APPROXIMATE MODELLING PROCEDURES FOR RAPID ANALYSIS AND DESIGN

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Overview

What is modelling?

How can approximate modelling methods be used to understand thermal behaviour in electronics applications

Modelling procedures: as applied to heat sinks

Other potential applications

Modelling Alternatives

Experimental Methods

- prototype testing
- empirically-based correlations

Numerical Methods

- approximate the governing equations
  over a finite, discretized domain

Analytical Methods

- closed form solutions
- approximate methods

Why Use Approximate Methods?

Fast, accurate and easy to use

Minimal hardware requirements

Ideal for preliminary design studies

- material selection
- component selection and placement
- trade-off studies

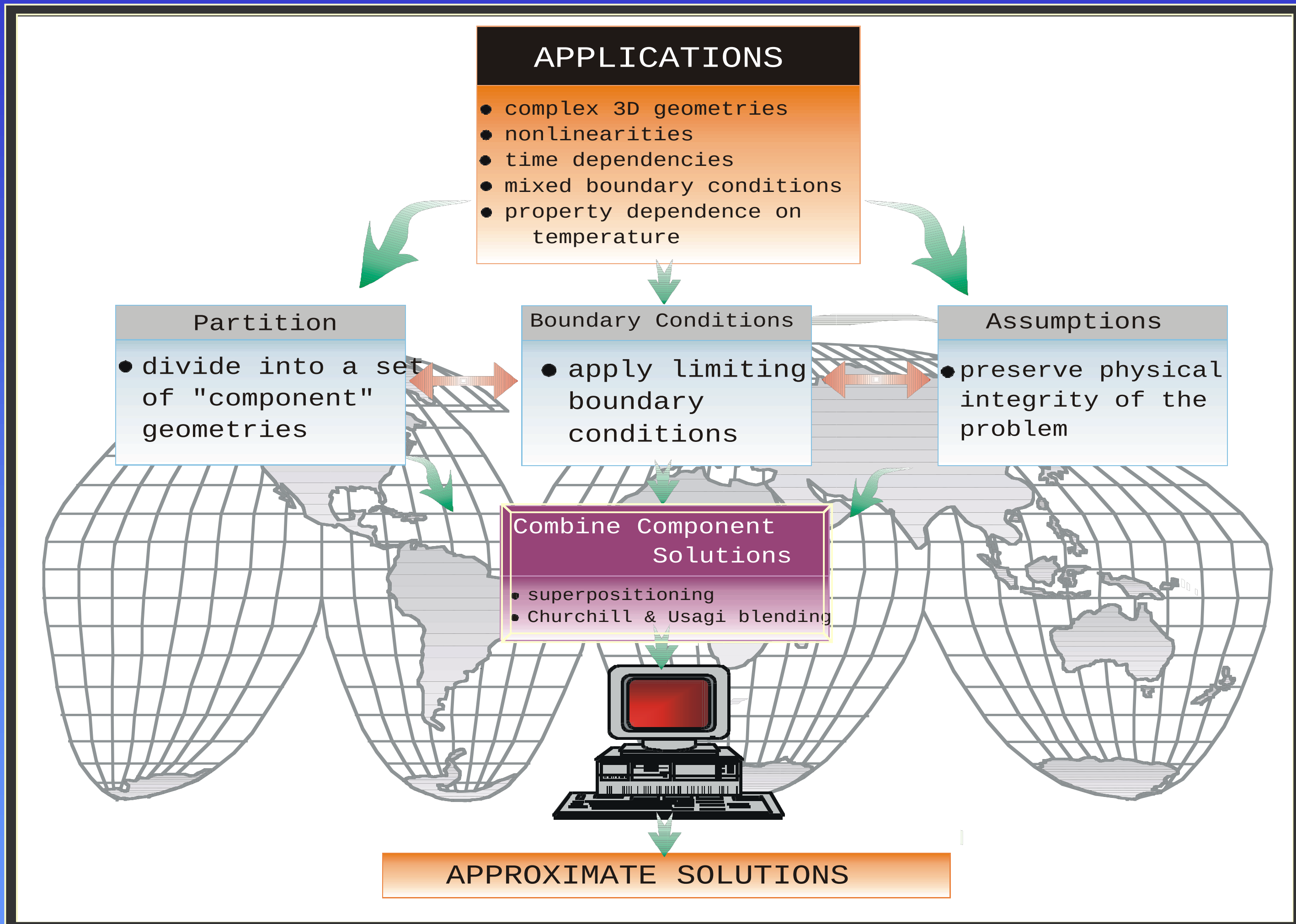
Optimization studies

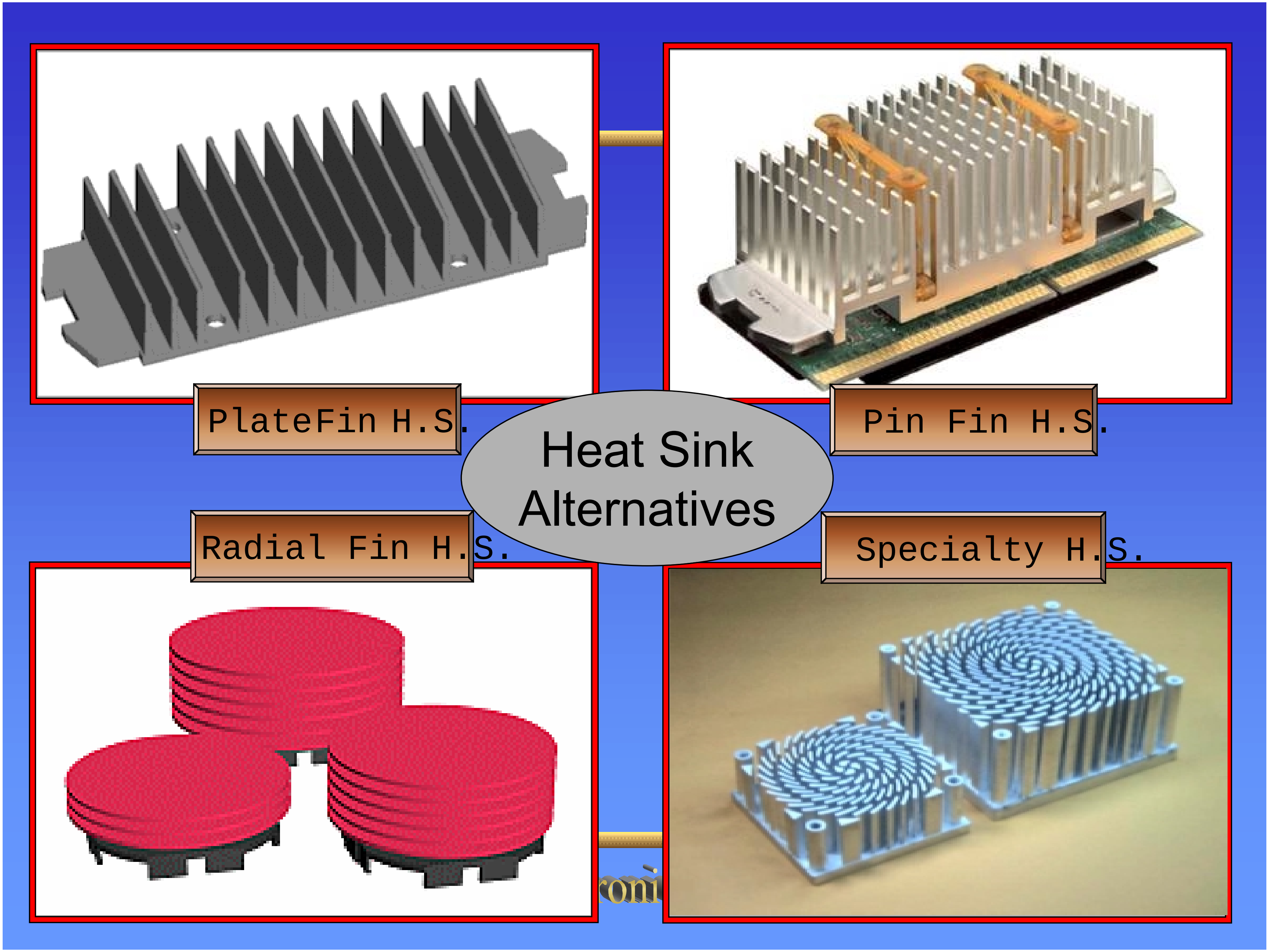
Concurrent design

Perceived Limitations

- Limited range of applications
- Cannot be used for complex geometries
- Cannot be used with mixed or non-uniform boundary conditions
- Simplifying assumptions provide inaccurate solutions

Modelling Approach

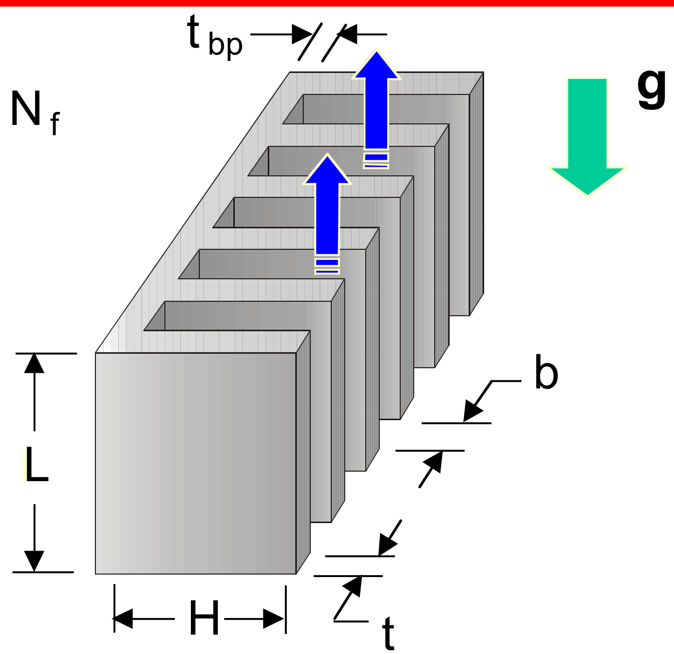



Heat Sink Alternatives
PlateFin H.S.
Pin Fin H.S.
Radial Fin H.S.
Specialty H.S.

