ON THE EFFECT OF SHAPE, ASPECT RATIO AND ORIENTATION UPON NATURAL CONVECTION FROM ISOTHERMAL BODIES OF COMPLEX SHAPE

M. M. Yovanovich, Professor
Microelectronics Heat Transfer Laboratory
Department of Mechanical Engineering
University of Waterloo
Waterloo, Ontario, Canada

Abstract

A simple, semi-empirical, correlation equation for natural convection from complex isothermal three-dimensional bodies is presented. It is based on the linear superposition of the theoretical diffusive and thin boundary layer asymptotic limits. One empirical parameter only is required when the Nusselt and Rayleigh numbers are based on the characteristic length proposed by Yovanovich for the diffusive limit. The correlations are shown to be in excellent agreement over several decades of the Rayleigh number with data obtained by several authors for spheres, oblate and prolate spheroids, bisphere, cubes and short cylinders in various orientations. The accuracy of the new correlations is, on average, equal to the accuracy of the complex correlation equations of Raithby-Hollands and Hassani-Hollands. The correlation equation can be modified to account for Pr different from air. Alternate approximate correlation coefficients are also presented.

Nomenclature

A	surface area of the body
$\sqrt{A}$	characteristic length of the body
AR	aspect ratio of oblate and prolate spheroids
B	correlation coefficient or major axis of spheroid
C	correlation coefficient or minor axis of spheroid
$C_{\sqrt{A}}$	correlation coefficient for air based on $\mathcal{L} = \sqrt{A}$
$\overline{C_l}$	Raithby-Hollands parameter
$\overline{C_t}$	Hassani-Hollands parameter
$\hat{C}_t$	$\overline{C_t}\sqrt{A}/H$
e	eccentricity of the oblate and prolate spheroids $[e = \sqrt{1-u^2}]$
f_o	conversion factor for oblate spheroids
f_p	conversion factor for prolate spheroids
$F(Pr)$	function of Prandtl number $[0.670/[1+(0.492/Pr)^{9/16}]^{4/9}]$
g	scalar gravitational acceleration
$G_{\sqrt{A}}$	laminar boundary layer body-gravity function $\left[G_{\sqrt{A}} = \left[\frac{1}{A}\iint_A \left(\frac{P}{\sqrt{A}}\sin\theta\right)^{1/3} dA\right]^{3/4}\right]$
$Gr_{\mathcal{L}}$	Grashof number $[g\beta(T_0 - T_\infty)\mathcal{L}^3/\nu^2]$
h	heat transfer coefficient
H	Hassani-Hollands characteristic length in Rayleigh number only
k	thermal conductivity
$\mathcal{L}$	characteristic length of the body
m	correlation coefficient
n	Churchill-Usagi parameter in Hassani-Hollands correlation
$Nu_{\mathcal{L}}$	Nusselt number $[Nu_{\mathcal{L}} = h\mathcal{L}/k]$
Nu_l	thin laminar boundary layer Nusselt number of Raithby-Hollands
Nu_t	turbulent boundary layer Nusselt number of Raithby-Hollands
Nu^∞	diffusive limit Nusselt number as $Ra \to 0$
$Nu^\infty_{\sqrt{A}}$	diffusive limit Nusselt number based on $\sqrt{A}$
p	Churchill-Usagi parameter in Hassani-Hollands correlation
P	local perimeter of the body
Pr	Prandtl number $[\nu/\alpha]$
$Ra_{\mathcal{L}}$	Rayleigh number $[Gr_{\mathcal{L}}Pr = g\beta(T_0 - T_\infty)\mathcal{L}^3/\nu\alpha]$
S	cube side
T_0	uniform boundary temperature
T_∞	fluid temperature remote from the body
u	ratio of minor to major axes of spheroids

Greek Letters

α	thermal diffusivity
β	thermal compressibility coefficient
ν	kinematic viscosity
θ	angle between outward body normal and gravity vector
ρ	mass density

Introduction

Natural convection heat transfer from isothermal three-dimensional bodies of various shapes and aspect ratios under different orientations relative to the gravity vector has been studied experimentally by numerous researchers over several decades beginning with the important work of Langmuir [1] in 1912. Over the next thirty years Saunders [2] and Elenbass [3] extended the experimental results over a wider range of the Rayleigh number, body shapes and orientations. They derived correlation equations specific to body shapes using the conventional characteristic lengths: body diameter or body length.

Yuge [4] examined free, forced and combined convection heat transfer from from a sphere into air, while Amato and Tien [5] measured and correlated natural convection from isothermal spheres into water. They used the diameter of the sphere as the characteristic length, and employed the diffusive limit in their correlation equations. Amato and Tien also compared their results to the numerous correlation equations which had been developed for heat and mass transfer from spheres.

During the past ten years at the Universities of Minnesota and Waterloo precise data have been obtained for a variety of body shapes, aspect ratios and orientations over a wide range of Rayleigh number for air and water. Raithby, Pollard, Hollands and Yovanovich [6] obtained data for isothermal oblate and prolate spheroids; Sparrow and Ansari [7] reported some data for an isothermal short cylinder of unit aspect ratio in the vertical orientation; Chamberlain, Hollands and Raithby [8] reported data for small and large isothermal spheres, an isothermal bi-sphere and an isothermal cube in three orientations; while Sparrow and Stretton [9] obtained data for two isothermal cubes in numerous orientations in natural convection to water or air. With the exception of the Sparrow-Stretton data, all other data were obtained in air.

These data were obtained for eight different body shapes having different sizes and quite different aspect ratios; which ranged from a low value of 0.1 for one of the two oblate spheroids up to a value of 2 for the prolate spheroid and the bi-sphere. The other oblate spheroid had an aspect ratio of 0.5, and the spheres, cube and short cylinder had an aspect ratio of unity. The cube in the various orientations can be considered to be some other body having aspect ratios greater than unity.

