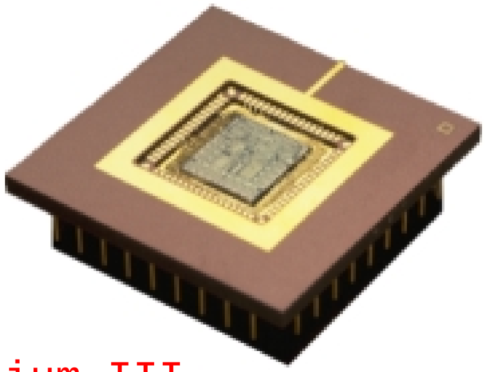
Thermal Design and Optimization of Heat Sinks

J. Richard Culham

Outline

- Background
- Modelling Approach
- Validation
- Optimization
- Future Work
- Summary

40 Watts! What's the big deal?



Pentium III

- 0.25 micron CMOS technology
- 9.5 million transistors
- 450 - 550 MHz

Light Bulb

- Power: 40 W
- Area: 120 cm^2
- Flux: 0.33 W/cm^2

Silicon

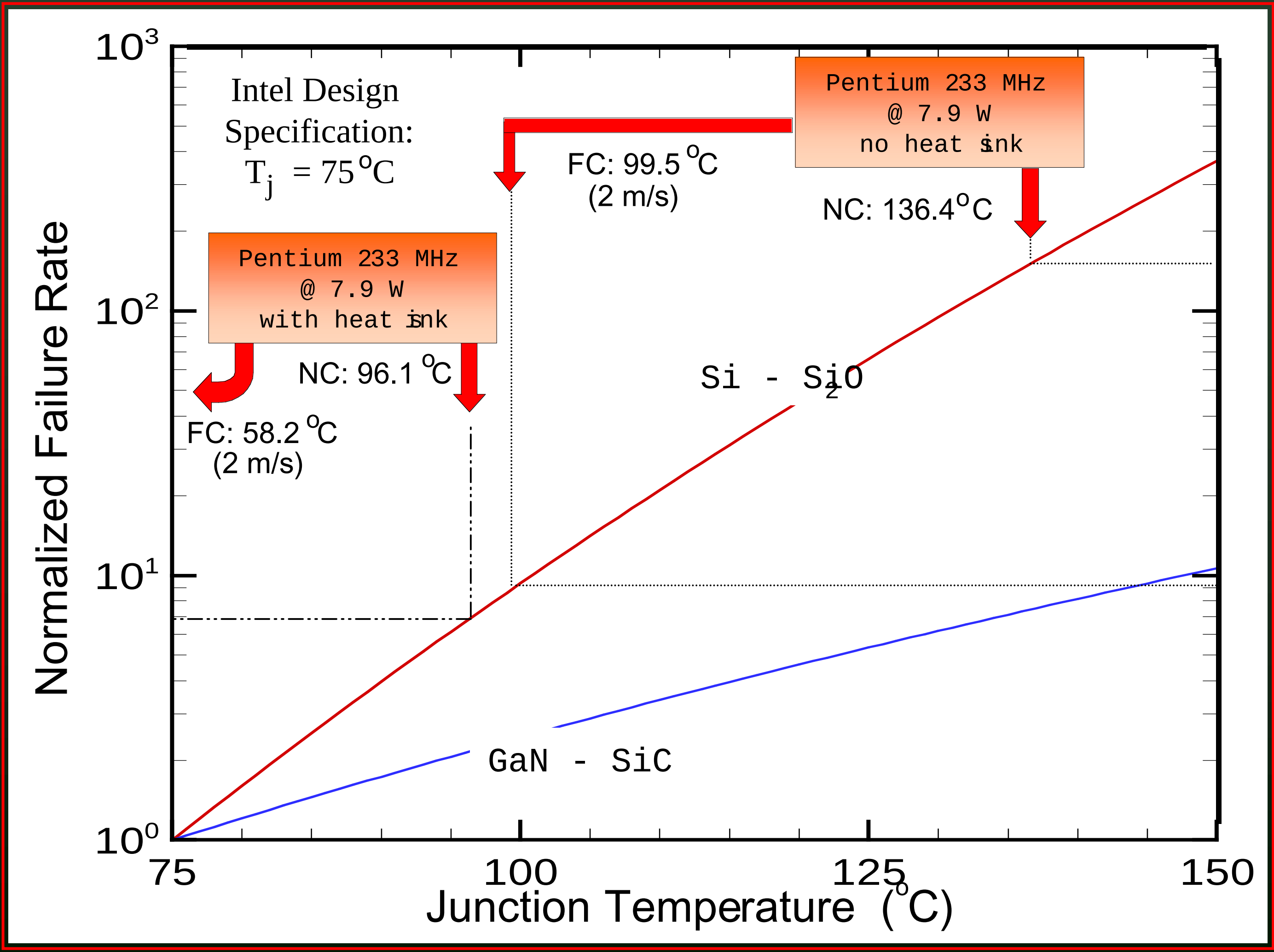
- Power: 40 W
- Area: 1.5 cm^2
- Flux: 26.7 W/cm^2

Package

- Rj-c: 0.94 C/W
- Rj-a: 6.8 C/W (no heat sink)
- Rj-a: 2.5 C/W (heat sink)

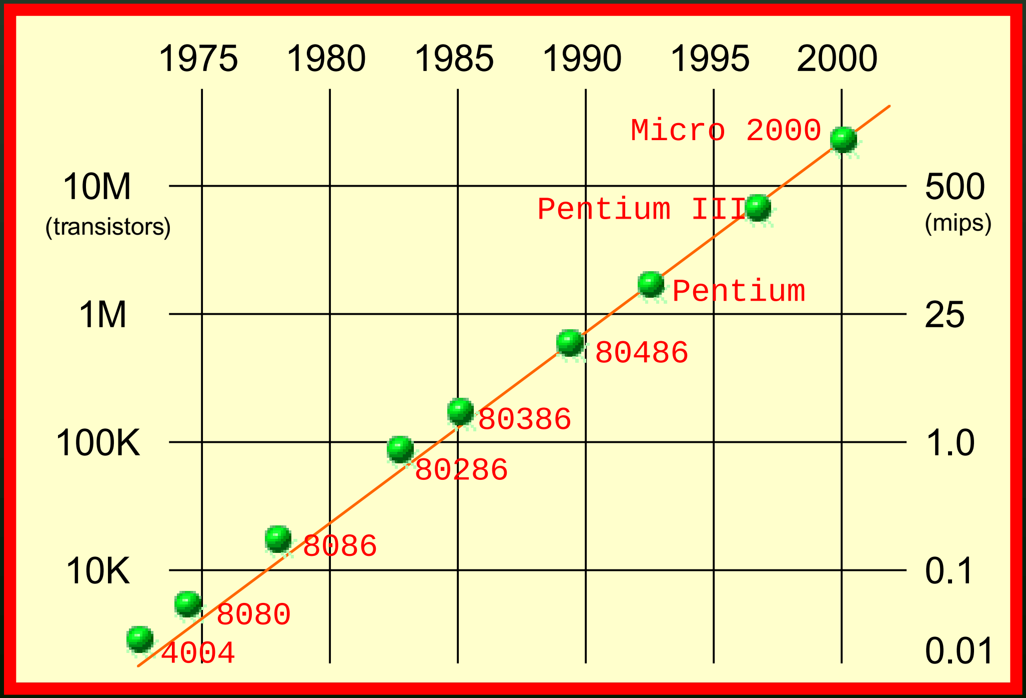
80 x

Component Failure Rate



Moore's Law (1965)

- each new chip contains roughly twice as much capacity as its predecessor
- a new generation of chips is released every 18 - 24 months



From: www.intel.com

- *in 26 years, the population of transistors per chip has increased by 3,200 times*

IC Trends: Past, Present & Future

	1980	1999	2003	2006	2012
Comp. Per Chip	0.2 M	6.2 M	18 M	39 M	100 M
Frequency (MHz)	5	1250	1500	3500	10000
Chip Area (sq. cm)	0.4	4.45	5.60	7.90	15.80
Max. Power (W)	5	90	130	160	175
Junction Temp. (C)	125	125	125	125	125

From: David L. Blackburn, NIST

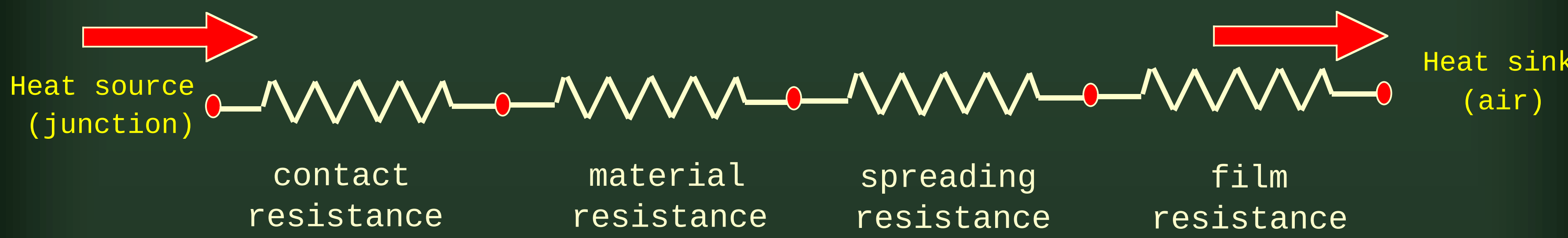
Why Use Natural Convection?

simplicity:
- low maintenance
- lower power consumption
- less space (notebook computers)

less noise

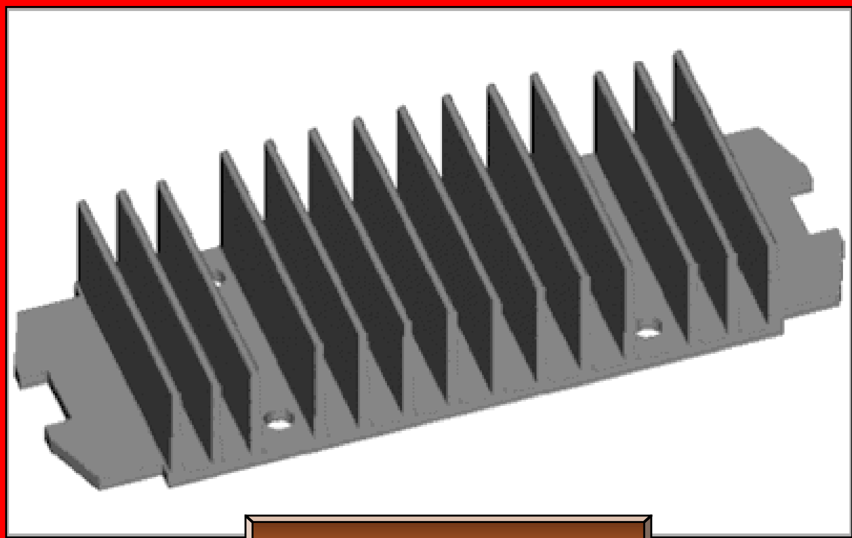
fail safe heat transfer condition

Thermal Resistance



$$\downarrow R_{film} \equiv \frac{1}{h \bullet A}\uparrow$$

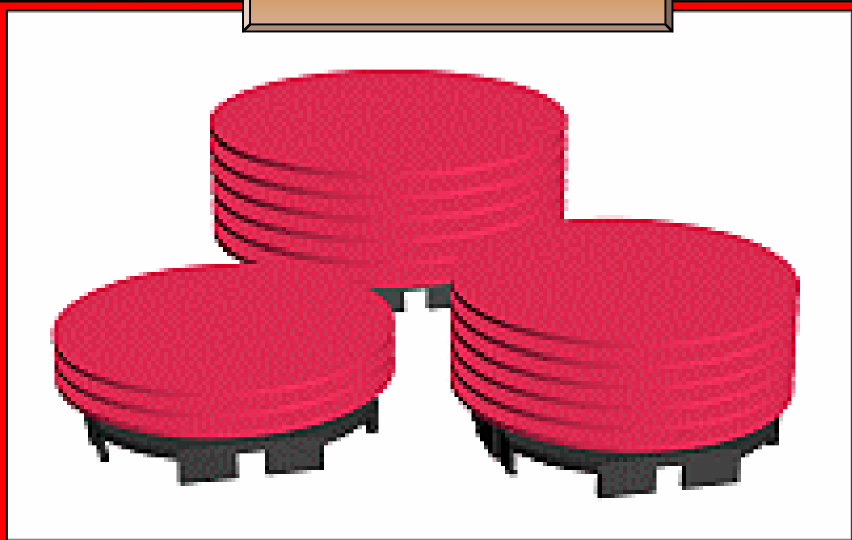
- increased heat transfer coefficient
  - immersion cooling (boiling)
  - impingement cooling
  - forced air
  - natural convection
- increased surface area
  - spreaders
  - heat sinks



PlateFin H.S.

