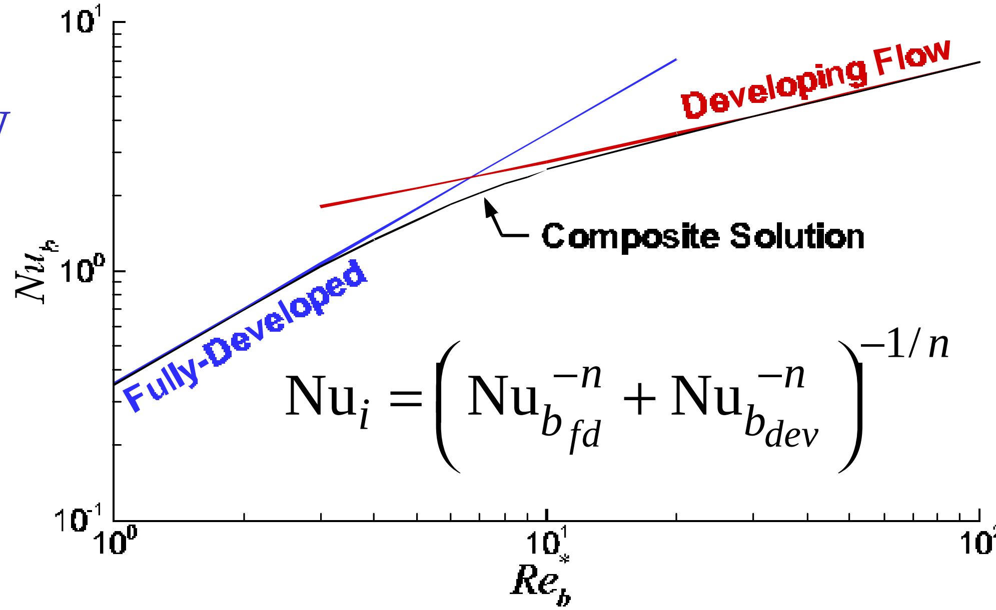
$$\mathrm{Nu}_b = \frac{Q\,b}{k\,A\left(T_s - T_a\right)}, \quad A = 2\,L\,H$$

Parallel Plate Channel Model

- Composite solution of 2 limiting cases (Teertstra et al, 1999)
  - Fully Developed Flow

$$\mathrm{Nu}_{b_{fd}} = \frac{\mathrm{Re}_b^* \, \mathrm{Pr}}{2}$$

$$\mathrm{Re}_b^* = \mathrm{Re}_b \cdot \frac{b}{L} = \frac{U\, b^2}{\nu\, L}$$



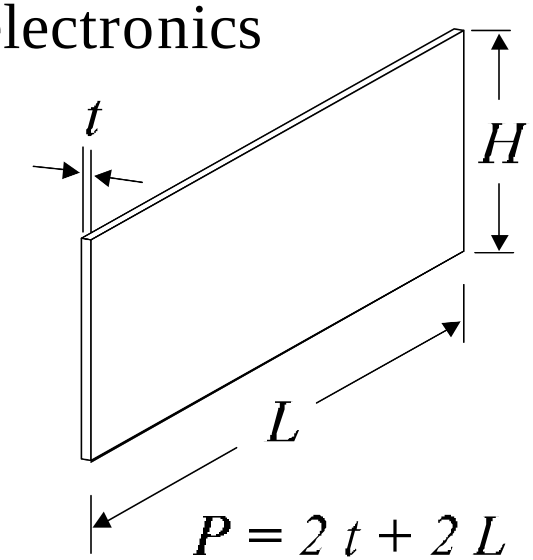
$$\mathrm{Nu}_i = \left(\mathrm{Nu}_{b_{fd}}^{-n} + \mathrm{Nu}_{b_{dev}}^{-n} \right)^{-1/n}$$

  - Developing Flow

$$\mathrm{Nu}_{b_{dev}} = 0.664 \sqrt{\mathrm{Re}_b^*} \, \mathrm{Pr}^{1/3} \left(1 + \frac{3.65}{\sqrt{\mathrm{Re}_b^*}} \right)^{1/2}$$

Plate Fin Heat Sink Model

- Fin effects included in heat sink model:
  - high aspect ratio heat sinks for power electronics
  - dense arrays of tall, thin fins
  - increased surface area for convection
  - efficiency reduced
- Fin efficiency: $\eta = \mathrm{Nu}_b / \mathrm{Nu}_i$
- Assume adiabatic condition at fin tip:



$$P = 2\,t + 2\,L$$
$$A_c = t\,L$$

$$\eta = \frac{\tanh(m\,H)}{m\,H}, \quad m = \sqrt{\frac{h\,P}{k\,A_c}}, \quad h = \mathrm{Nu}_i \cdot \frac{k_f}{b}$$