Heat Sink Model

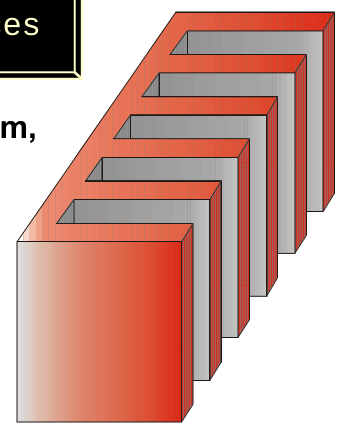
- Plate fin heat sink
- Natural convection
- Vertical orientation
- Isothermal
- Steady state
- Working fluid is air

Modelling Procedure



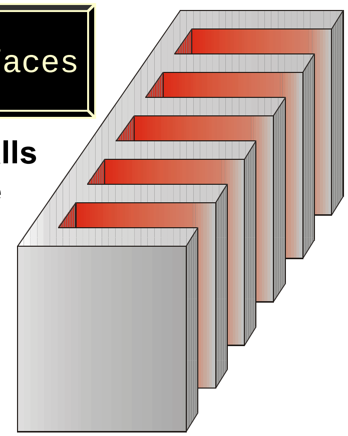
Exteriorsurfaces

- **fins : top, bottom, ends & tip**
- **base plate: top, bottom, ends and back**