Pin Fin H.S.

Radial Fin H.S.



Specialty H.S.

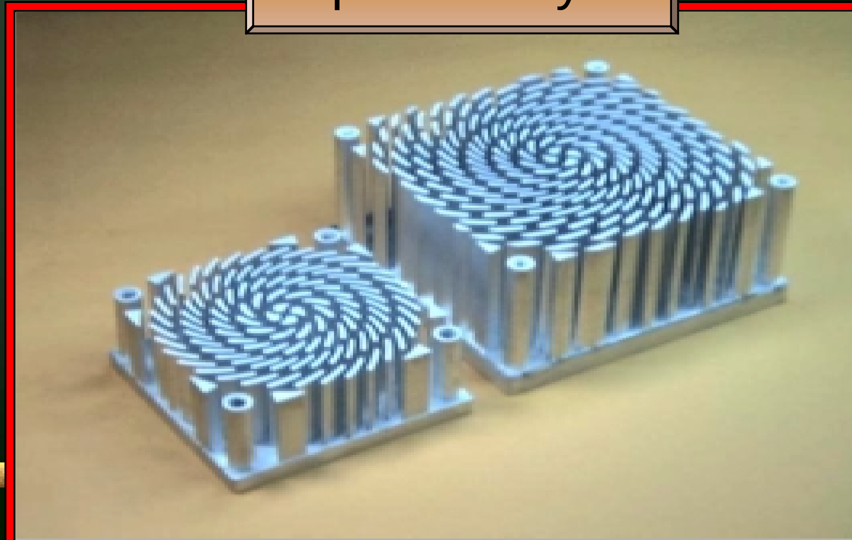
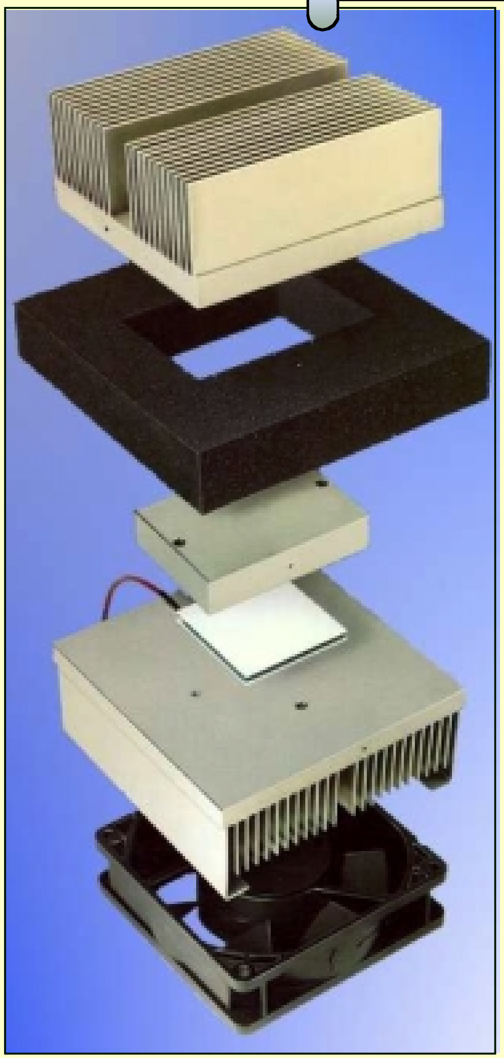
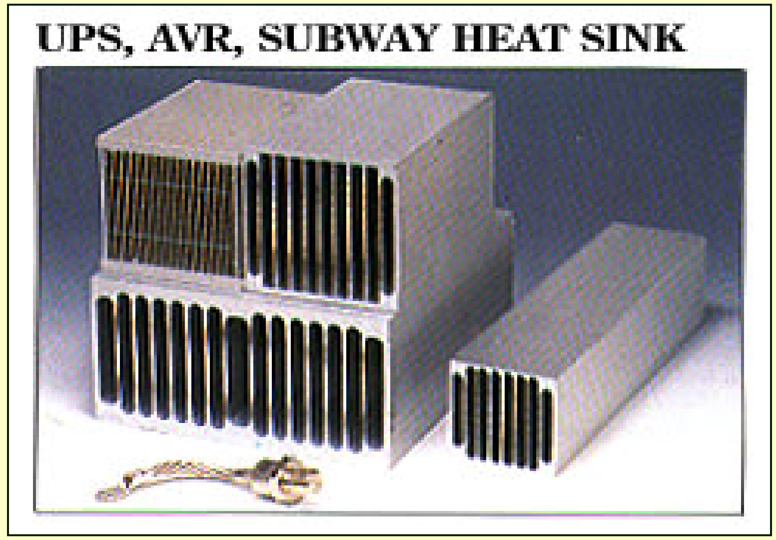
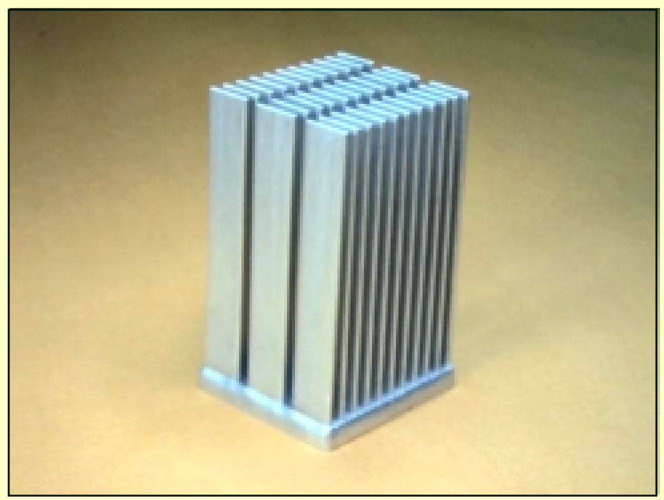