Raithby, Pollard, Hollands and Yovanovich [6] used the major axes of the oblate and prolate spheroids to reduce their data. The Nusselt number versus Rayleigh number data for the respective bodies plotted as three separate curves requiring separate correlation equations. The conclusions drawn from these spheroidal results are that natural convection from spheroids is dependent upon shape and aspect ratio.

Chamberlain, Hollands and Raithby [8] used the diameter for the two spheres and the bi-sphere, and the side of the cube as the characteristic length for these bodies. Again, the reduced data plotted as separate curves requiring separate correlation equations, and the conclusion derived from this work is that natural convection from spheres, bi-sphere and cubes is dependent upon shape and aspect ratio. The cube data [8,9] for the three different orientations showed a nominal effect of orientation. The maximum difference was found to be approximately 5% at a Rayleigh number of the order of 10^7.

Sparrow and Ansari [7] showed that the characteristic lengths proposed by King [10] and Lienhard [11] do not yield good results when the short cylinder reduced data were compared with the sphere correlation. When they used the diameter of the short cylinder they found significant differences between the sphere and short cylinder results. Their conclusion is that natural convection from spheres and short cylinders in the vertical orientation is somewhat dependent upon shape.

Sparrow and Stretton [9] obtained data for two isothermal cubes in air and water, and examined the effect of using various characteristic lengths in the Nusselt and Rayleigh numbers. They found that the lengths which they examined gave significantly different correlation equations for their cube data. They introduced a new definition of the characteristic length which depends on the total surface area and the square root of the projected area of the body which blocks the natural convection flow. This characteristic length brought the cube, short cylinder and sphere results together and allowed them to develop a single correlation of Nu versus Ra for body shapes having an aspect ratio of unity. When they used the new characteristic length for the oblate, prolate and bi-sphere data [16], they observed that the data plotted as separate curves requiring four separate correlation equations. They found that their general correlation underpredicted the vertical bi-sphere data on average by 12% and the prolate spheroid data by 19%. The difference between their predictions and the data for the thin oblate spheroid data was found, on average, to be −32%, but the data for the oblate spheroid of aspect ratio of 0.5 was in excellent agreement with their predictions. They concluded that natural convection is quite dependent on body aspect ratio. Sparrow and Stretton [9] observed a 5% maximum difference in their air data and a 12% maximum difference in their water data for the two extreme orientations of the cube. They also observed that the effect of orientation was somewhat dependent on the magnitude of the Rayleigh number.

Raithby and Hollands [12] proposed an approximate method for predicting natural convection heat transfer from isothermal bodies immersed in an extensive, stagnant fluid. Their method which accounts for thick layer effects, reduces the natural convection problem to an equivalent simpler, conduction problem, by surrounding the isothermal body with a stationary fluid layer of variable thickness and then solving the conduction problem through this conduction layer as first suggested by Langmuir [1]. The approximate method was used to obtain solutions for numerous two-dimensional and axisymmetric bodies, including the spheroids and the bi-sphere [6,8]. The effect of variable Prandtl number and turbulent flow was also taken into account by means of the Churchill-Usagi [14] method of *blending* asymptotic solutions. They used the diameter for the spheres and bi-sphere, and the side of the cube as the characteristic length. They reported very good agreement between their predictions and data over a wide range of Rayleigh number ($100 < Ra < 10^8$); but this modified method requires another correlation parameter which also appears to depend on the body shape, orientation and aspect ratio.

Recently Hassani and Hollands [17,18] modified the Raithby-Hollands approximate method by using the diffusive limit proposed by Yovanovich [19] which is based on the square root of the total heat transfer area as the characteristic length of the body. They also introduced another characteristic length into the Rayleigh number, and they showed [18] that this length is closely related to the diffusive length of Yovanovich [19]. The difference between the Hassani-Hollands length and the Yovanovich length is less than 4% for most body shapes except for the horizontal circular and square disks and the thin oblate spheroid where the difference is approximately 40%. They also used the Churchill-Usagi blending method twice in the development of another general correlation equation which is applicable for a variety of complex shapes. The new correlation equation requires another correlation coefficient which is shown to be dependent on body shape, aspect ratio and the orientation of the body relative to the gravity vector. The Hassani-Hollands correlation equation requires the evaluation of their proposed characteristic length, the laminar and turbulent Rayleigh number coefficients, and two Churchill-Usagi coefficients. Tables are presented [18] for these parameters for the various body shapes considered to-date.

This paper presents a simple correlation equation which is based on the linear superposition of the diffusive limit ($Ra \to 0$) and the boundary layer limit ($10^4 < Ra < 10^8$). The characteristic body length used in the Nusselt and Rayleigh numbers is the square root of the total heat transfer area which has been demonstrated by Yovanovich to be the *best* characteristic body length for pure conduction from isothermal bodies. The boundary layer limit is based on the results of similarity analysis valid for $Pr \to \infty$ and the Raithby-Hollands approximate method which is valid for all Pr.

Theoretical Considerations

The earliest natural convection heat transfer correlations were developed for isothermal bodies losing heat to an extensive, stagnant fluid for Rayleigh numbers of the order of 10^5 to 10^8. For this range of Rayleigh number the correlation equation which adequately predicts the data has the simple form

$$Nu = C Ra^m \tag{1}$$

where the correlation coefficients C and m are found to be approximately 0.45 and 1/4 respectively for a sphere in air [4,6,12] when the sphere diameter is used as the characteristic length in the Nusselt and Rayleigh numbers.