Interior surfaces

- **fins : side walls**
- **channel base**



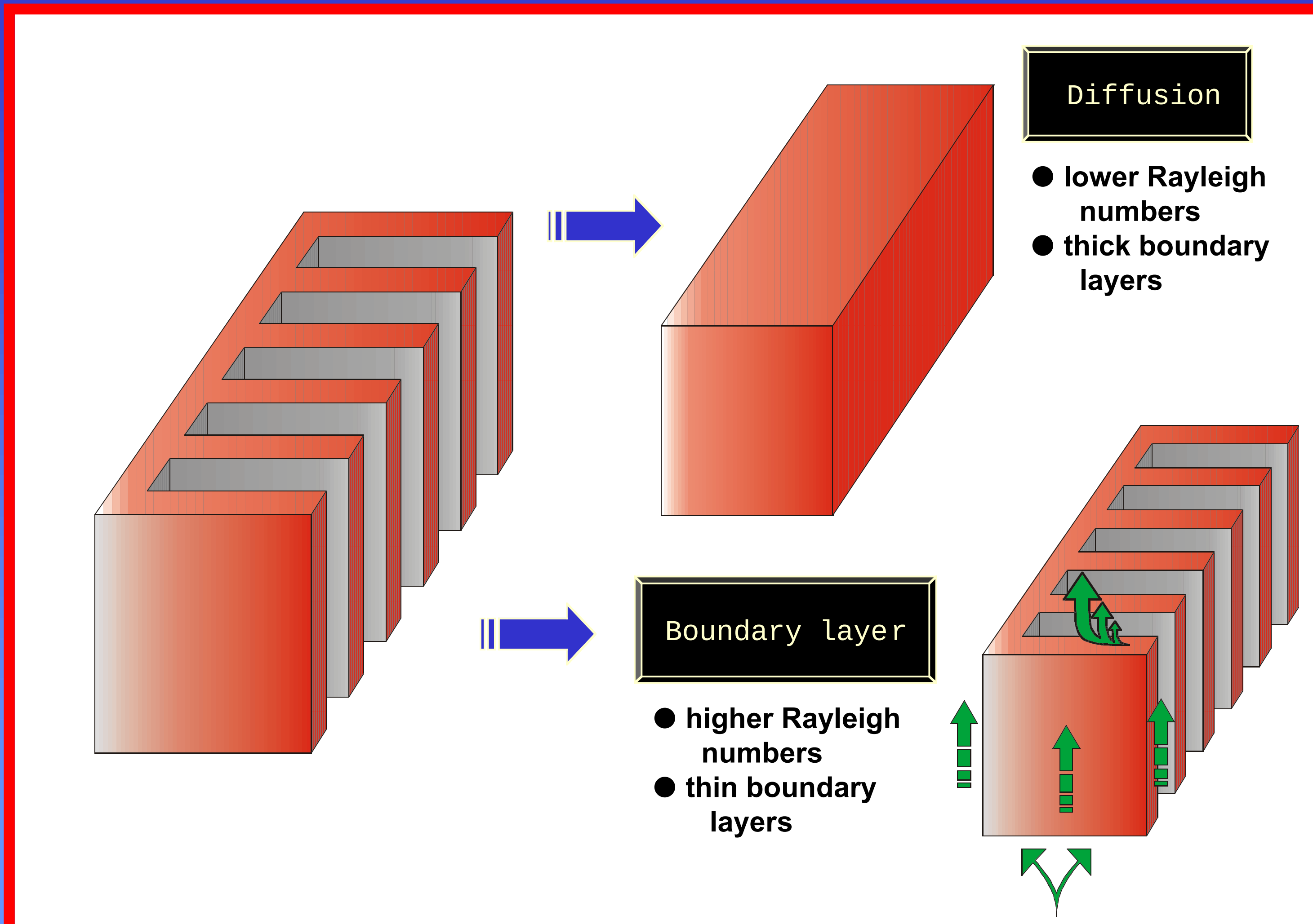
Given: **dimensions & temperature**

Find: Nu_b *vs.* Ra_b

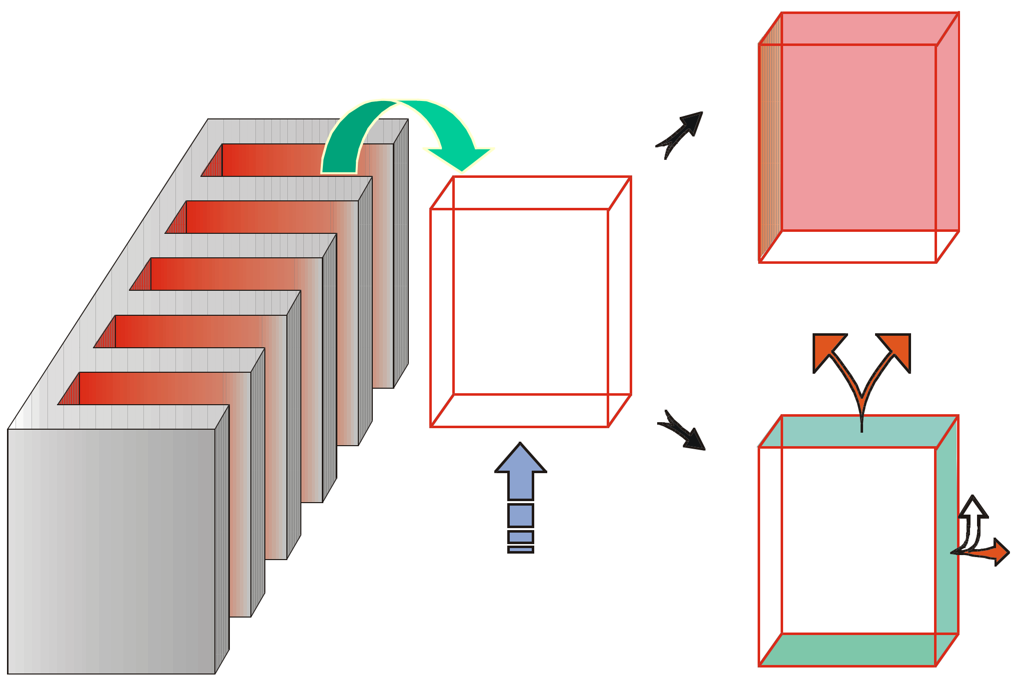
$$Ra_b = \frac{g \beta \Delta T b^3}{\alpha \nu} \bullet \frac{b}{L}$$

$$Nu_b = \frac{h b}{k_f}$$

Exterior Surfaces



Interior Surfaces



Channel flow

- **Elenbaas model with adjustment for end wall**
- **combined flow : developing + fully developed**