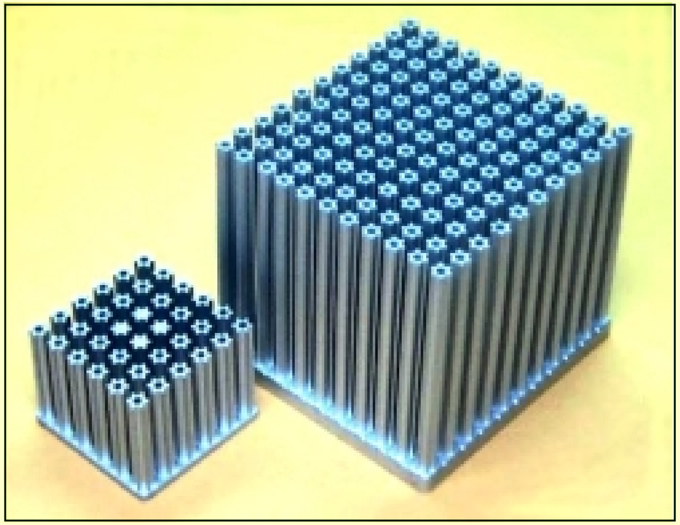
Plate Fin Heat Sinks

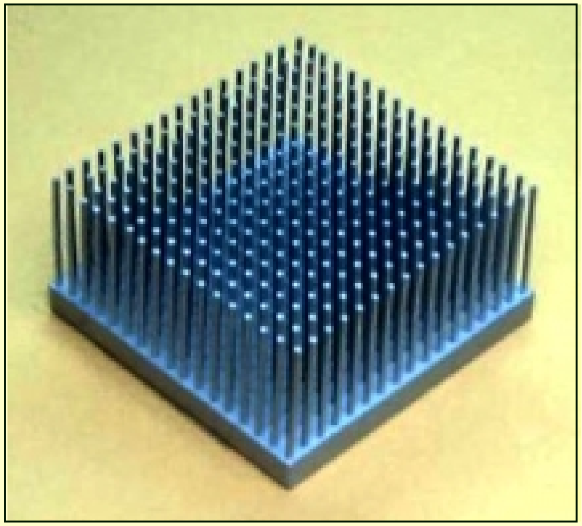


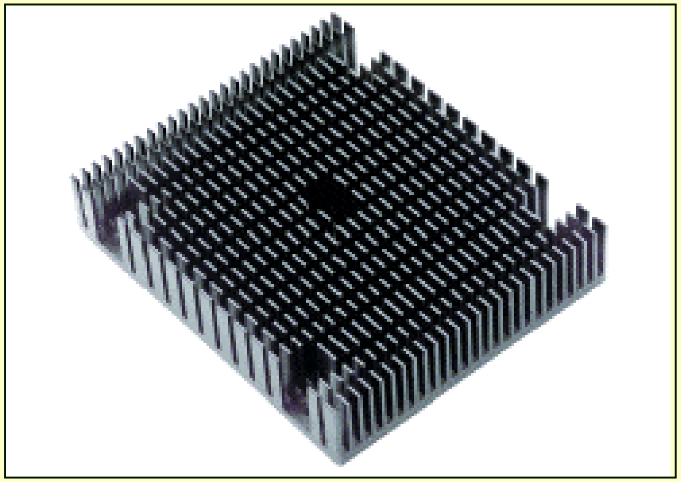




Pin Fin Heat Sinks

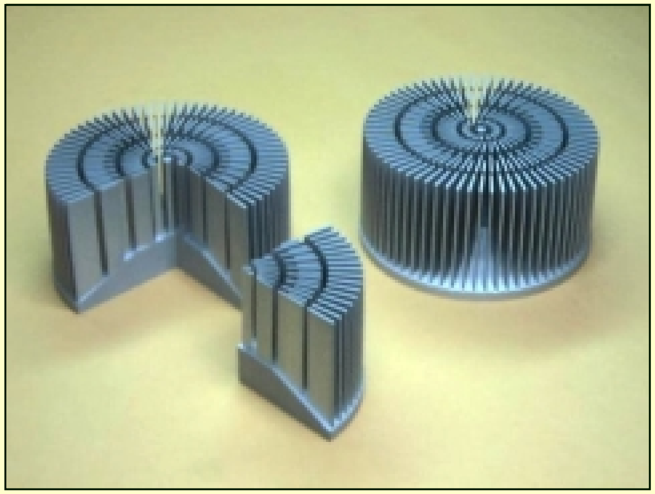




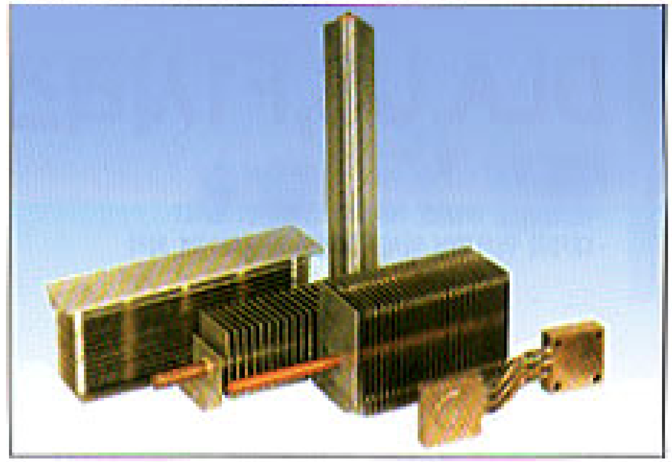


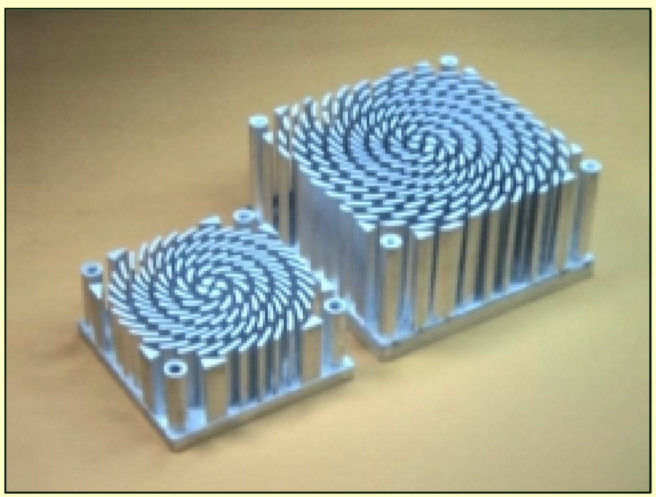
Radial Fin Heat Sinks

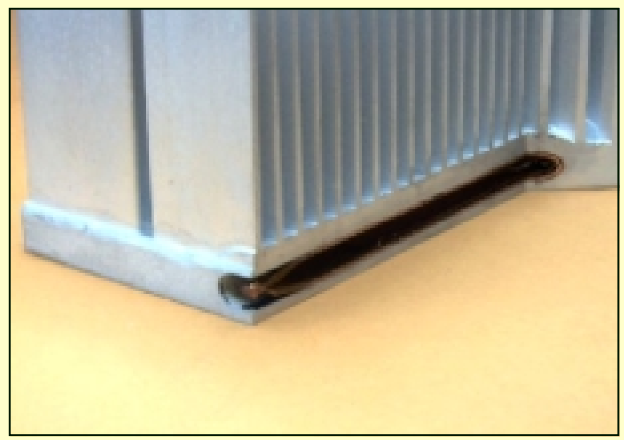
Specialty Heat Sinks



HEAT PIPE HEAT SINK







Heat Sink Model

Plate fin heat sink

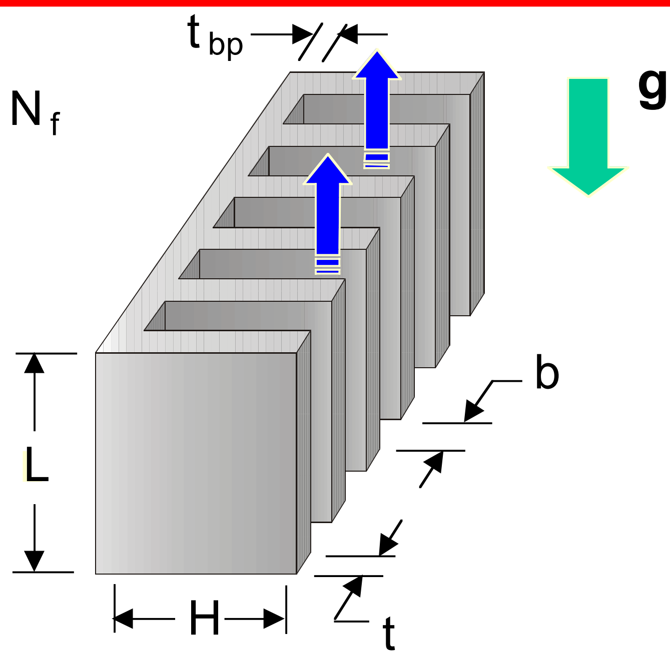
Natural convection

Isothermal

Steady state

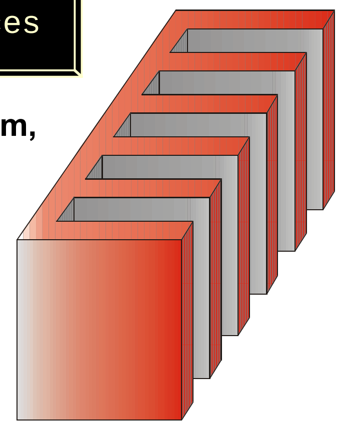
Working fluid is air i.e. $Pr = 0.71$

Modelling Procedure



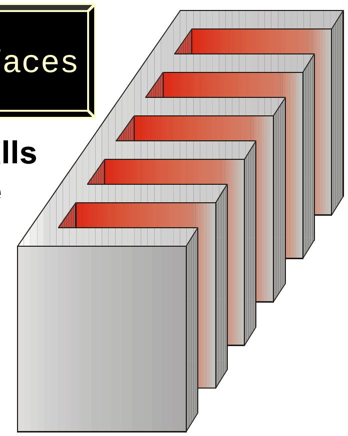
Exteriorsurfaces

- fins : top, bottom, ends & tip
- base plate: top, bottom, ends and back



Interior surfaces

- fins : side walls
- channel base



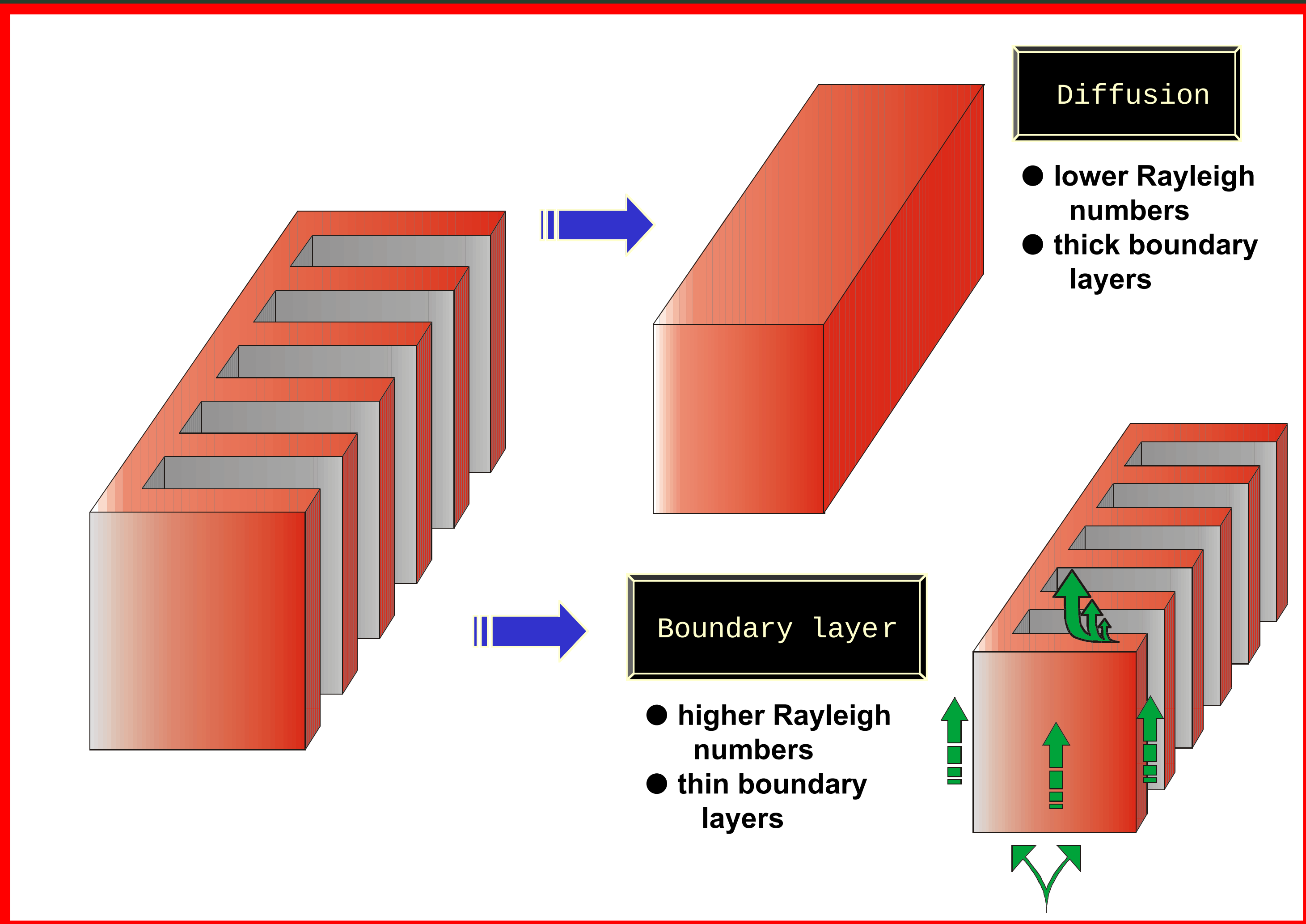
Given: **dimensions & temperature**

Find: Nu_b *vs.* Ra_b

$$Ra_b = \frac{g\,\beta\,\Delta T\,b^3}{\alpha\,\nu} \bullet \frac{b}{L}$$

$$Nu_b = \frac{h\,b}{k_f}$$

Exterior Surfaces



Diffusion Model

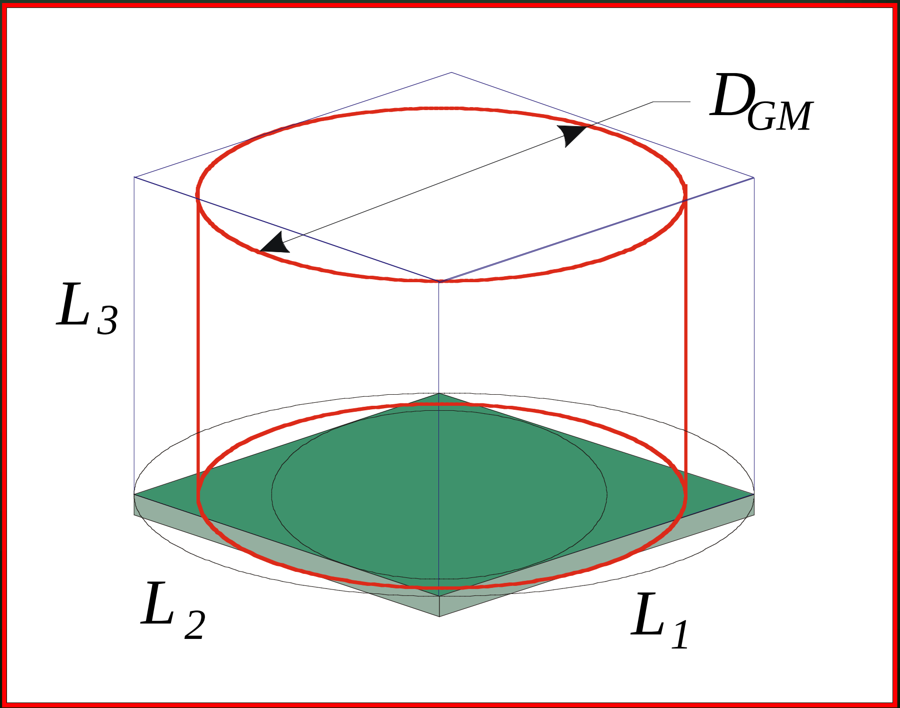
$$Nu_0 = S^*_{\sqrt{A}} = \left[S^*_{\sqrt{A}}\right]_{plate} \left(\frac{1 + 0.8688\left(L_3/D_{GM}\right)^{0.76}}{\sqrt{1 + 2\, L_3/D_{GM}}} \right)$$

$1.0 \le L_1/L_2 \le 5.0$

$$\left[S^*_{\sqrt{A}}\right]_{plate} = \frac{\sqrt{2/\pi}\left(1 + \sqrt{L_1/L_2}\right)^2}{\sqrt{L_1/L_2}}$$