The failure of the simple form to correlate data in the extended range $1 < Ra < 10^8$ has lead researchers to consider the usefulness of the following correlation equations:

$$Nu = B + C Ra^m \tag{2}$$

and

$$Nu = Nu^\infty + C Ra^m \tag{3}$$

The parameters B, C and Nu^∞ in the above equations are dependent on the Rayleigh number range, the body characteristic length and whether the parameter m has been set to the theoretical boundary layer value of 1/4. The parameter B in Eq. (2) represents the intercept value resulting from linear least-squares data fitting, whereas Nu^∞ in Eq. (3) represents the contribution of molecular diffusion into an infinite, quiescent fluid corresponding to $Ra \to 0$; it will be denoted the diffusive limit. Some researchers have reported a very wide range of values for B; including negative values which are physically unacceptable.

Amato and Tien [5] report correlation equations developed for heat and mass transfer from spheres into a variety of fluids. One set of correlations is based on $\mathcal{L} = D$, $Nu^\infty = 2$, $m = 1/4$ and the fitted coefficient C was found to lie in the range $0.399 \le C \le 0.59$. Two correlations are based upon $\mathcal{L} = D$, $m = 1/4$ and $B = 5.4$ or 2.3 with $C = 0.44$ or 0.585. The third set of correlations are based upon $\mathcal{L} = D$, $m = 1/4$, $B = 0$ and the fitted coefficient C was reported to lie in the range $0.51 \le C \le 0.56$. Amato and Tien [5] used Eq. (3) with $\mathcal{L} = D$, $Nu^\infty = 2$, $m = 1/4$ and found $C = 0.500$ for heat transfer into water. They reported a mean deviation of less than 11% provided $3 \times 10^5 \le Ra_D \le 8 \times 10^8$.

Schlichting [24] reports that Shell calculated the mean value of Nu for a single sphere in air and found that $Nu_D = 0.429\, Gr_D^{1/4}$ was confirmed by measurements in air.

Churchill and Chu [13] have used the diffusive limit and the boundary layer limit in the following *blended* form to increase the accuracy of the correlation equation:

$$Nu = [B^n + (C Ra^{1/4})^n]^{1/n} \tag{4}$$

The parameter m was set to the boundary layer value of 1/4. They found that the flat plate and horizontal circular cylinder data could be predicted accurately with $n = 1$ provided $Ra < 10^8$.

Raithby and Hollands [12] have used the Churchill-Usagi [14] method of *blending* limiting solutions to correlate free convection from bodies of complex shape. They recommend the equation

$$Nu = (Nu_l^n + Nu_t^n)^{1/n} \tag{5}$$

where Nu_l and Nu_t are the Raithby-Hollands method solutions for the thin laminar and turbulent boundary layers respectively. This method was used with success [8,15] to correlate air data for two spheres, an aligned bi-sphere and a cube in three orientations. As examples of this method, the sphere and bi-sphere correlation equations are presented here to illustrate several points discussed above. The sphere correlation equation is

$$Nu = [(2 + 0.452 Ra^{1/4})^6 + (0.099 Ra^{1/3})^6]^{1/6} \tag{6}$$

where the characteristic length is the sphere diameter, $Nu^\infty = 2$, and the Churchill-Usagi parameter $n = 6$. For the bi-sphere they obtained the correlation equation

$$Nu = [(1.38 + 0.378 Ra^{1/4})^{4.8} + (0.104 Ra^{1/3})^{4.8}]^{1/4.8} \tag{7}$$

where the diameter was selected as the characteristic length, $Nu^\infty = 1.386$, and the Churchill-Usagi parameter $n = 4.8$. They reported excellent agreement between these correlation equations and the air data of Chamberlain [8]. The cube data for the three orientations require separate correlation equations with $Nu^\infty = 1.386$ based on the cube side as the body characteristic length, and the laminar Rayleigh number coefficient having the values 0.343, 0.444 and 0.449 respectively for the three body orientations. The Churchill-Usagi parameter n ranged from 2.45 to 16 depending strongly on the cube orientation. The Churchill-Usagi parameter used by Raithby and Hollands is determined empirically to give the best fit to the data.

Comparing the Raithby-Hollands correlation equations for the spheres, the aligned bi-sphere and the cube, it can be seen that the choice of the characteristic length has a significant effect on the values of the diffusive limit, Nu^{∞}, the laminar Rayleigh number coefficient, C, and the Churchill-Usagi parameter, n. A more physically appropriate characteristic length should minimize the differences between these correlation coefficients.

In a recent paper Hassani and Hollands [18] have modified and simplified the Raithby-Hollands method by introducing the diffusive limit $Nu^{\infty}_{\sqrt{A}}$ of Yovanovich [19] and the characteristic length, $\sqrt{A}$, proposed by Yovanovich [20] in the Nusselt numbers which appear in their correlation equation. They also introduced another characteristic length, H, in the laminar and turbulent Rayleigh numbers which appear in their correlation equation. This new length is seen to be closely related to the diffusive length proposed by Yovanovich for all body shapes where $\sqrt{A}/H = \pm 5\%$, and 9% for the bi-sphere. The largest difference of 50% is observed for the horizontal, thin oblate spheroid. They propose the correlation equation:

$$Nu_{\sqrt{A}} = \left[\left[[\overline{C_l} Ra_H^{1/4}]^p + [\hat{C}_t Ra_H^{1/3}]^p\right]^{n/p} + [Nu_{\sqrt{A}}]^n\right]^{1/n} \qquad (8)$$

where n and p are a new set of Churchill-Usagi parameters. As anticipated the choice of $\sqrt{A}$ as the characteristic length has reduced the range of the parameter n from 1.01 to 1.14 for the variety of body shapes considered in their paper. The other parameter p is determined by means of another correlation equation developed by Hassani and Hollands [18]. The laminar Rayleigh number coefficient, $\overline{C_l}$, is reported to be a constant for all body shapes, and the turbulent Rayleigh number coefficient, $\hat{C}_t = \overline{C_t}\sqrt{A}/H$, has values ranging between 0.090 for the cube in orientation 1 and 0.114 for the bi-sphere.