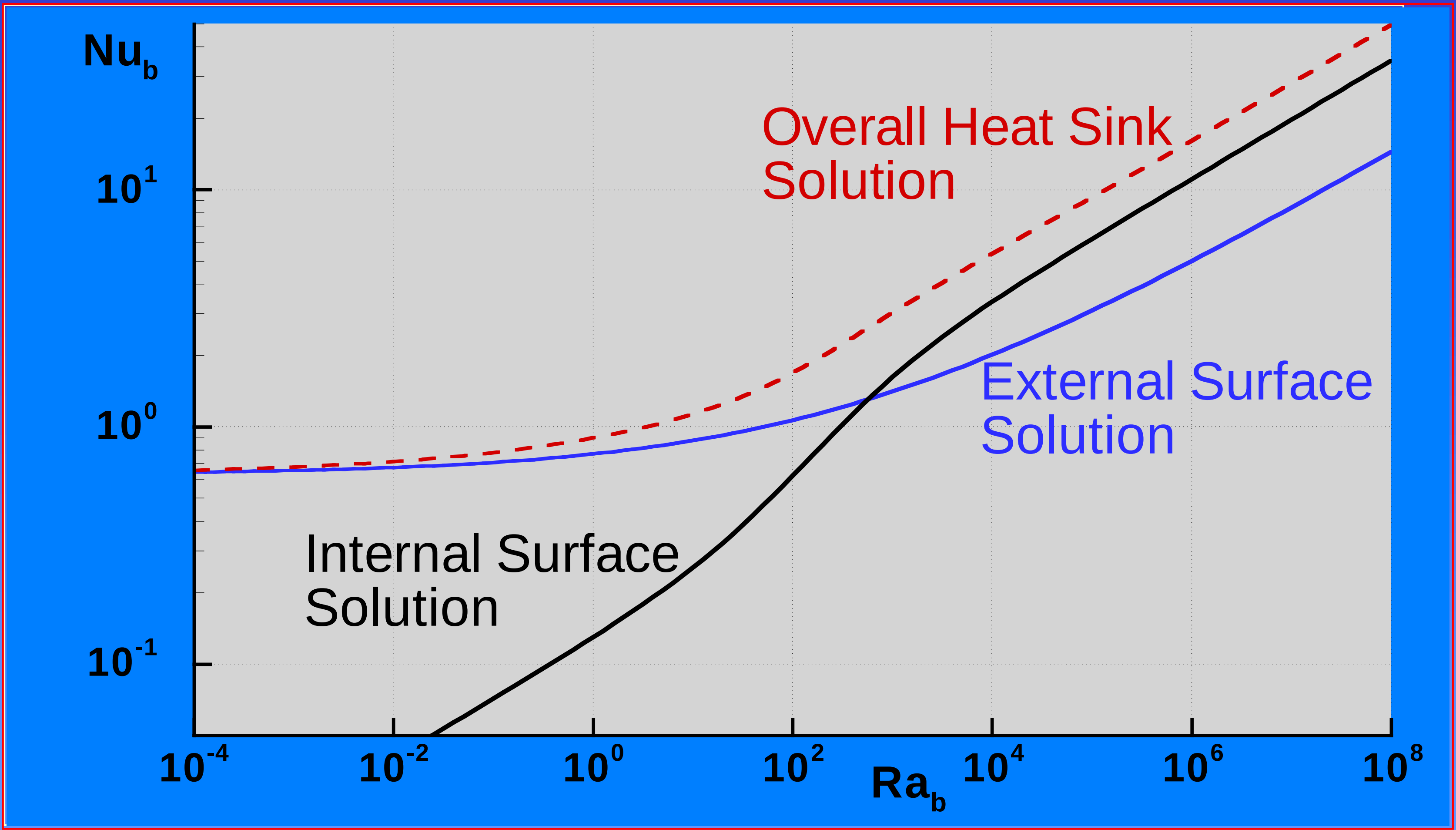
Control surfaces

- **open surfaces with energy migration**

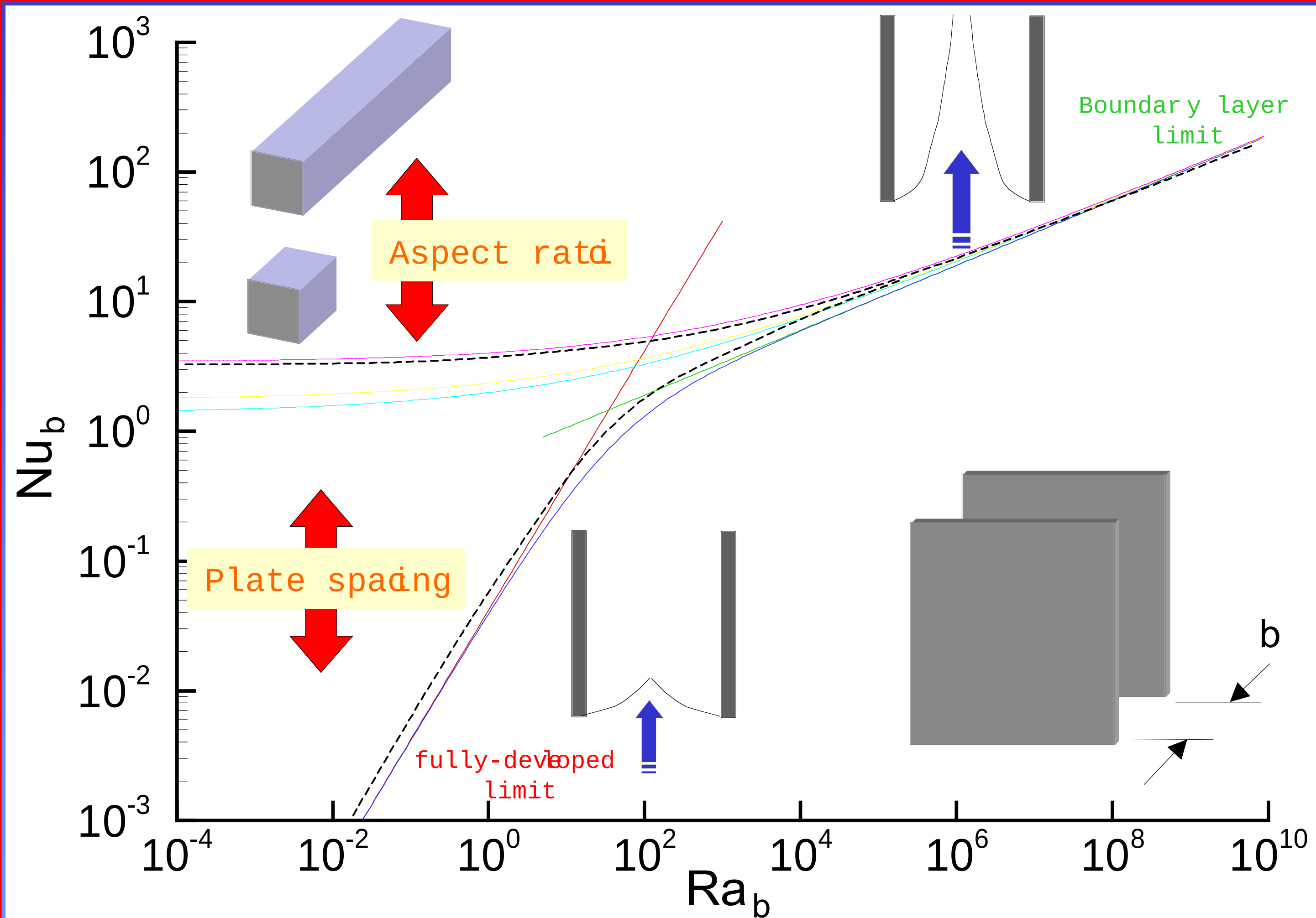
Comprehensive Model

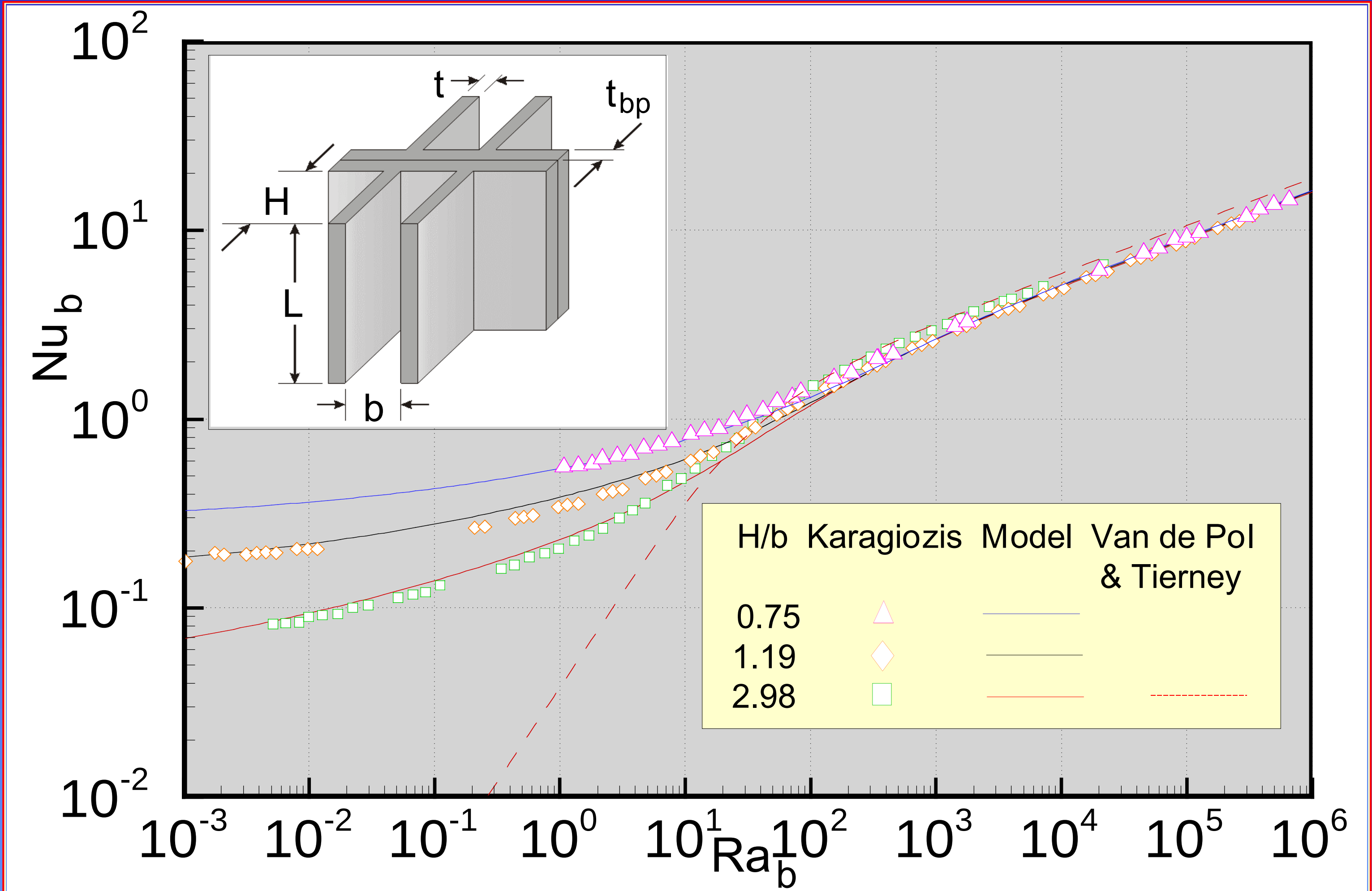
$$Nu = \underbrace{Nu_0}_{\text{diffusion}} + \underbrace{\frac{1}{\dfrac{1}{Nu_2} + \dfrac{1}{Nu_3 + Nu_4}}}_{\text{channel flow}} + \underbrace{Nu_1}_{\text{external boundary layer flow}}$$