Plate Fin Heat Sink Model

- Model Summary

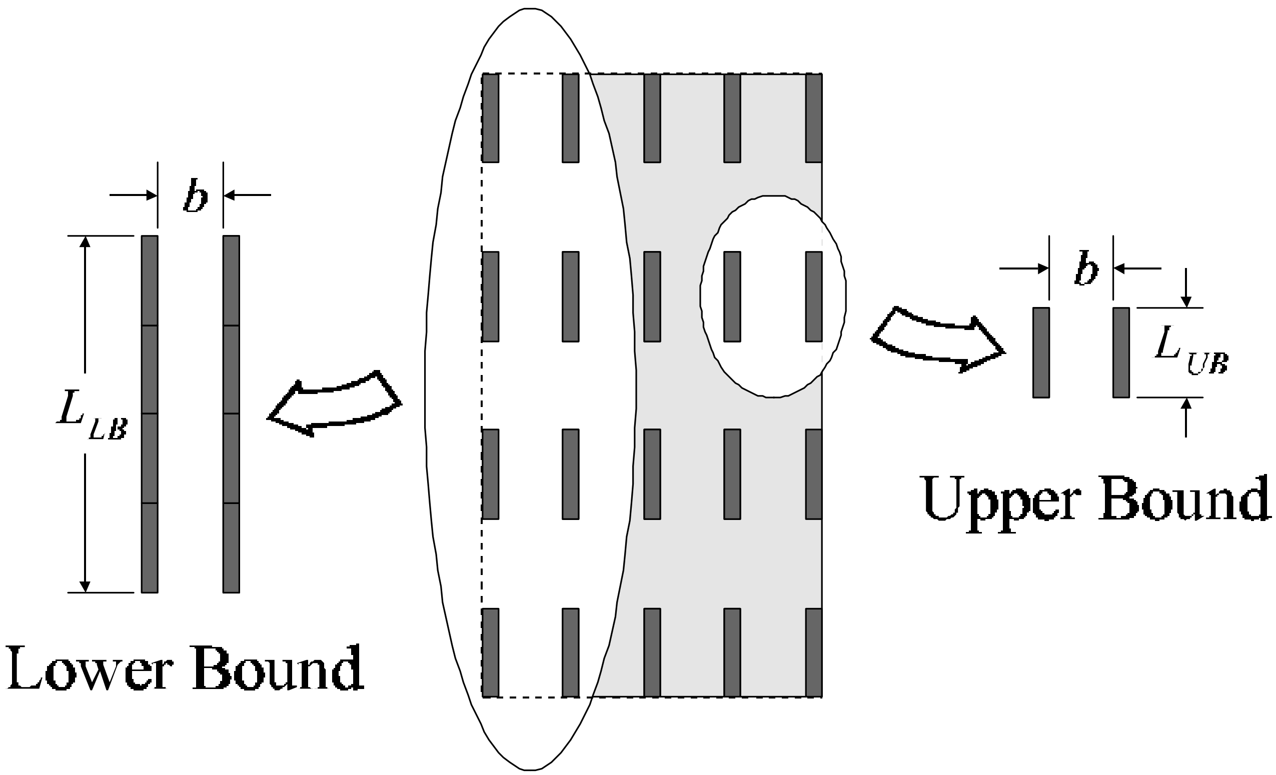
$$Q = N \cdot \frac{k_f A(T_s - T_a)}{b} \cdot \eta \cdot \mathrm{Nu}_i$$

$$\mathrm{Nu}_i = \left[\left(\frac{\mathrm{Re}_b^* \, \mathrm{Pr}}{2} \right)^{-3} + \left(0.664 \sqrt{\mathrm{Re}_b^*} \, \mathrm{Pr} \left(1 + \frac{3.65}{\sqrt{\mathrm{Re}_b^*}} \right)^{1/2} \right)^{-3} \right]^{-1/3}$$

$$\eta = \frac{\tanh \sqrt{2 \, \mathrm{Nu}_i \frac{k_f}{k} \frac{H}{b} \frac{H}{t} \left(\frac{t}{L} + 1 \right)}}{\sqrt{2 \, \mathrm{Nu}_i \frac{k_f}{k} \frac{H}{b} \frac{H}{t} \left(\frac{t}{L} + 1 \right)}}$$