There is at present no theoretical basis to support the use of one correlation equation over another for a range of body shapes, aspect ratio, orientation and a wide range of the Rayleigh number. It is, therefore, proposed to compare the simplest correlation equation which is based on the linear superposition of the Yovanovich diffusive limit [19] and the laminar boundary layer limit based on the Yovanovich characteristic length [19,20] with air data for a variety of body shapes, aspect ratio and orientation. The proposed, simple correlation equation for three-dimensional bodies is

$$Nu_{\sqrt{A}} = Nu^{\infty}_{\sqrt{A}} + C_{\sqrt{A}} Ra^{1/4}_{\sqrt{A}} \qquad (9)$$

where $Nu^{\infty}_{\sqrt{A}}$ is the diffusive limit [19] and $C_{\sqrt{A}}$ is an empirical correlation coefficient to be determined from the air data for $0 < Ra_{\sqrt{A}} \leq 10^8$.

The simple correlation equation can be modified to account for Prandtl numbers different from the air value by the use of the Prandtl number function $F(Pr)$ defined as [9,12,14]

$$F(Pr) = \frac{0.670}{[1 + (0.492/Pr)^{9/16}]^{4/9}} \qquad (10)$$

which was developed by Churchill and Chu [13] and used effectively by Sparrow and Stretton [9] to correlate their air and water data. This function has the value of 0.514 for $Pr = 0.71$. The laminar Rayleigh number coefficient, $C_{\sqrt{A}}$, [20] is related to

Table 1: Conversion Factors for Nusselt and Rayleigh Numbers for Various Body Shapes

Body Shape	Characteristic Length	Conversion Factors: Nu	Conversion Factors: Ra
Sphere	Diameter, D	$\sqrt{\pi}$	$\pi^{3/2}$
Bi-sphere	Diameter, D	$\sqrt{2\pi}$	$(2\pi)^{3/2}$
Cube	Cube Side, S	$\sqrt{6}$	$6^{3/2}$
Cylinder	Diameter, D	$\sqrt{3\pi/2}$	$(3\pi/2)^{3/2}$

the laminar boundary layer body-gravity function $G_{\sqrt{A}}$ which is dimensionless and the Prandtl number function, Eq. (10), [20]:

$$C_{\sqrt{A}} = F(Pr) G_{\sqrt{A}} \qquad (11)$$

where

$$G_{\sqrt{A}} = \left[\frac{1}{A}\int\int_A \left(\frac{P}{\sqrt{A}}\sin\theta\right)^{1/3} dA\right]^{3/4} \qquad (12)$$

which can be derived from the boundary layer equations by similarity methods for $Pr \to \infty$ for any arbitrary body shape which does not possess horizontal planes, corners, or surface depressions. This new function can also be derived from the relationship developed by the Raithby-Hollands approximate method [12] by assuming the diffusive characteristic length and performing the integration over the total body surface. The geometric parameter P which appears in the integrand is the local perimeter of the body, and θ is the local angle between the outward normal to the body surface and the direction of the gravity vector. This function can be evaluated analytically for the oblate and prolate spheroids, and some other simple body shapes. It is noted [20] that it is a relatively weak function of the body shape, aspect ratio and orientation for a wide range of these geometric parameters [21].

Results and Discussion

The air data of Nu and Ra for isothermal spheres [8,15], bi-sphere [8,15], cube [8,9,15], oblate and prolate spheroids [6,22], and short cylinders [7,18] are converted to $Nu_{\sqrt{A}}$ and $Ra_{\sqrt{A}}$ data by means of the conversion factors given in Table 1.

The relationships for conversion of the oblate spheroidal data are [19,20]:

$$Nu_{\sqrt{A}} = f_o(u) Nu_B \qquad (13)$$

and

$$Ra_{\sqrt{A}} = f_o^3(u) Ra_B \qquad (14)$$

where

$$f_o(u) = \sqrt{\frac{\pi}{2}}\left[1 + \frac{(1-e^2)}{2e}\ln\left(\frac{1+e}{1-e}\right)\right]^{1/2} \qquad (15)$$

and

$$e = \sqrt{1-u^2} \qquad (16)$$

$$u = \frac{C}{B} = AR \qquad (17)$$

The relationships for the conversion of the prolate spheroidal data are [19,20]:

$$Nu_{\sqrt{A}} = u f_p(u) Nu_B \tag{18}$$

and

$$Ra_{\sqrt{A}} = u^3 f_p^3(u) Ra_B \tag{19}$$

where

$$f_p = \sqrt{\frac{\pi}{2}} \left[1 + \frac{\sin^{-1} e}{e\sqrt{1-e^2}}\right]^{1/2} \tag{20}$$

and e and u are defined in Eqs. (16) and (17), but for prolate spheroids the aspect ratio, $AR = 1/u$.

Log-log plots of $Nu_{\sqrt{A}}$ versus $Ra_{\sqrt{A}}$ for the various body shapes (see Figures 1 and 2) are presented in Figures 3-10. The data points for $10^4 \le Ra_{\sqrt{A}} \le 10^8$ were used with the following equation to obtain a set of values of the laminar boundary layer coefficients:

$$\left(C_{\sqrt{A}}\right)_i = \frac{(Nu_{\sqrt{A}})_i - Nu^{\infty}_{\sqrt{A}}}{\left(Ra^{1/4}_{\sqrt{A}}\right)_i} \tag{21}$$

where i denotes the ith data point and $Nu^{\infty}_{\sqrt{A}}$ is the diffusive limit [19] reported in Table 2.

It can be seen that the largest difference in the values of $Nu^{\infty}_{\sqrt{A}}$ is between the thin oblate spheroid of aspect ratio $AR = 0.1$ and the prolate spheroid of aspect ratio $AR = 1.93$ and this relative difference is approximately 6.7%. The arithmetic average of the $(C_{\sqrt{A}})_i$ for each body shape is given in Table 3.