Total Composite Solution

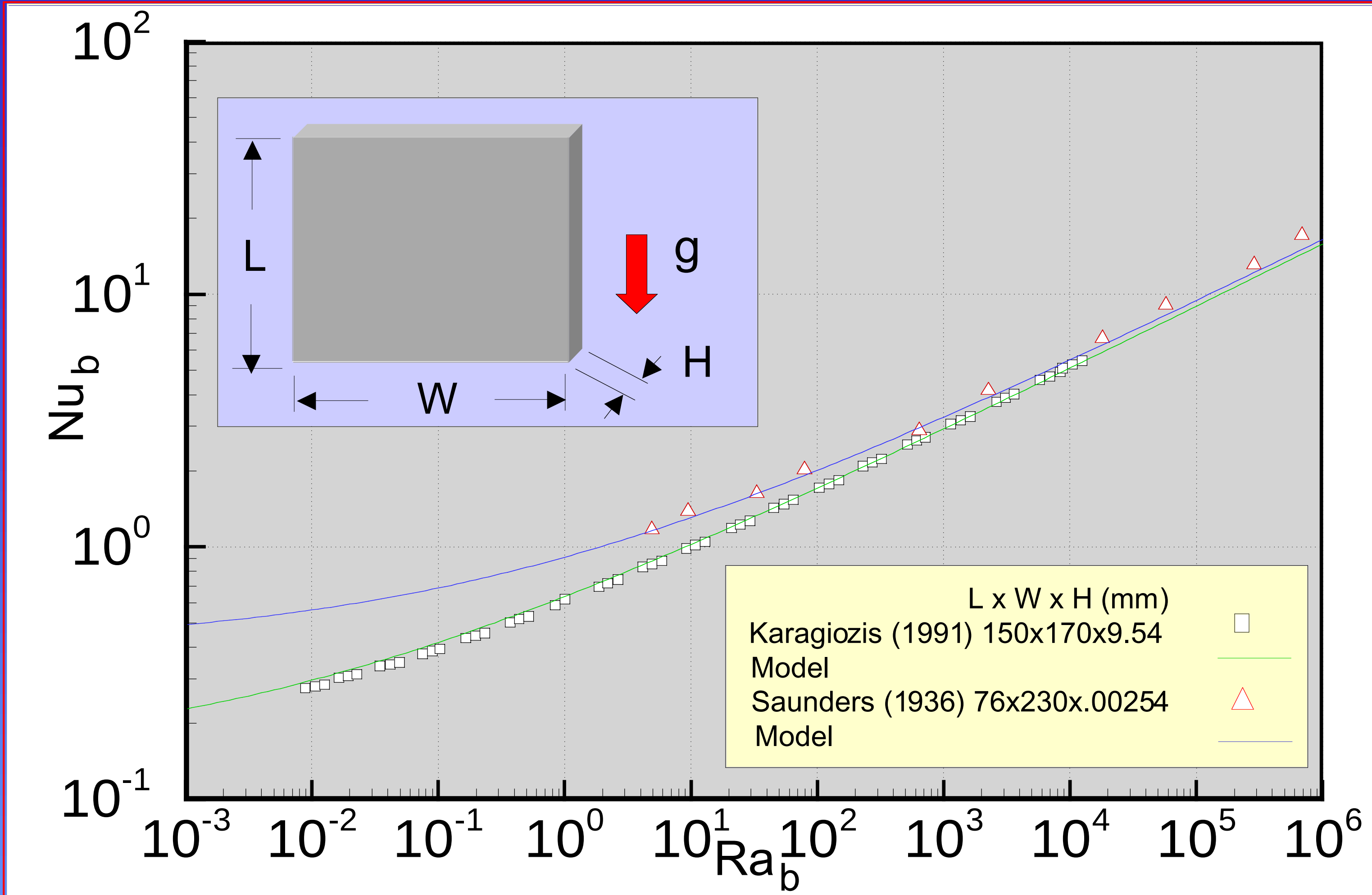


Modelling Domain





$$Nu = Nu_0 + \left\{ Nu_2^{-2} + \left[Nu_3 + Nu_4 \right]^{-2} \right\}^{-1/2} + Nu_1$$