$5.0 < L_1/L_2 < \infty$

$$\left[S^*_{\sqrt{A}}\right]_{plate} = \frac{2\sqrt{2\pi}\,\sqrt{L_1/L_2}}{\ln\left(4\, L_1/L_2\right)}$$



Exterior Boundary Layer Model

$$Nu_{\sqrt{A}} = G_{\sqrt{A}} \bullet F(\Pr) \bullet Ra_{\sqrt{A}}^{1/4}$$

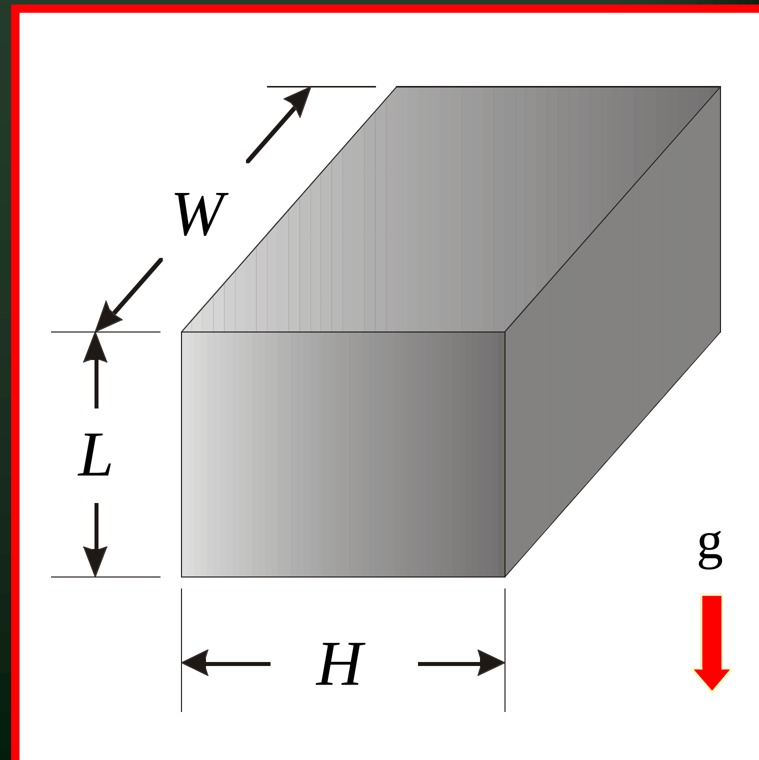
(in terms of the surface area)

Where:

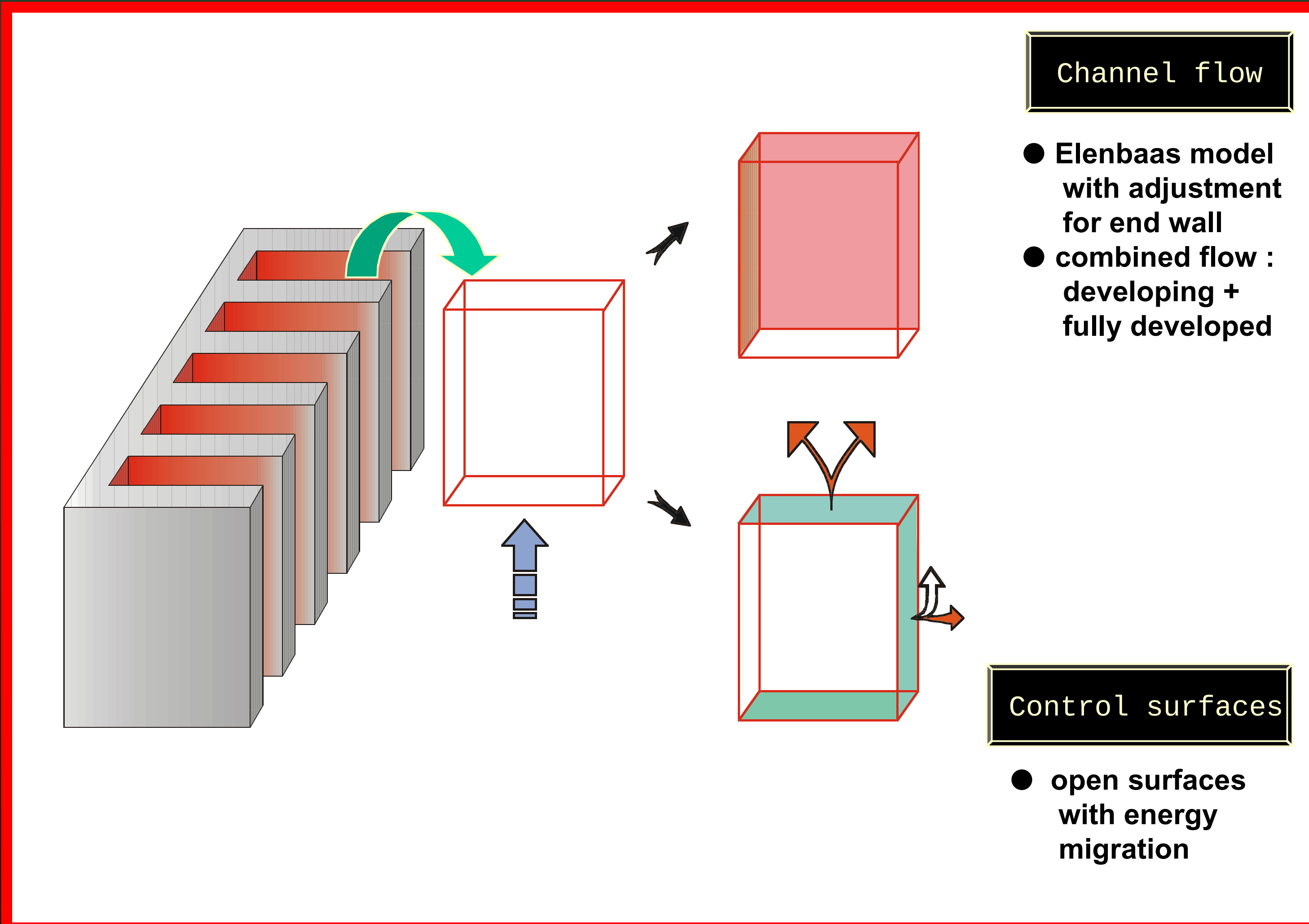
$$G_{\sqrt{A}} = 2^{1/8}\left[\frac{0.625H^{4/3}W + L(H+W)^{4/3}}{\left(L \bullet W + L \bullet H + H \bullet W\right)^{7/6}}\right]^{3/4}$$

$$F(\Pr) = \frac{0.670}{\left[1+(0.5/\Pr)^{9/16}\right]^{4/9}}$$

$$Ra_{\sqrt{A}} = \frac{g\beta\Delta T\left(\sqrt{A}\right)^3}{\alpha\nu}$$



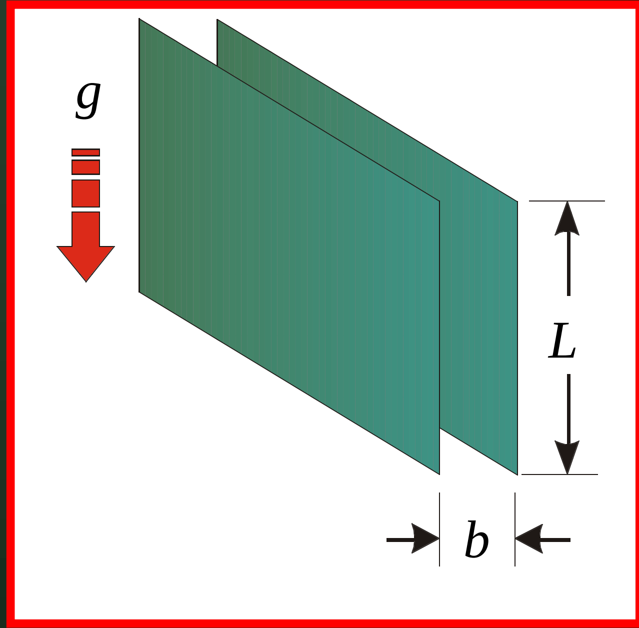
Interior Surfaces



Parallel Plates Model

Elenbaas, 1941

$$Nu_b = \frac{1}{24} Ra_b \left\{ 1 - \exp\left(-35 / Ra_b \right) \right\}^{3/4}$$



Churchill, 1977

$$Nu_b = \left\{ Nu_{fd}^{-m} + Nu_{dev}^{-m} \right\}^{-1/m}$$

$$Nu_{fd} = \frac{1}{24} Ra_b$$

$$Nu_{dev} = G_{\sqrt{A}} \bullet F(\Pr) \bullet Ra_b^{1/4}$$

fd - fully developed

dev - developing flow

$G_{\sqrt{A}}$ - body gravity function

$F(\Pr)$ - Prandtl number function

Comprehensive Model

$$Nu = \underbrace{Nu_0}_{\text{diffusion}} + \underbrace{\frac{1}{\dfrac{1}{Nu_2} + \dfrac{1}{Nu_3 + Nu_4}}}_{\text{channel flow}} + \underbrace{Nu_1}_{\text{external boundary layer flow}}$$

diffusion

channel flow

external boundary layer flow

Model Validation

Limiting Cases

cuboids

- ❶ plate - Karagiozis (1991), Saunders (1936)
- ❷ cube - Chamberlain (1983), Stretton (1988)
- ❸ rectangular prism - Clemes (1990)

parallel plates

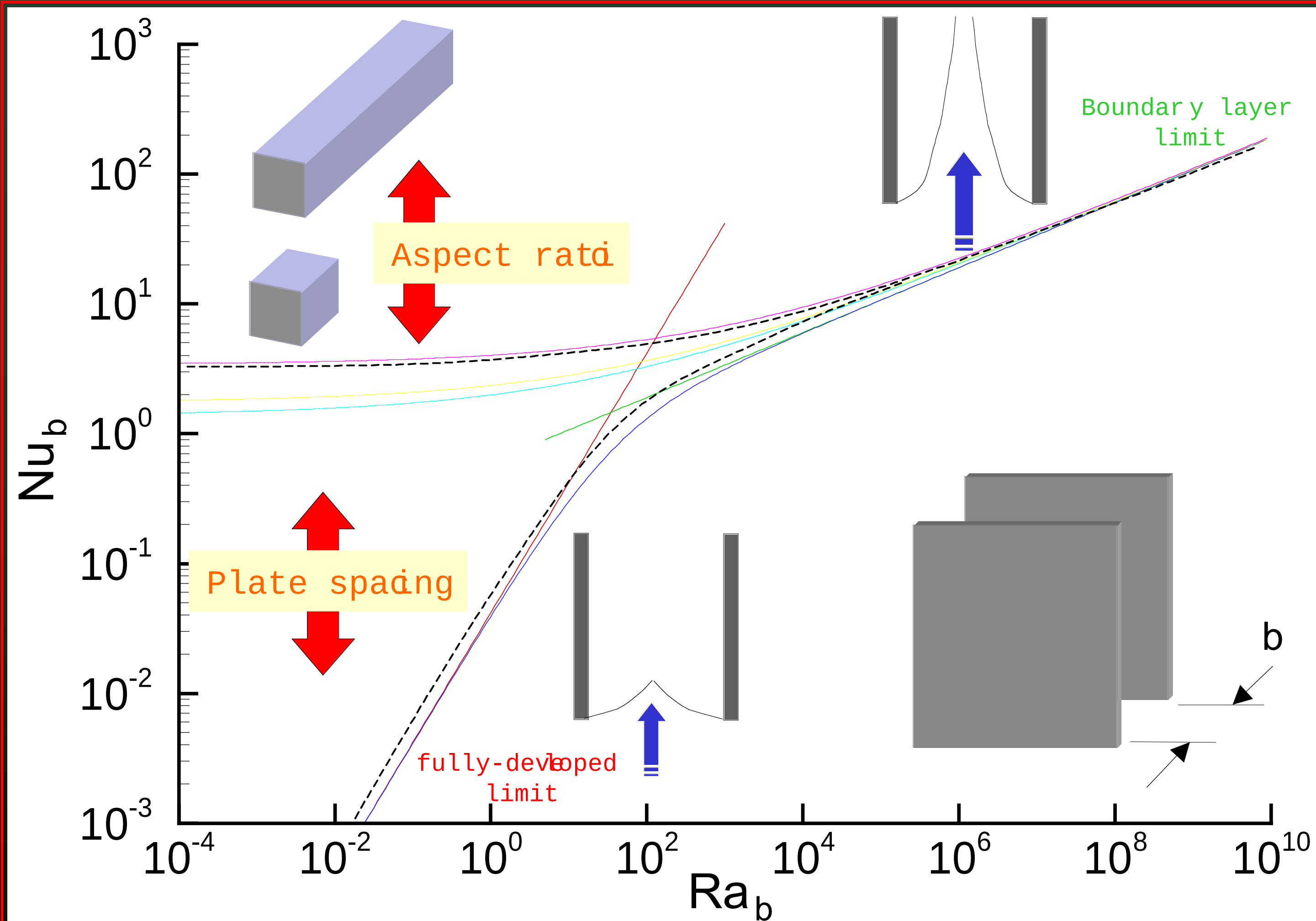
- ❶ Elenbaas (1942), Aihara (1973), Kennard (1941)