A least-squares fit to all data for $0 < Ra_{\sqrt{A}} < 10^8$ for all body shapes was conducted, and the correlation coefficients $C_{\sqrt{A}}$ were found by means of the following matrix [23]:

$$\begin{bmatrix} N & \sum_1^N (Ra^{1/4}_{\sqrt{A}})_i \\ \sum_1^N (Ra^{1/4}_{\sqrt{A}})_i & \sum_1^N (Ra^{1/4}_{\sqrt{A}})_i^2 \end{bmatrix} \begin{Bmatrix} Nu^{\infty}_{\sqrt{A}} \\ C_{\sqrt{A}} \end{Bmatrix} = \begin{Bmatrix} \sum_1^N (Nu_{\sqrt{A}})_i \\ \sum_1^N (Nu_{\sqrt{A}} Ra^{1/4}_{\sqrt{A}})_i \end{Bmatrix}$$

The coefficients were found to lie within ±1% of those based upon the averaged values of Eq. (21). Since the averaged values produce slightly smaller RMS percent differences, they are given in Table 3.

Table 2: Diffusive Nusselt Numbers for Various Three-Dimensional Body Shapes [19]

Body Shape	$Nu^{\infty}_{\sqrt{A}}$
Sphere	3.545
Bi-sphere	3.475
Cube 1	3.388
Cube 2	3.388
Cube 3	3.388
Vertical Cylinder	3.444
Horizontal Cylinder	3.444
Cylinder at 45°	3.444
Prolate Spheroid (AR = 1.93)	3.566
Oblate Spheroid (AR = 0.5)	3.529
Oblate Spheroid (AR = 0.1)	3.342

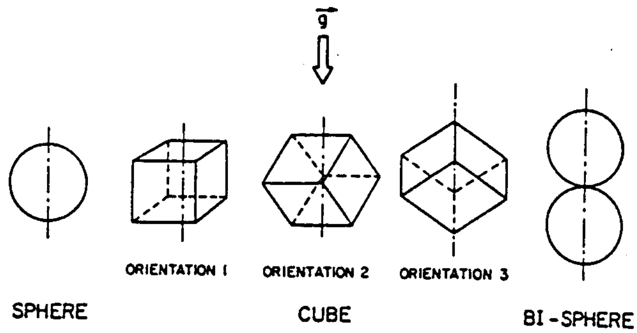

Figure 1. Body Shapes and Orientation

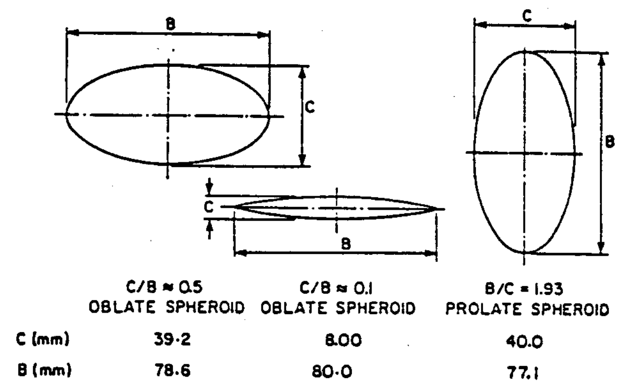

Figure 2. Spheroids and Orientation

Table 3: Average Correlation Coefficients for Various Three-Dimensional Body Shapes

Body Shape	$C_{\sqrt{A}}$	$G_{\sqrt{A}}$
Spheres	0.526	1.023
Bi-sphere	0.477	0.928
Cube 1	0.489	0.951
Cube 2	0.509	0.990
Cube 3	0.521	1.014
Vertical Cylinder	0.497	0.967
Cylinder at 45°	0.516	1.004
Horizontal Cylinder	0.524	1.019
Old Prolate Spheroid (AR = 1.93)	0.560	1.089
Old Oblate Spheroid (AR = 0.5)	0.522	1.016
Old Oblate Spheroid (AR = 0.1)	0.419	0.815
New Prolate Spheroid (AR = 1.93)	0.520	1.012
New Oblate Spheroid (AR = 0.5)	0.500	0.973
New Oblate Spheroid (AR = 0.1)	0.395	0.768

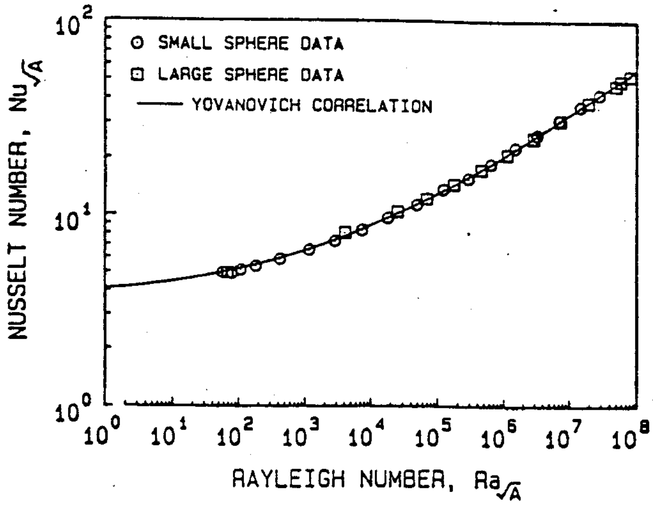

Figure 3. Comparison Between Correlation and Data for Spheres [15]