Slotted Fin Heat Sink - Model Bounds

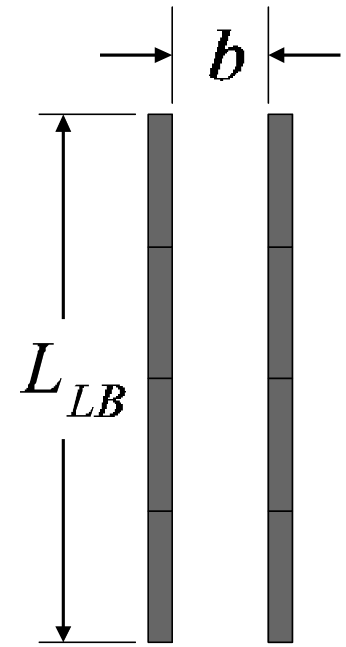
- Complex problem where exact solution not possible
- Upper and lower bounds from plate fin heat sink model:



Slotted Fin Heat Sink - Lower Bound

- No new boundary layers formed
- Modeled using equivalent fin length:

$$L_{LB} = L - N_S \cdot S = L\left(1 - S/P\right)$$



- Lower bound expressions:

$$\mathrm{Re}^*_{LB} = \frac{\mathrm{Re}^*_b}{1 - S/P} \quad \rightarrow \mathrm{Nu}_i$$

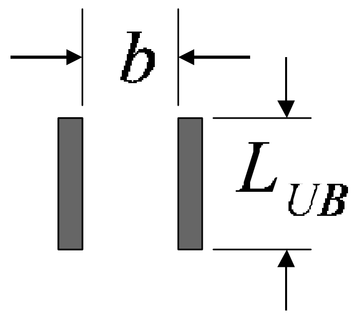
$$\frac{t}{L_{LB}} = \frac{t/L}{1 - S/P} \quad \rightarrow \eta$$

Slotted Fin Heat Sink - Upper Bound

- New thermal boundary layer formed at each fin section with no upstream effects
- Modeled using equivalent fin length:

$$L_{UB} = P - S = L(P/L)(1 - S/P)$$

- Upper bound expressions:



$$Re^*_{UB} = \frac{Re^*_b}{(P/L)(1 - S/P)} \quad \rightarrow Nu_i$$

$$\frac{t}{L_{UB}} = \frac{t/L}{(P/L)(1 - S/P)} \quad \rightarrow \eta$$

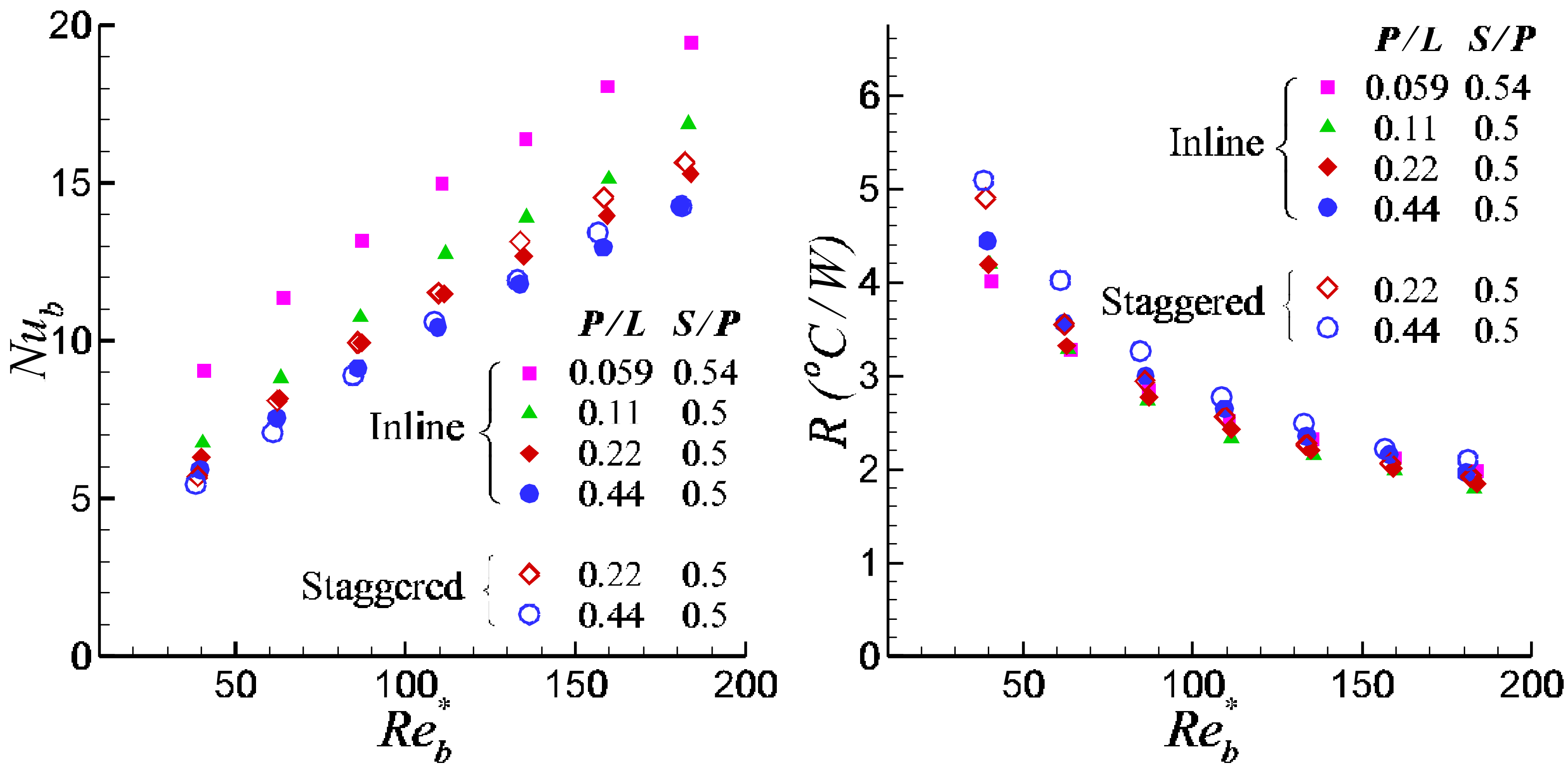
Experimental Apparatus

- High aspect ratio, $H/b \approx 20$
- Various slot configurations
- Back-to-back arrangement
- Mounted in Plexiglas shroud
- Approach velocity measured with hot wire anemometer

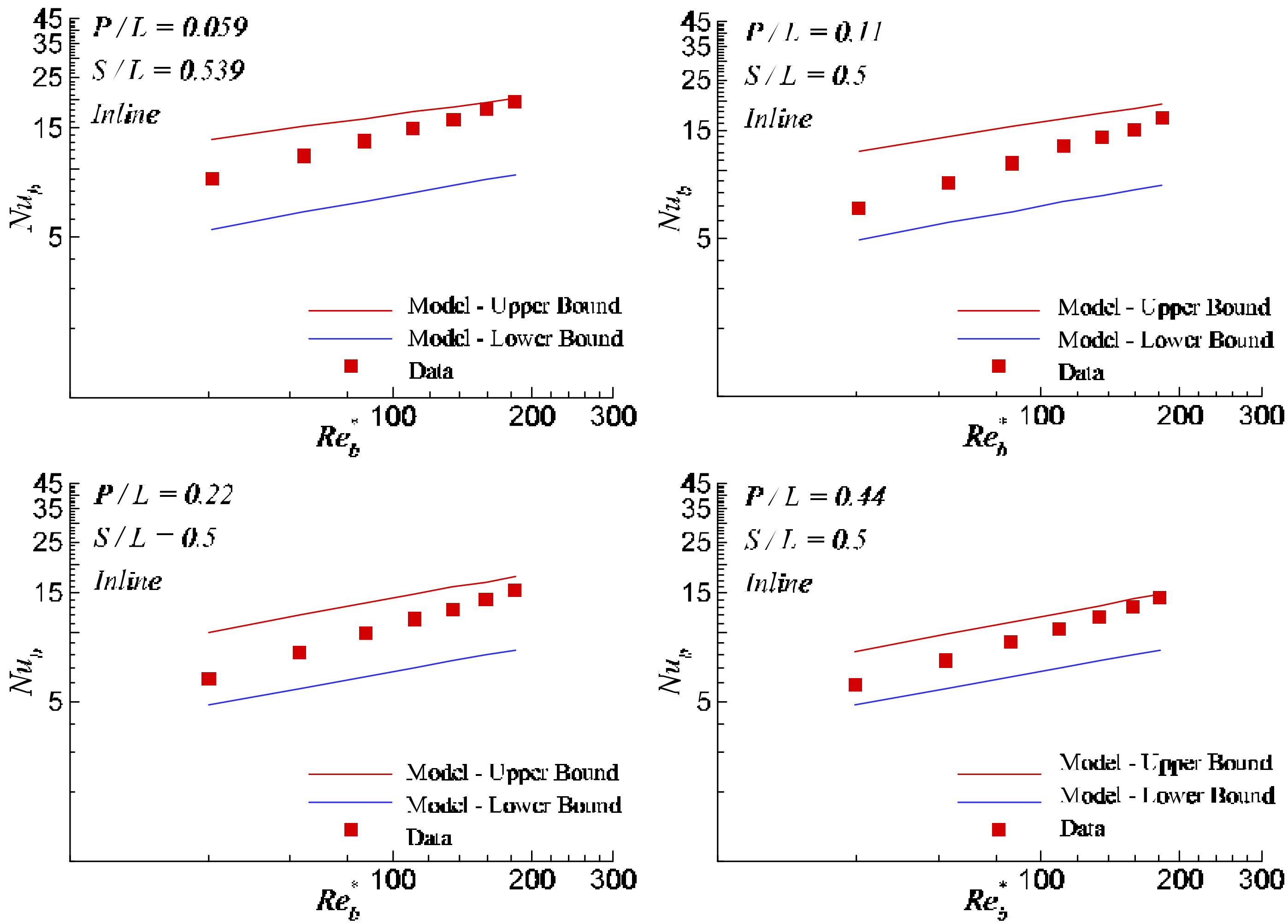
P/L	S/P	Slots
0.059	0.54	inline
0.11	0.5	inline
0.22	0.5	inline
0.44	0.5	inline
0.22	0.5	staggered
0.44	0.5	staggered