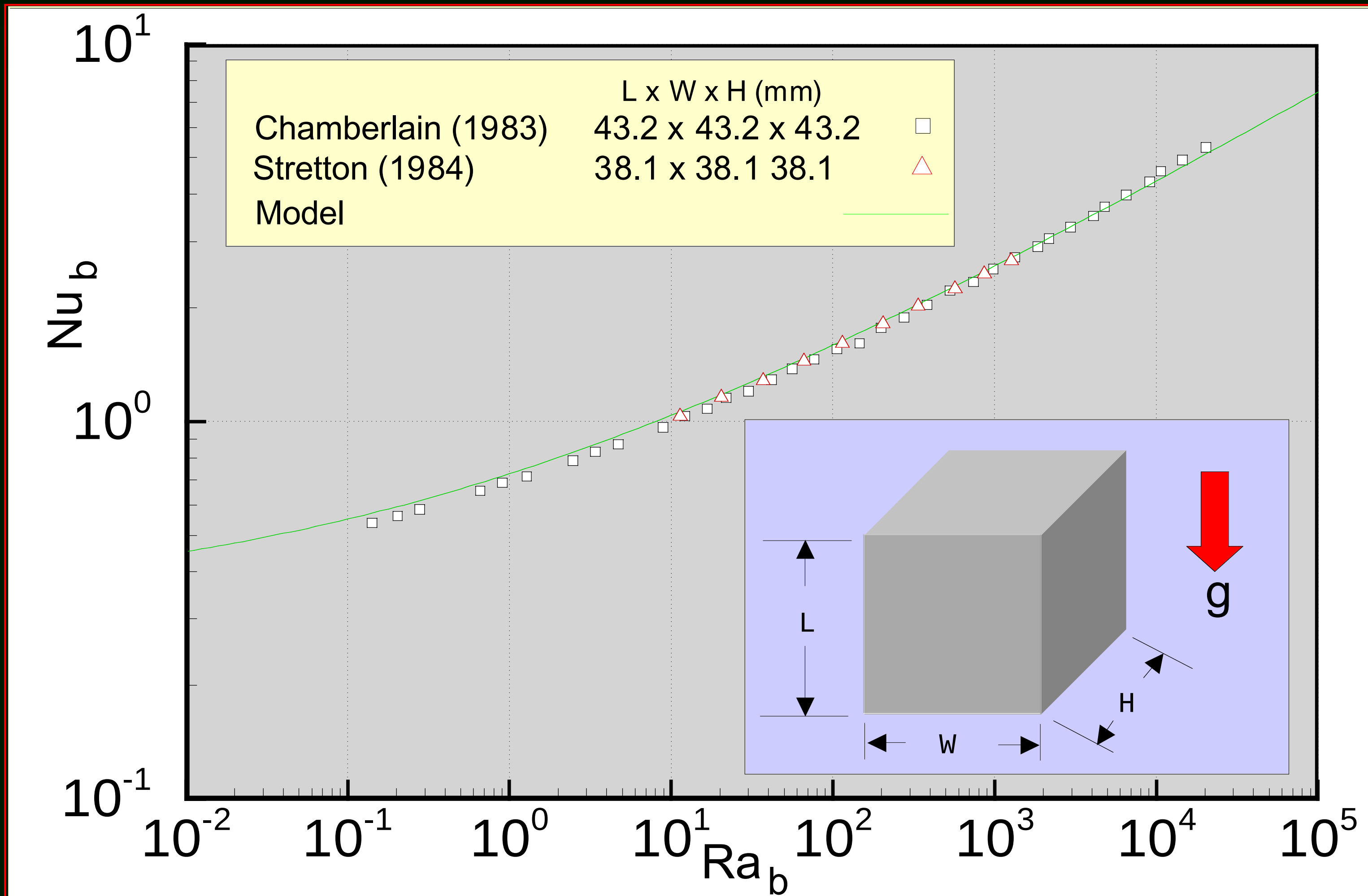
Heat Sinks

Karagiozis (1991)

Van de Pol & Tierny (1978)

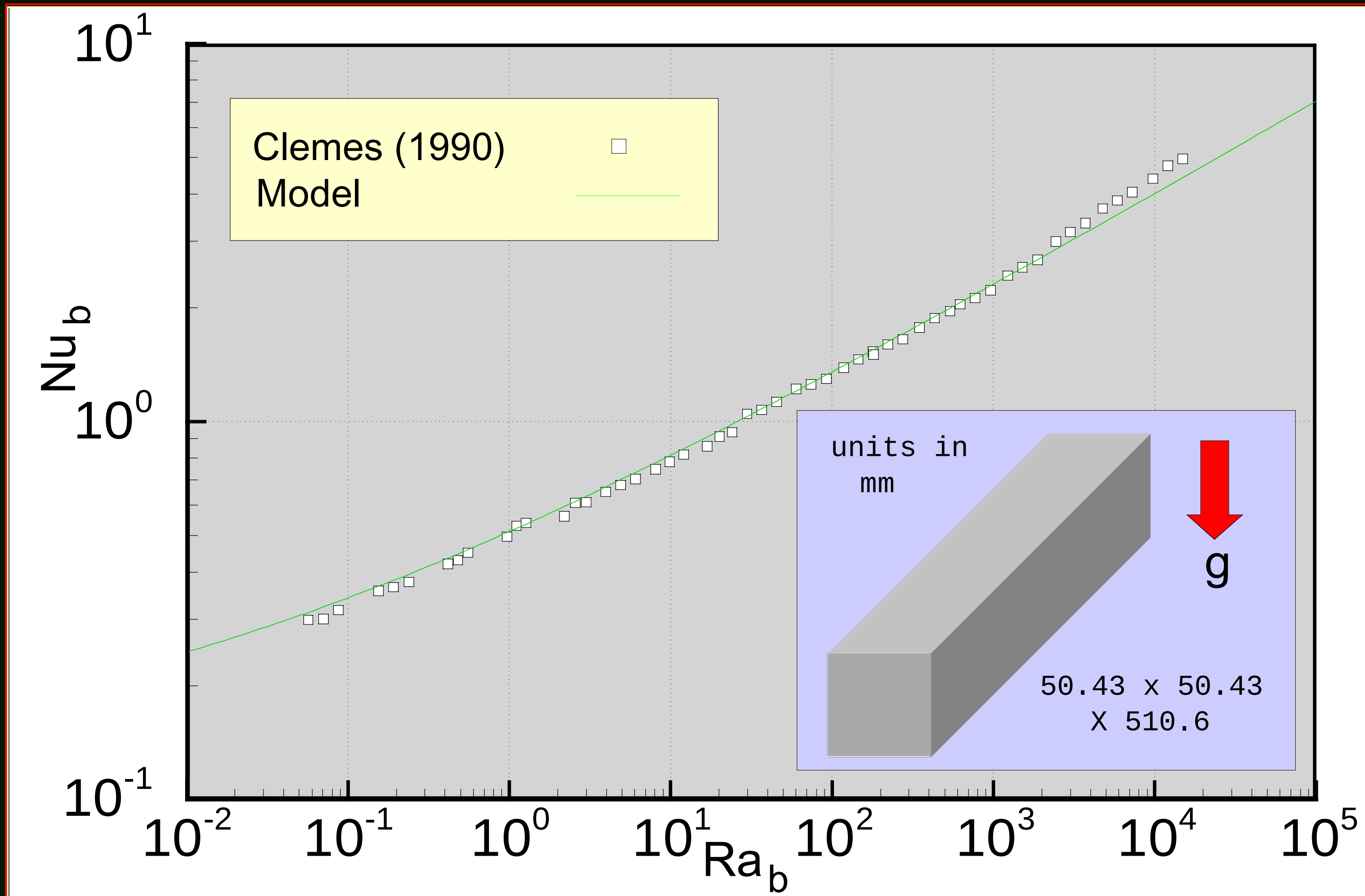
Modelling Domain





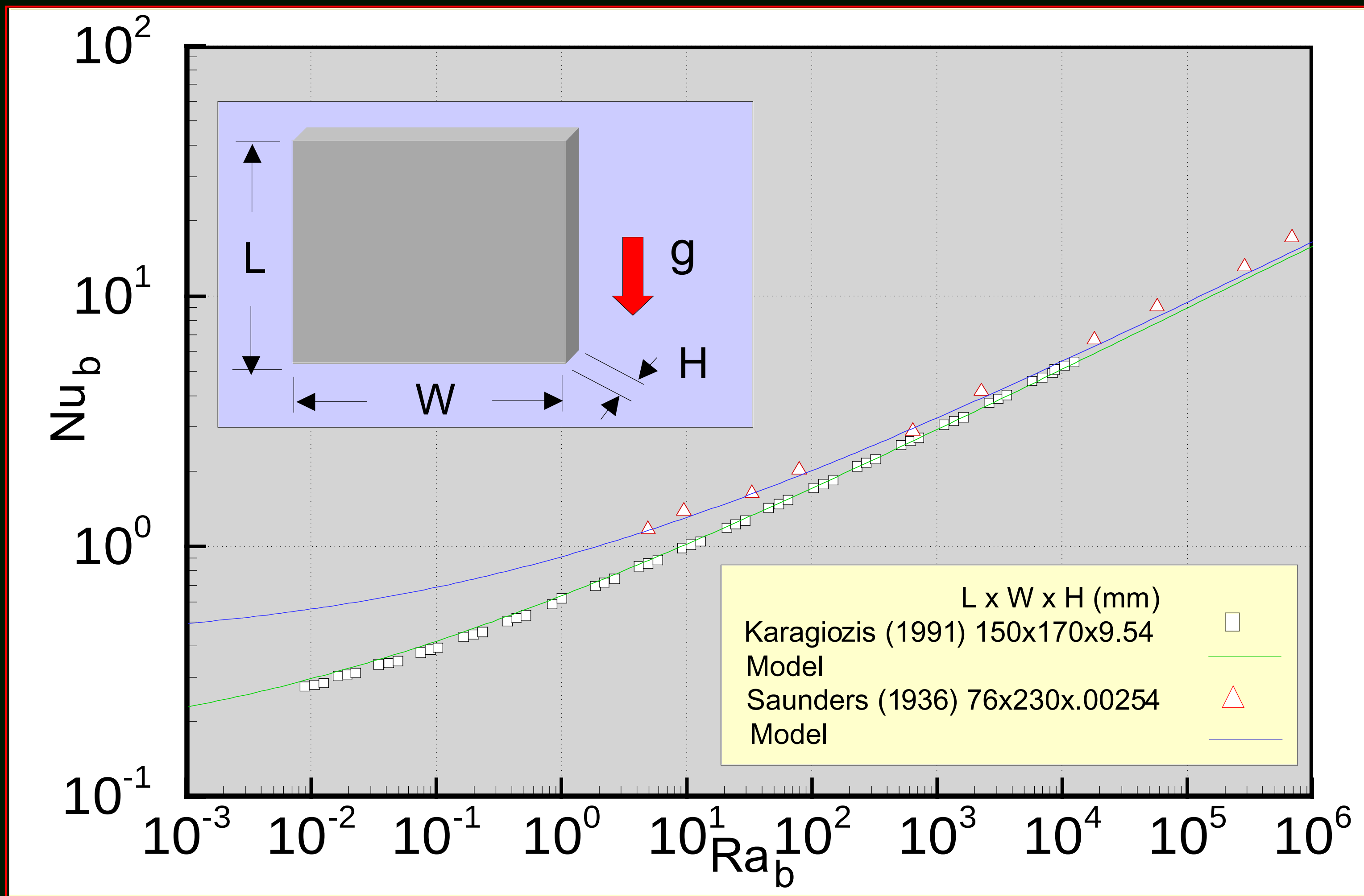
$$Nu = Nu_0 + \left\{ Nu_2^{-2} + \left(Nu_3 + Nu_4 \right)^{-2} \right\}^{-1/2} + Nu_1$$