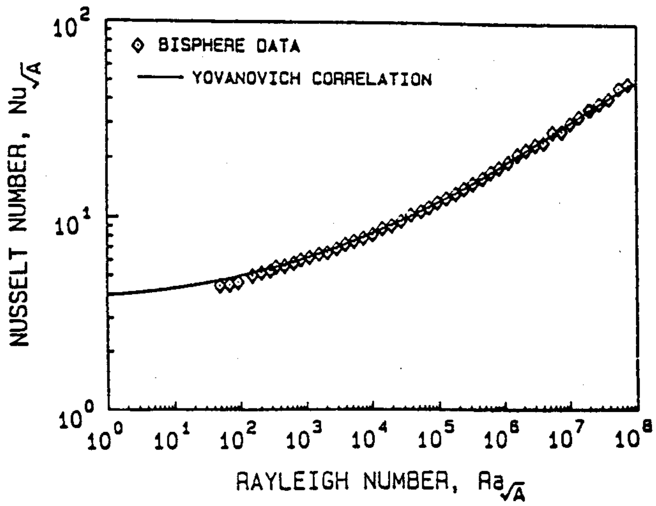

Figure 4. Comparison Between Correlation and Data for Bi-sphere [15]

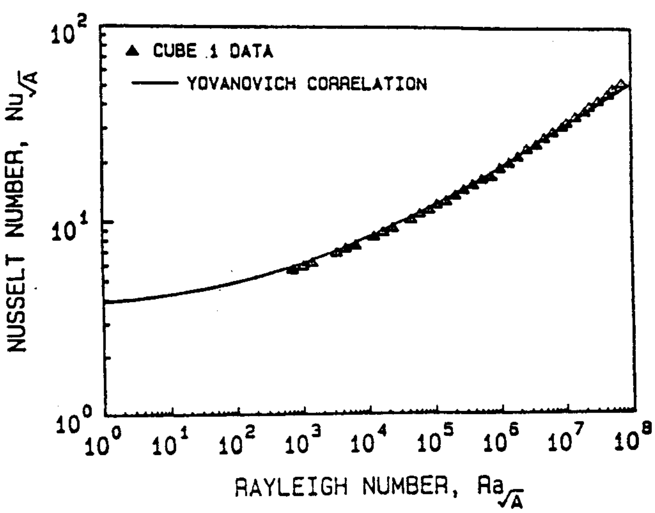

Figure 5. Comparison Between Correlation and Data for Cube 1 [15]

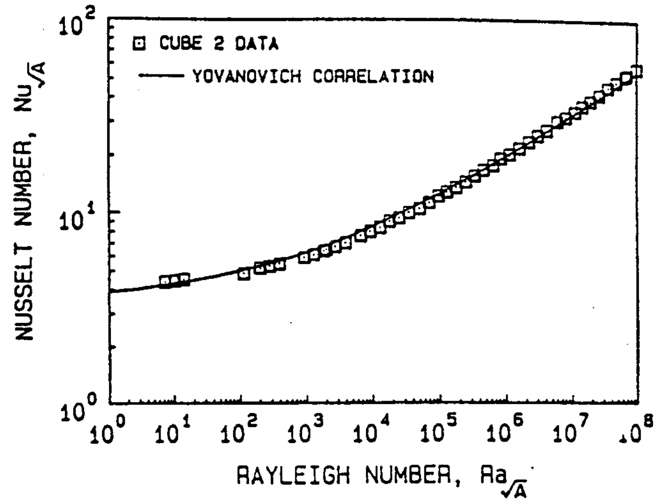

Figure 6. Comparison Between Correlation and Data for Cube 2 [15]

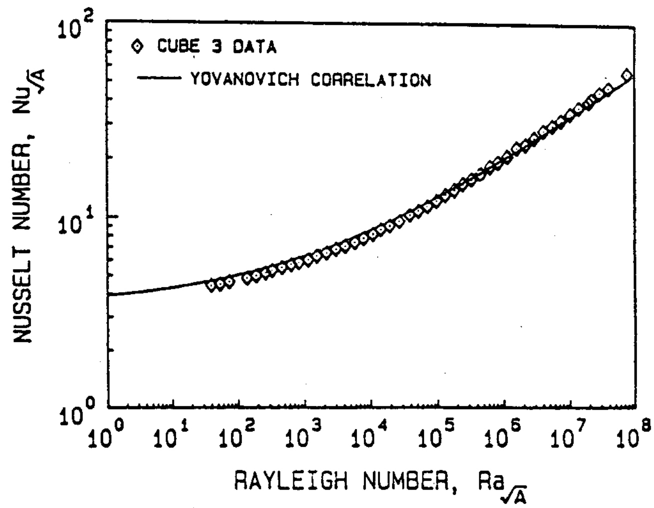

Figure 7. Comparison Between Correlation and Data for Cube 3 [15]

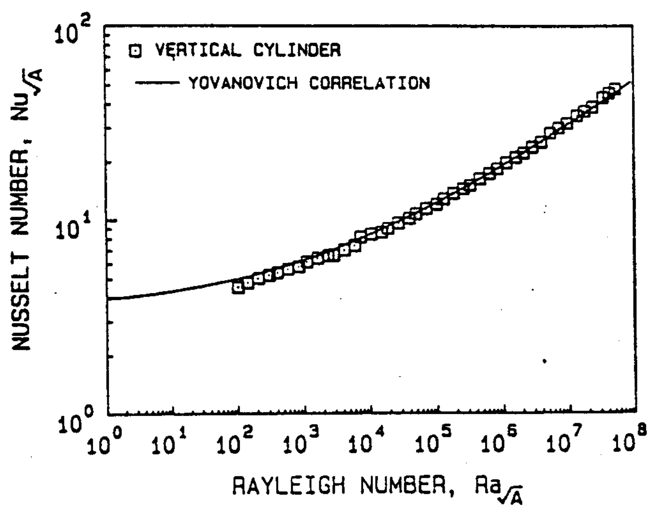

Figure 8. Comparison Between Correlation and Data for Vertical Cylinder [22]