- Temperatures measured at 4 locations on baseplate
- Radiation losses measured in separate experiment

Experimental Results

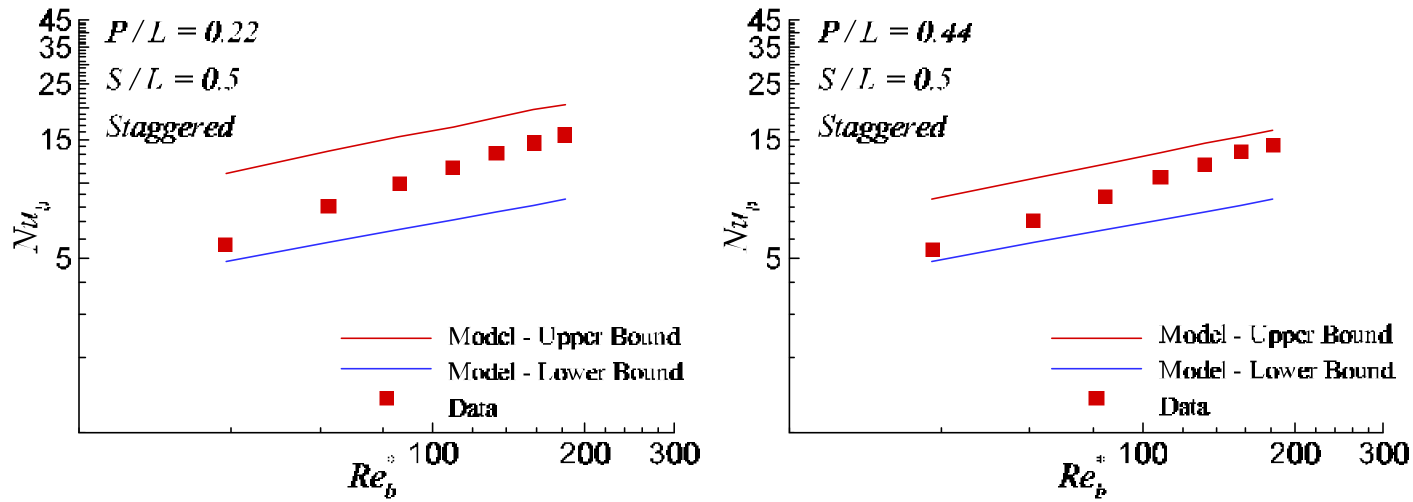


Model Validation



Microelectronics Heat Transfer Laboratory
University of Waterloo

Model Validation



- Arithmetic mean of bounds within 12% RMS of data

$$\mathrm{Nu}_b = \frac{Nu_{LB} + Nu_{UB}}{2}$$

$$40 \le \mathrm{Re}_b^* \le 180$$

$$0.11 \le P/L \le 0.44$$

$$S/P = 0.5$$

Summary and Conclusions

- Models developed for upper and lower bounds for slotted fin heat sinks
- Experimental data within bounds for full range of test conditions
- Arithmetic mean of bounds predicts Nu_b within 12% RMS over range of test conditions
- Reliable optimization procedure cannot be determined from the limited range of S/P values
- Additional study and data are required

Acknowledgements

The authors gratefully acknowledge the continued financial support of R-Theta Inc. and Materials and Manufacturing Ontario.