CUBE | PRISM | FLAT PLATE | ‰ ‰ PLATES | HEAT SINK



$$Nu = Nu_0 + \left\{ Nu_2^{-2} + \left[Nu_3 + Nu_4 \right]^{-2} \right\}^{-1/2} + Nu_1$$

CUBE | PRISM | FLAT PLATE | ‰ ‰ PLATES | HEAT SINK



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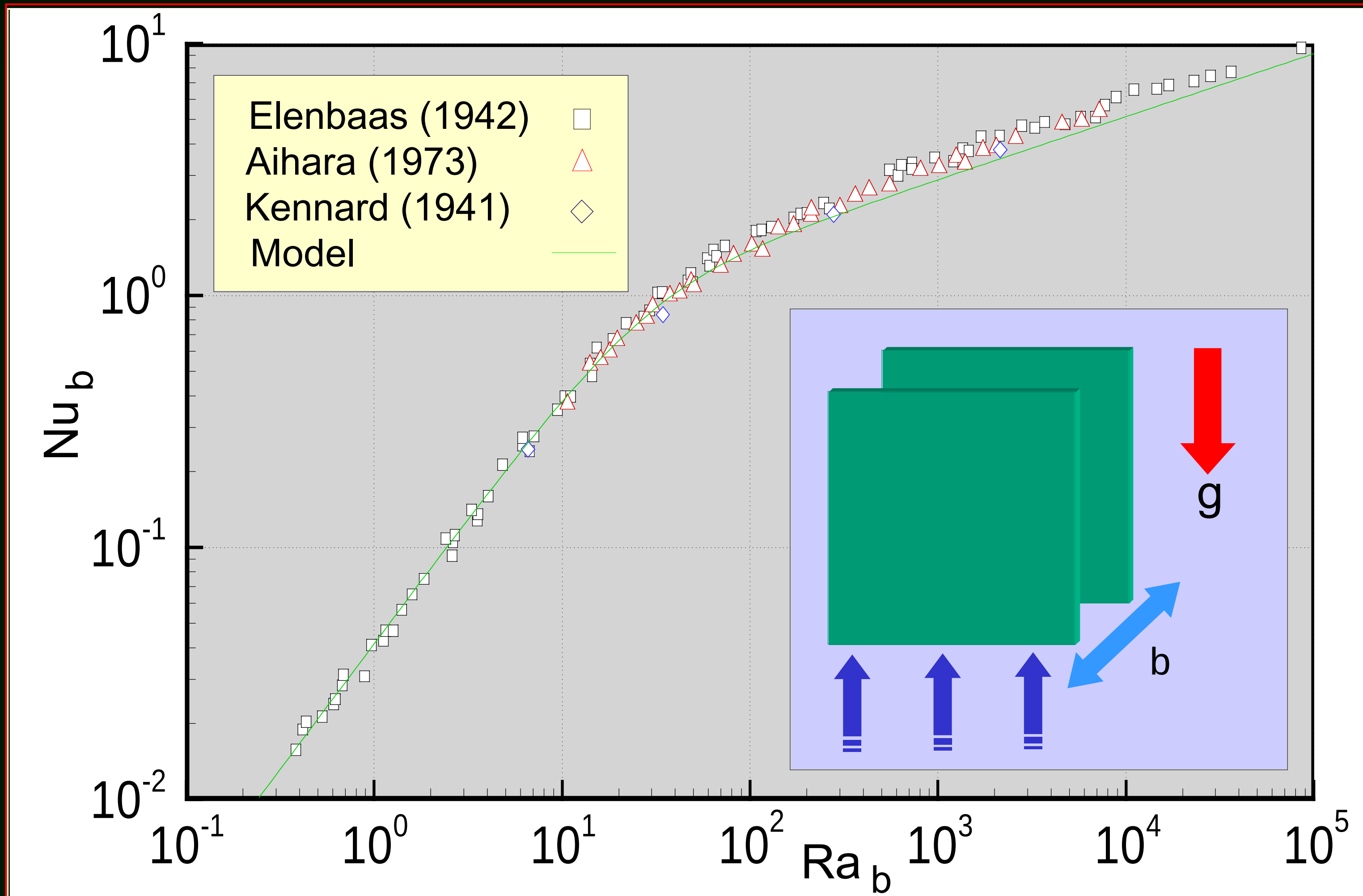
CUBE | PRISM | FLAT PLATE | ‰ ‰ PLATES | HEAT SINK



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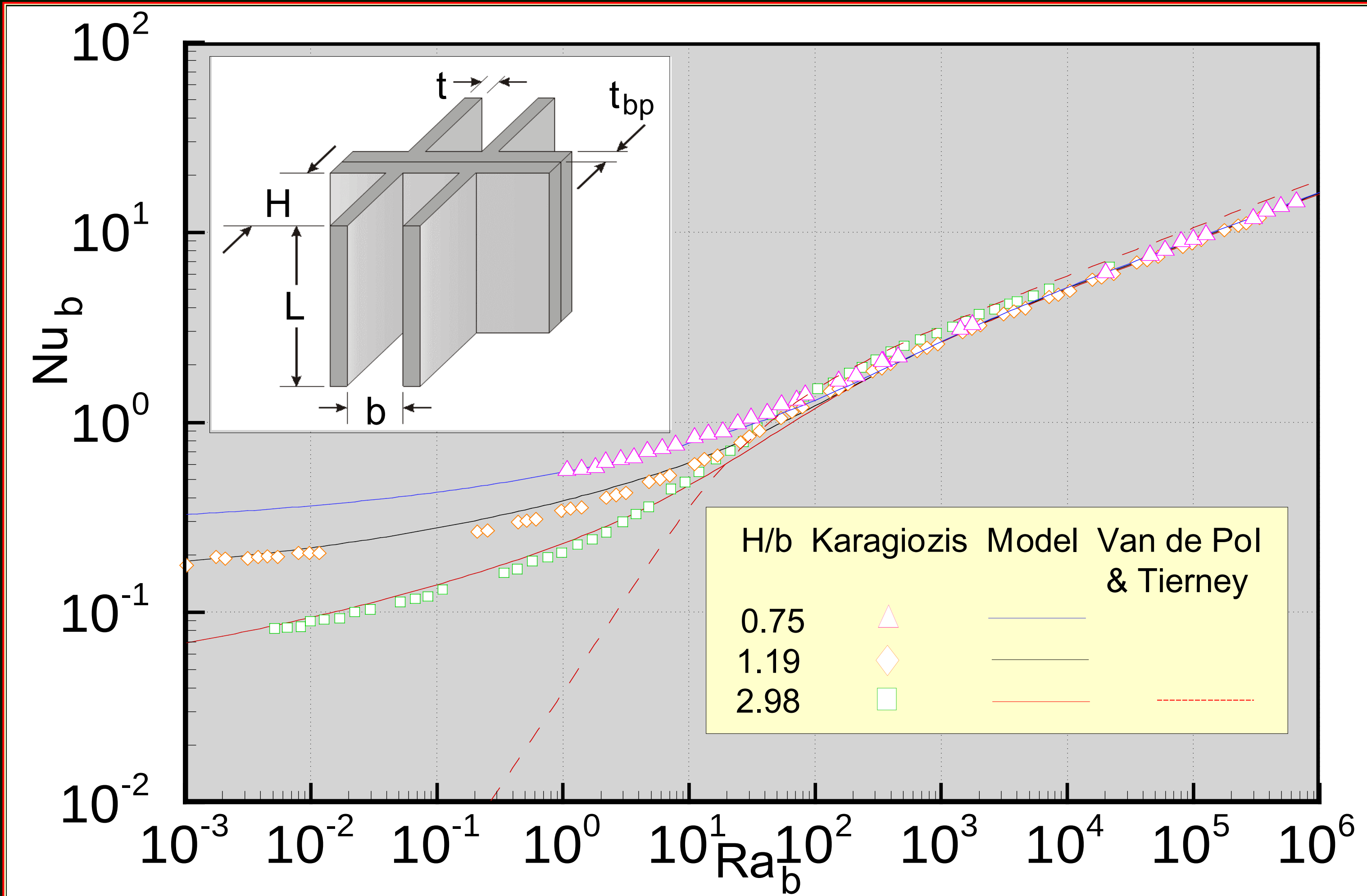
CUBE | PRISM | FLAT PLATE | ‰ ‰ PLATES | HEAT SINK



$$Nu = Nu_0 + \left\{ Nu_2^{-2} + \left[Nu_3 + Nu_4 \right]^{-2} \right\}^{-1/2} + Nu_1$$

CUBE | PRISM | FLAT PLATE | ‰ ‰ PLATES | HEAT SINK

Which is the Right Tool?

Analysis Tool vs. **Design Tool**

Analysis Tool

- design is known a priori
- used to calculate the performance of a given design, i.e. Nu vs. Ra
- cannot guarantee an optimized design

Design Tool

- used to obtain an optimized design for a set of known constraints

i.e. given:
- heat input
- max. temp.
- max. outside dimensions

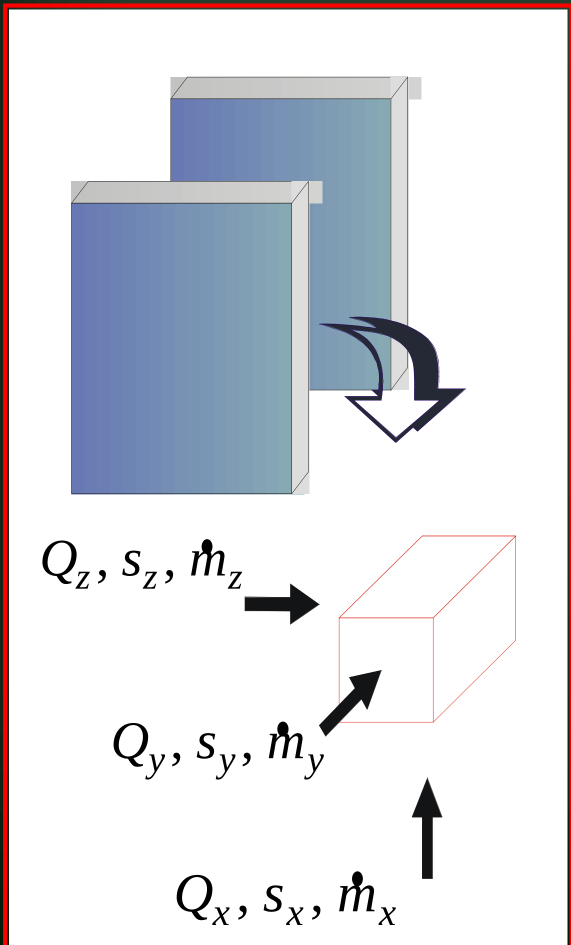
find: the most efficient design

Optimization Using EGM

Why use Entropy Generation Minimization?