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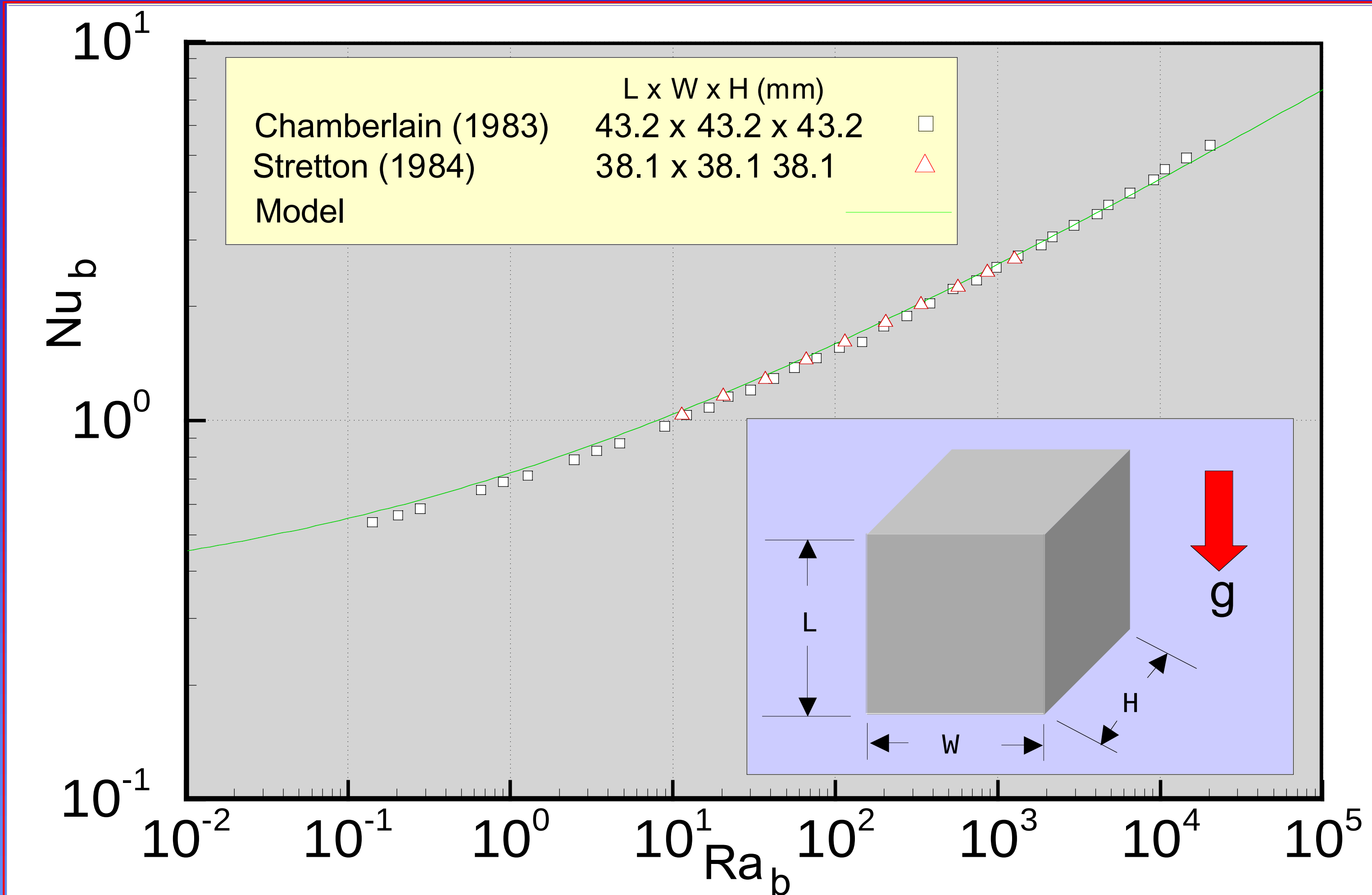

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How Do We Use These Results?

dimensionless heat transfer coefficient

dimensionless flow parameter

$$Nu_b = \frac{hb}{k_f} = \frac{Q}{\Delta T \bullet A} \bullet \frac{b}{k_f} \propto \frac{g\beta\Delta T b^3}{\alpha\nu} \bullet \frac{b}{L} = Ra_b$$

If max. ΔT changes ⇨ what is the max. Q dissipated

For a given Q ⇨ what is the max. ΔT of the heat sin
or the package junction

How do changes in geometry affect ⇨ ΔT and Q

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Future Work

Goal: Develop a comprehensive model to find the best heat sink design given a limited set of design constraints

Physical Design

¥ heat sink type
¥ material
¥ weight
¥ dimensions
¥ surface finish

Thermal

¥ maximum volume
¥ boundary conditions
¥ max. allowable temp.
¥ orientation
¥ flow mechanism

Cost

¥ labour
¥ manufacturing
¥ material

Standards

¥ noise
¥ exposure to touch

Other Examples of Approximate Models

Applications	*Asymptotic Limits*	
Heat & Mass Transfer		
• ***Boundary layer flow***	laminar	turbulent
• ***Channel flow***	fully developed flow	boundary layer flow
• ***External flow***	diffusion	boundary layer flow
• ***Internal flow***	fully developed flow	developing flow
• ***Enclosures***	diffusion	boundary layer flow
• ***Transient conduction***	short time	steady state
• ***Radiation***	opaque	transparent
• ***Steady conduction at nano-scales***	rarefied	continuum
Moving Sources	stationary	fast moving
Elasto-plastic contacts	elastic	plastic

Summary

Approximate models offer superior speed of execution and ease of use over most conventional modelling methods

Analytical modelling can be used for a wide range of applications previously considered to be too complex

The End