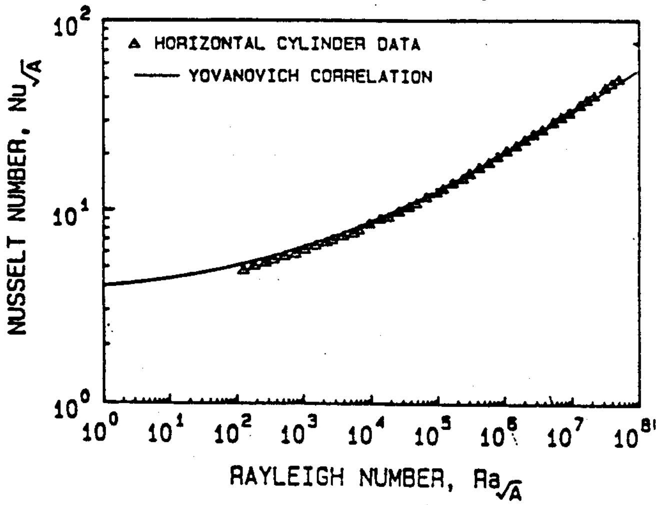

Figure 9. Comparison Between Correlation and Data for Horizontal Cylinder [22]

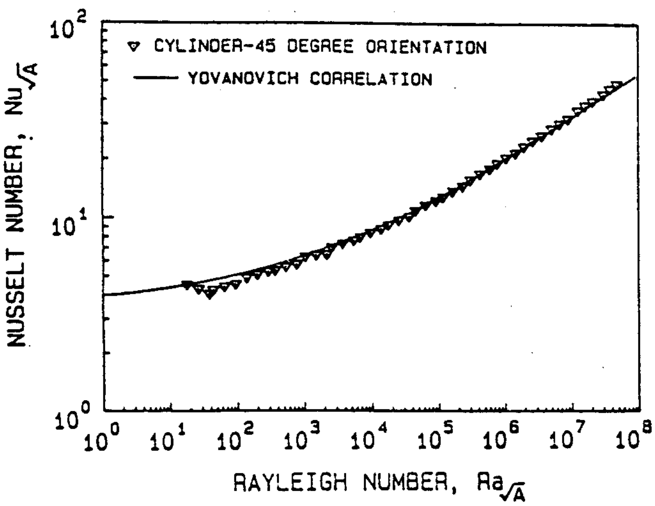

Figure 10. Comparison Between Correlation and Data for 45° Cylinder [22]

These empirically determined coefficients are remarkably similar considering the variety of body shapes, their aspect ratios and orientations. As anticipated the largest value is associated with the prolate spheroid which is a *streamlined* body and the smallest value is associated with the thin oblate spheroid which is a blunt body. The prolate coefficient is approximately 32% larger than the thin oblate coefficient. Excluding the thin oblate spheroid, the coefficients lie in the range $0.477 \le C_{\sqrt{A}} \le 0.560$ where the lower bound corresponds to the bi-sphere and the upper bound to the prolate spheroid. Two sets of values of $C_{\sqrt{A}}$ are reported for the spheroids. The old limited spheroid data [6] and the new, more precise data [22] applicable over a wider range of Ra were used to generate the $C_{\sqrt{A}}$ values in Table 3. The less precise Nu data (approximately 4.5% higher) produced coefficients which are approximately 4% higher; the new values are recommended. The first set designated as *old* was obtained for the data of Raithby et al [6] and the second set designated as *new* was obtained from the more precise and extensive data of Hassani [22] using the bodies of reference [6]. The higher precision in the *new* data is due to better estimates of the body specific heat and emissivity as well as a significant reduction in the lead wire, thermocouple and support wire conduction losses. An examination of the cube coefficients shows that they lie in the range $0.489 \le C_{\sqrt{A}} \le 0.521$, and there is only a 6.5% difference between the minimum and maximum values corresponding to orientations 1 and 3 respectively. At $Ra_{\sqrt{A}} = 10 \times 10^7$ the difference in the Nusselt number for these orientation extremes is approximately 5.4%. Sparrow and Stretton [9] reported orientation-related variations in the Nusselt number to be in the 2−5% range for air and 10−12% range for water. The coefficients for the short cylinder of unity aspect ratio lie in the range $0.497 \le C_{\sqrt{A}} \le 0.524$, and there is only a 5.4% difference between the minimum and maximum values corresponding to the vertical and horizontal orientations. The Sparrow-Ansari vertical cylinder data [7] in the range $3 \times 10^3 \le Ra_D \le 1.3 \times 10^5$ ($3.1 \times 10^4 < Ra_{\sqrt{A}} < 1.4 \times 10^6$) are in good agreement with the data in Figure 8. The Sparrow-Ansari correlation equation is based on Eq. (1) and a least-squares fit of the data; they found $C = 0.775$ and $m = 0.208$. Their correlation equation converts to $Nu_{\sqrt{A}} = 1.037 Ra_{\sqrt{A}}^{0.208}$ which is valid in the narrow range of Rayleigh number $1.4 \times 10^5 < Ra_{\sqrt{A}} < 1.3 \times 10^6$, and they reported no difference between the data and the prediction. A comparison of the predictions of the converted Sparrow-Ansari correlation equation with the data shown in Figure 8 over the applicable Rayleigh number range shows the data to lie approximately 3.6 − 6.2% above the predictions indicating very good agreement between the two sets of data. It can also be seen that the choice of $\mathcal{L} = D$ and $m = 0.208$ in Eq. (1) has a significant effect on the value of C.

When the coefficients for the cube and the short cylinder are compared at comparable orientations, for example cube 1 and the vertical cylinder, the difference between the coefficients is less than 1%. Comparing cube 2 and the 45° cylinder one finds a difference of approximately 1.4%, and for cube 3 and the horizontal cylinder the difference is approximately 0.6%.