- entropy production amount of energy degraded to a form unavailable for work
- lost work is an additional amount of heat that could have been extracted
- degradation process is a function of thermodynamic irreversibilities e.g. friction, heat transfer etc.
- minimizing the production of entropy, provides a concurrent optimization of all design variables

Entropy Balance (local)



$$\dot{S}'''_{gen} dV = \left(\dot{m} s + \frac{Q}{T_0}\right)_{out} - \left(\dot{m} s + \frac{Q}{T_0}\right)_{in} + \frac{dS_{cv}}{dt}$$

$$\dot{S}'''_{gen} = \frac{1}{T_0} \nabla \bullet Q'' - \frac{1}{T_0^2} Q'' \bullet \nabla T + \rho \frac{Ds}{Dt}$$

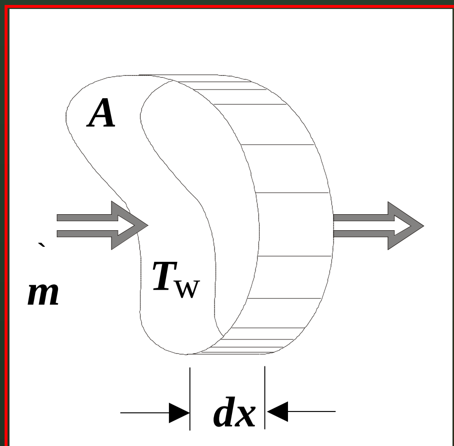
conservation of mass + 1st law of thermodynamics + Gibb's Equation

$$\dot{S}'''_{gen} = \frac{1}{T_0^2} k (\nabla T)^2 + \frac{1}{T_0} \mu \phi$$

heat transfer   viscous dissipation

Entropy Balance (external & internal)

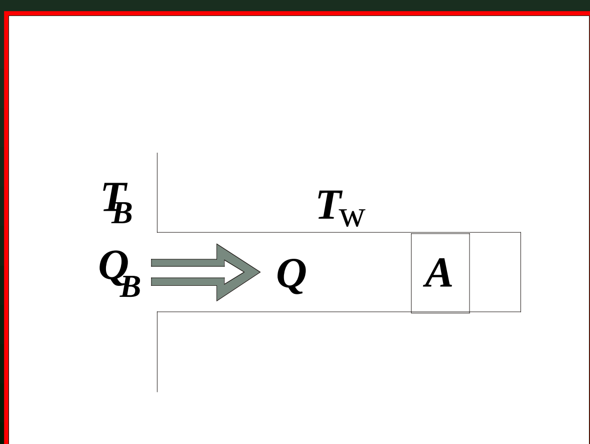
Passage geometr y



$$\dot{S}'_{gen} = \frac{Q' \Delta T_w}{T_0^2} + \frac{\dot{m}}{\rho T_0}\left(-\frac{dP}{dx}\right)$$

irreversibilities due to: wall-fluid ˘T | fluid friction

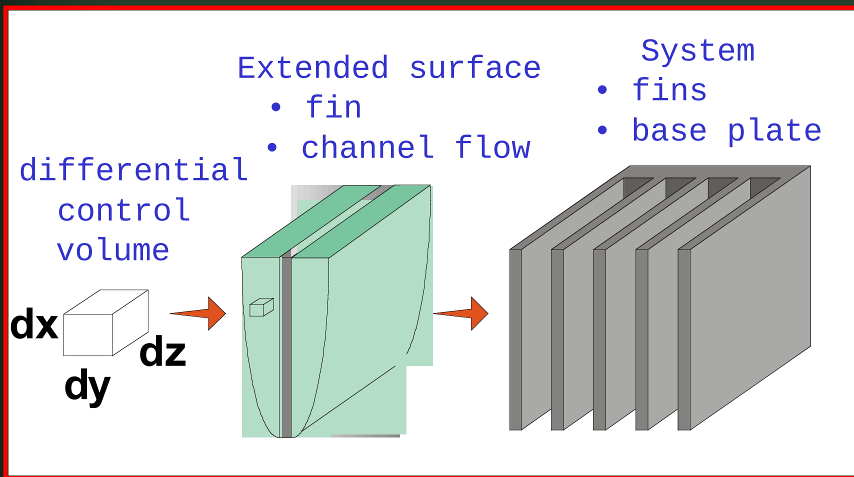
Extended surface



$$\dot{S}_{gen} = \iint_A \frac{Q''}{T_w} d\sigma - \frac{Q_B}{T_B}$$

irreversibilities due to base-wall ˘T

Total Entropy Generation



$$\dot{S}_{gen} = \underbrace{\sum}_{\text{component level}} \; \underbrace{\sum}_{\text{elemental level}} \; \underbrace{\sum \dot{S}_{gen}}_{\text{differential level}}$$

$$= \frac{Q_B \, \vartheta_B}{T_0^2} + \frac{F_d U}{T_0}$$

$$= \frac{Q_B^2 \, R_{total}}{T_0^2} + \frac{F_d U}{T_0}$$

where:

Q_B – **base heat flow rate**

ϑ_B – **base - stream temp. difference**

T_0 – **ambient temperature**

F_D – **drag force**

R_{total} – **total fin resistance**

U **- specified**

- fan curve

- buoyancy induced

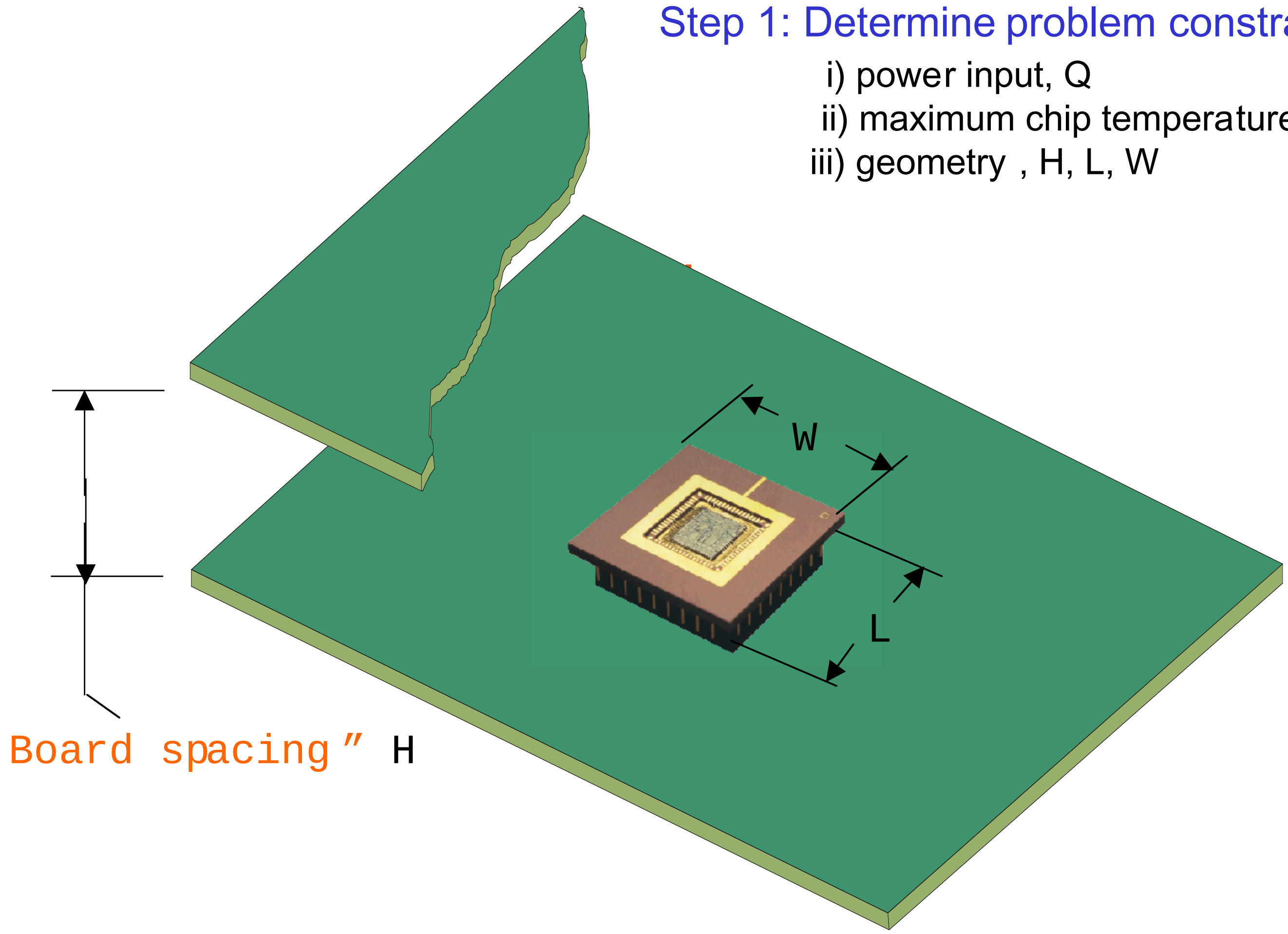
Example: Heat Sink Optimization

Step 1: Determine problem constraints

i) power input, Q

ii) maximum chip temperature, Tmax

iii) geometry , H, L, W

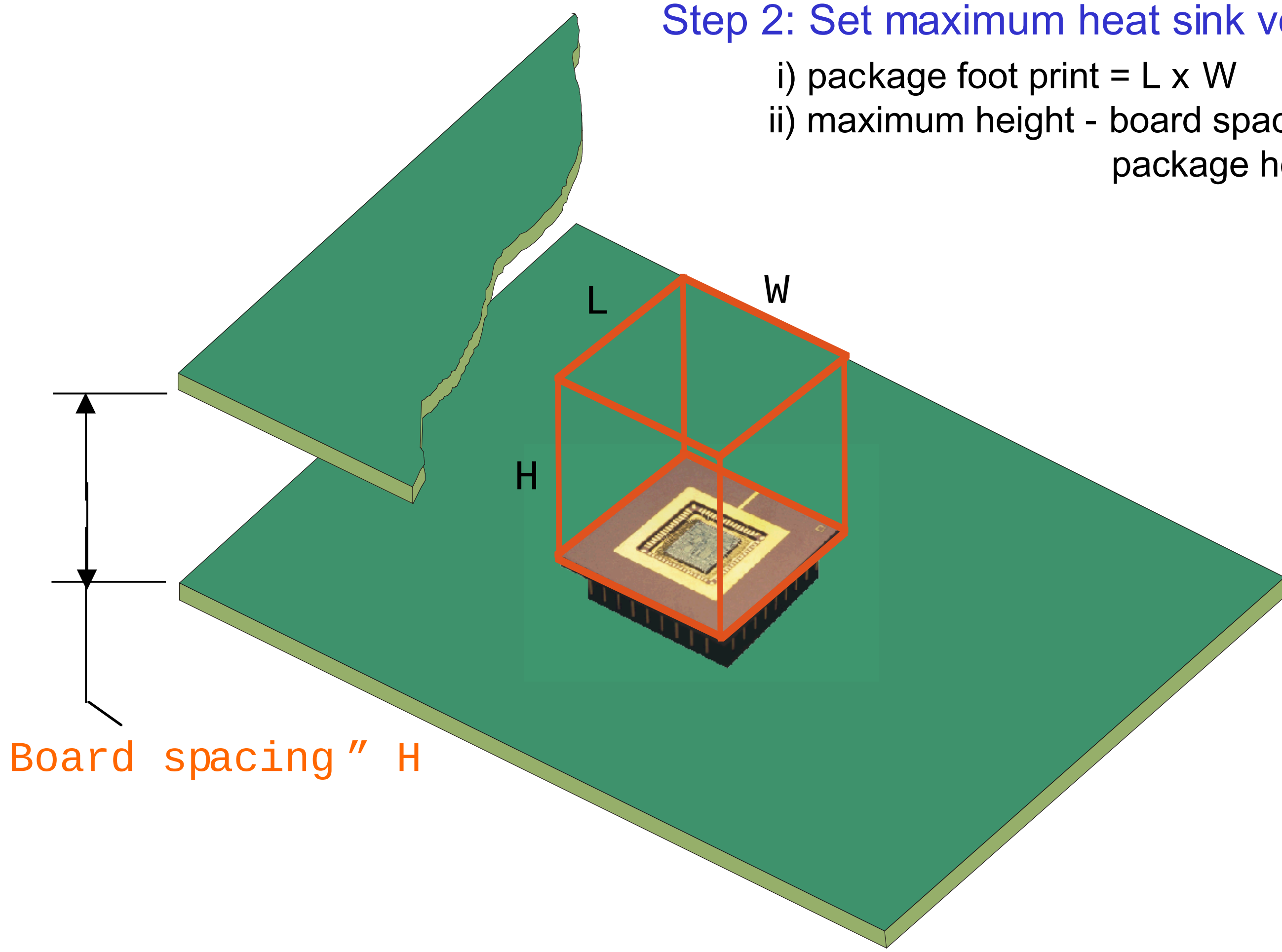


Example: Heat Sink Optimization

Step 2: Set maximum heat sink volume

i) package foot print = L x W

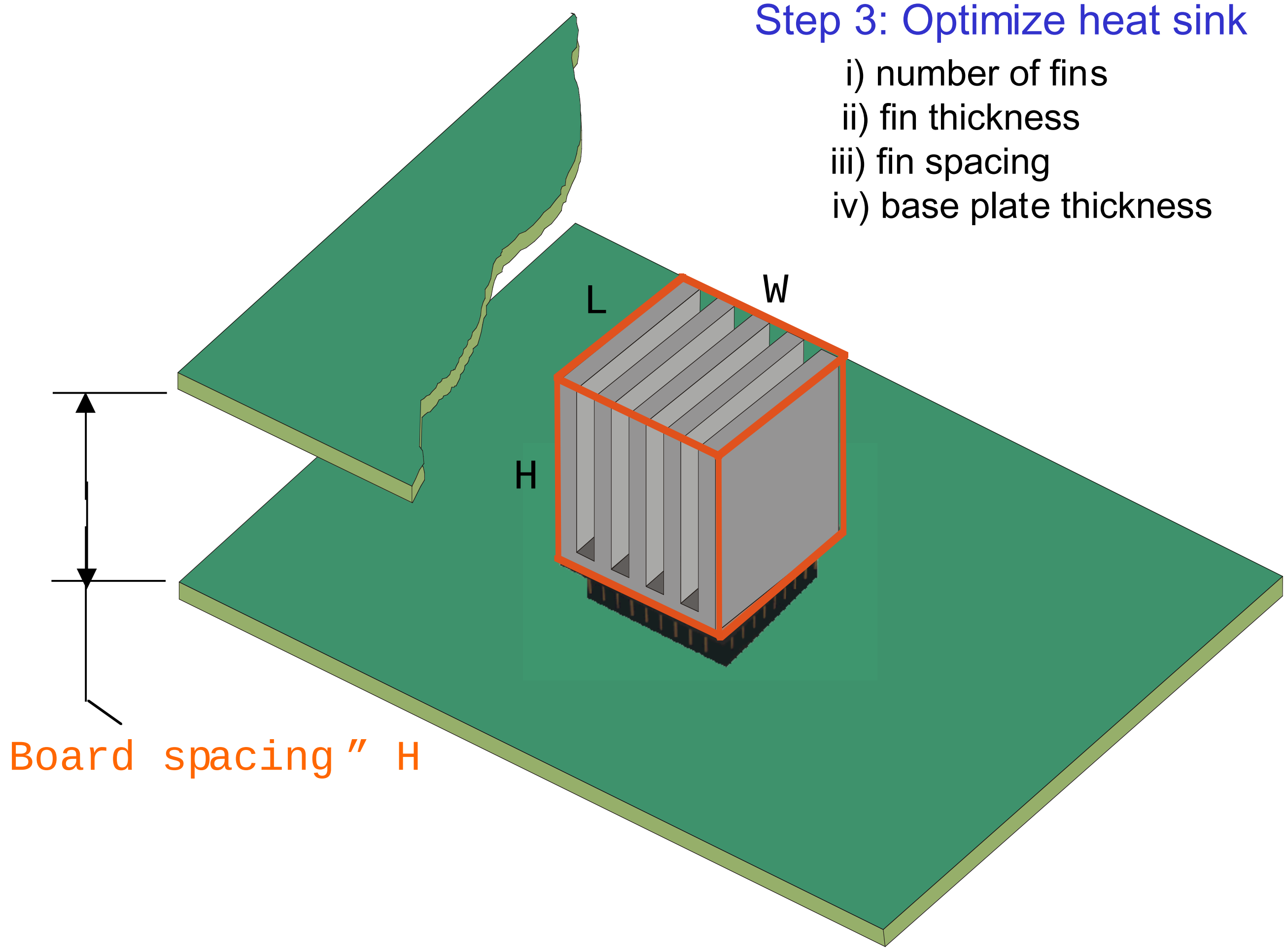
ii) maximum height - board spacing minus package height



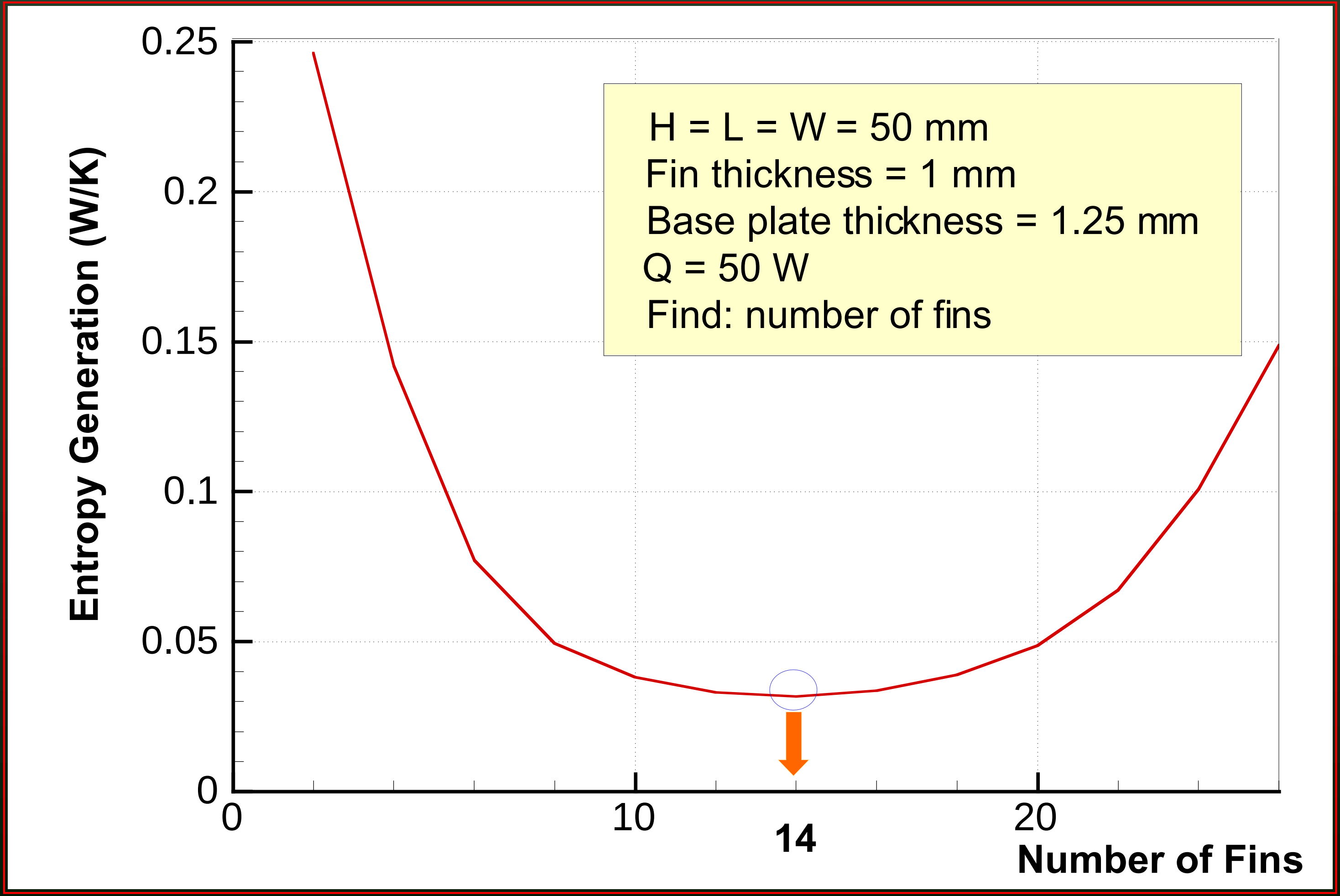
Example: Heat Sink Optimization

Step 3: Optimize heat sink

i) number of fins
ii) fin thickness
iii) fin spacing
iv) base plate thickness



Single Parameter EGM



Multi-Parameter Minimization Procedure

$$\dot{S}_{gen} = f\left(x_1, x_2, x_3, \mathrm{K}\ , x_N\right)$$

$$\frac{\partial \dot{S}_{gen}}{\partial x_i} = 0 = g_i()\,, \qquad i = 1, 2, 3, \mathrm{K}\ , N$$

Newton-Raphson Method with Multiple Equations and Unknowns

$$\begin{bmatrix} \partial g_1/\partial x_1 & \partial g_1/\partial x_2 & \partial g_1/\partial x_3 \\ \partial g_2/\partial x_1 & \partial g_2/\partial x_2 & \partial g_2/\partial x_3 \\ \partial g_3/\partial x_1 & \partial g_3/\partial x_2 & \partial g_3/\partial x_3 \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix}$$

where: $g_i(guess) \approx g_i(actual) + g_i'(guess) \bullet \delta x_i$

iterate until $\partial x_i \to 0$

Future Work

Goal: Develop a comprehensive model to find the best heat sink design given a limited set of design constraints

Physical Design

¥ heat sink type
¥ material
¥ weight
¥ dimensions
¥ surface finish

Thermal

¥ maximum volume
¥ boundary conditions
¥ max. allowable temp.
¥ orientation
¥ flow mechanism

Cost

¥ labour
¥ manufacturing
¥ material

Standards

¥ noise
¥ exposure to touch

Summary

Heat sink design requires both a selection tool & an analysis tool

Selection is based on:

- physical constraints - geometry, material, etc.
- thermal-fluid conditions - bc's, properties, etc.
- miscellaneous conditions - cost, standards etc.

Analysis is based on simulating a prescribed design

The End

Karagiozis Heat Sink Model

$$Nu = \left[\left(Nu_{cub} \bullet A_{f_{cub}} + Nu_{ch}^{fd} \bullet A_{f_{ch}} \right)^{-n_1} + \left(\overline{C}_{l_m} \left\{ Ra_b^* \left(1 + \left(\frac{C}{Ra_b^*} \right)^{m_1} \right) \right\}^{1/4} \right)^{-n_1} \right]^{-1/n_1}$$

where:

$$\overline{C}_{l_m} = \left[0.509 + (0.0135)\frac{H}{b}, \, 0.6 \right]_{\min}$$

$$C = 12.5 \frac{1}{(H/b)^{3.17}} (H/b), \; H/b \geq 1$$

$$= 141 \frac{1}{(H/b)^{3.17}} \quad , H/b < 1$$

$$m_1 = \left[1.2 \, , \, 0.64 + 0.56 \frac{H}{b} \right]_{\min}$$

$$n_1 = 1.20 \rightarrow 1.95 \quad at \quad t = 4.96 \; mm$$

$$= 1.57 \rightarrow 3.0 \quad at \quad t = 9.67 \; mm$$

$$= 1.44 \rightarrow 2.23 \quad at \quad t = 14.96 \; mm$$

Modified flat plate model → correction term at low Ra