An examination of the $C_{\sqrt{A}}$ values in Table 3 for the sphere, the new prolate spheroid, the horizontal cylinder, and the corner-over-corner cube shows a maximum difference of 1%. A value of 0.523 would accurately predict the Nusselt number for these *steamlined* body shapes with aspect ratios between 1 and $\sqrt{3}$, and having different orientations. The thick oblate spheroid ($AR = 0.5$) and the vertical cylinder have correlation coefficients which differ by 1% and they lie approximately 5% below the sphere, etc. These bodies can be considered to be *semi-blunt* bodies.

The correlation coefficients for the cube 2 and the 45° cylinder lie very close to the arithmetic average of the *streamlined* and *semi-blunt* bodies.

The laminar boundary layer coefficients were used in the correlation equation and the predictions are compared with the data in the figures where it can be seen that agreement is excellent for all body shapes, aspect ratios, and orientations over the full range of Ra. The RMS percent difference be-

Table 4: Comparison of Simple Correlation with Air Data and Other Correlations

		RMS Percent Difference		
Body Shape	AR	R-H	H-H	MMY
Spheres	1	0.84	3.67	0.92
Bi-sphere	2	3.63	2.26	2.38
Prolate Spheroid	1.93	1.90	3.56	1.83
Oblate Spheroid	0.50	3.75	4.09	2.60
Oblate Spheroid	0.10	6.90	–	2.34
Cube 1	1	6.16	5.62	3.13
Cube 2	$\sqrt{2}$	5.20	3.45	3.93
Cube 3	$\sqrt{3}$	4.08	2.89	4.80
Vertical Cylinder	1	–	2.71	3.70
Cylinder at 45°	$\sqrt{2}$	–	3.70	4.96
Horizontal Cylinder	1	–	2.06	3.56

tween the data and the corresponding predictions for all body shapes are given in Table 4. The largest differences of 4.80% and 4.96% occur for cube 3 and the 45° cylinder. The data and the predictions for these two cases are shown in Figures 7 and 10 respectively where it can be seen that the correlation equations overpredict the data at the lower Rayleigh numbers and underpredict the data at the higher Rayleigh numbers. The cube 3 data are approximately 5.7% below the predictions and approximately 5.9% above the predictions at the low and high Rayleigh numbers respectively where there is also slightly more scatter in the data. For the 45° cylinder there are 4 data points which lie more than 10% below the predictions while all other data points in the neighborhood of these points are only 6% below the predictions. These data points at the low Rayleigh number are clearly seen in Figure 10. At high Rayleigh numbers the data are approximately 5% above the predictions.

The best agreement between the data and the correlation equation occurs with the sphere where the RMS percent difference is less than 1%. The coefficient based on the air data of Chamberlain [8] is approximately 6.3% larger than the coefficient reported by Yuge [4]. For all other body shapes the RMS percent difference is less than 4%. The RMS percent differences between the data and the predictions of the more complex correlation equations of Raithby-Hollands [12] and Hassani-Hollands [18] are also given in Table 4. Overall the RMS percent differences of the simple correlation equation are as small or smaller than those of the more complex correlation equations of Raithby-Hollands [6] and Hassani-Hollands [18] which purportedly account for turbulence effects at high Rayleigh number.

An examination of Tables 2 and 3 suggests that approximations of $C_{\sqrt{A}}$ can be developed. For the sphere, prolate spheroid ($AR = 1.93$), oblate spheroid ($AR = 0.50$) and the bi-sphere we have

$$C_{\sqrt{A}} = Nu^{\infty}_{\sqrt{A}} - 3 \tag{22}$$

The coefficients for the oblate spheroid ($AR = 0.1$) and the cube can be approximated by

$$C_{\sqrt{A}} = \left[1 + \frac{(AR-1)}{10}\right](Nu^{\infty}_{\sqrt{A}} - 2.9) \tag{23}$$

where $AR(0.1 \le AR \le \sqrt{3})$ is the nominal aspect ratio given in Table 4.

When correlation coefficients determined by the above equations are used in Eq. (9), one finds natural convection from the spheres, bi-sphere, oblate and prolate spheroids, and the cubes can be predicted to an RMS percent difference of approximately 8%. A general expression based on the average values of $Nu^{\infty}_{\sqrt{A}}$ and $C_{\sqrt{A}}$:

$$Nu_{\sqrt{A}} = 3.470 + 0.510 Ra^{1/4}_{\sqrt{A}} \tag{24}$$

is recommended for "rough" estimates of free convection from isothermal, three-dimensional bodies of complex shape for $0 < Ra_{\sqrt{A}} < 10^8$.

Conclusions

A simple, but accurate, correlation equation is developed for isothermal three-dimensional bodies of complex shape, aspect ratio and orientation. The linear superposition of the diffusive and the boundary layer limits for the the prediction of the Nusselt is supported by very good agreement between the theory and air data over the full range of Rayleigh number, body shapes, aspect ratios and orientation.

The contribution of the diffusive limit to the total Nusselt number is significant in the laminar boundary layer regime. For example, at $Ra_{\sqrt{A}} = 10^4$, the diffusive limit contributes 40.7% and 45.8% to the total for the prolate and thin oblate spheroids. At $Ra_{\sqrt{A}} = 10^8$, the fraction falls to 6.4% and 7.8% respectively.

The characteristic length based upon the square root of the total surface area appears to be an appropriate length for correlating natural convection from complex bodies.

The effect of shape on the correlation coefficient is of the order of 5% when bodies of similar aspect ratios are compared, for example, the sphere, cube and short cylinder. The effect of aspect ratio is of the order of 4% provided $0.5 \le AR \le 2$. The effect of body orientation was found to be relatively small when similar bodies are compared, for example, the cube and short cylinder.

Simple equations are presented for estimating the single semiempirical parameter with acceptable accuracy.

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