BOUNDS ON LAMINAR NATURAL CONVECTION FROM ISOTHERMAL DISKS AND FINITE PLATES OF ARBITRARY SHAPE FOR ALL ORIENTATIONS AND PRANDTL NUMBERS

M.M. Yovanovich* and Khosrow Jafarpur†

Microelectronics Heat Transfer Laboratory
Department of Mechanical Engineering
University of Waterloo
Waterloo, Ontario, Canada N2L 3G1

ABSTRACT

Upper and lower bounds on Nusselt-Rayleigh, $Nu - Ra$, correlation equations are developed for natural convection heat (mass) transfer from isothermal (isopotential) square plates or circular disks which have one-to-two sides active, facing upward, downward and vertical, with and without adiabatic extensions of negligible or very large extent. The upper bound corresponds to a horizontal plate or disk with one side active facing upward located in a quiescent medium of infinite extent; and the lower bound correponds to a horizontal plate or disk with one side active located in an infinite adiabatic plane in contact with a quiescent medium of semi-infinite extent. The $Nu - Ra$ results for vertical or inclined plates or disks with both sides active lie between the proposed bounds. The bounds, developed for $Pr = Sc = 0.71$, are extended to include all values of Prandtl or Schmidt numbers. The proposed upper and lower bounds $Nu - Ra$ correlation equations are shown to be in complete agreement with all previously published correlation equations, and the proposed bounds remove the apparent discrepancies between them.

NOMENCLATURE

A	=	surface area of the body; m^2
$\tilde{A}$	=	area fraction
$\tilde{A}_i$	=	area fraction of the ith component
$\sqrt{A}$	=	characteristic length of the body proposed by Yovanovich (1987a); m
AR	=	aspect ratio of body
C, C'	=	laminar and turbulent correlation constants
c_p	=	specific heat at constant pressure; $J/kg{\cdot}K$
d	=	disk diameter, m
$F(Pr)$	=	Prandtl number function (Churchill and Churchill,1975), $\left[0.670/[1+(0.50/Pr)^{9/16}]^{4/9}\right]$
g	=	scalar gravitational acceleration; m/s^2
$G_{\sqrt{A}}$	=	laminar boundary layer body-gravity function based on $\sqrt{A}$
$Gr_{\sqrt{A}}$	=	Grashof number, $g\beta(T_s - T_\infty)(\sqrt{A})^3/\nu^2$
h	=	heat transfer coefficient; $W/m^2{\cdot}K$
H	=	plate height; m
k	=	thermal conductivity; $W/m{\cdot}K$
$\mathcal{L}$	=	charactristic length of the body; m
m	=	constant exponent
n	=	constant exponent
$\vec{n}$	=	outward surface normal vector
$Nu_{\sqrt{A}}$	=	Nusselt number, $h\sqrt{A}/k$
$Nu^{\infty}_{\sqrt{A}}$	=	diffusive limit;

*Professor and Director, Fellow ASME

†Graduate Research Assistant, Currently Assistant Professor, University of Shiraz, Iran

$Nu^{\infty}_{\sqrt{A}} = S^{\star}_{\sqrt{A}}$

$P(\theta)$ = local perimeter; m

P, P_{max} = perimeter of projected area onto a horizontal plane, maximum perimeter; m

Pr = Prandtl number; ν/α

R = thermal resistance; K/W

$R^{\star}_{\sqrt{A}}$ = dimensionless thermal resistance

$Ra_{\sqrt{A}}$ = Rayleigh number, $Gr_{\sqrt{A}}Pr$

RMS = Root-Mean-Square value

Sc = Schmidt number

$Sh_{\mathcal{L}}$ = Sherwood number based on $\mathcal{L}$

T = temperature; K

T_f = film temperature, $(T_s + T_\infty)/2$; K

W = plate width; m

Subscripts

$\sqrt{A}$ = based on $\sqrt{A}$, as the characteristic length

$\mathcal{L}$ = based on $\mathcal{L}$, as the characteristic length

∞ = at a remote point from the body

Superscripts

$\rightarrow$ = vector quantity

∞ = estimated at $Ra \rightarrow 0$

$\tilde{}$ = dimensionless quantity

$*$ = dimensionless quantity

GreekSymbols

α = thermal diffusivity, $k/\rho c_p$; m

β = volumetric expansion coefficient; K^{-1}

θ = angle between gravity vector and outward normal to surface; rad

ν = kinematic viscosity, μ/ρ; m^2/s

ρ = density; kg/m^3

Miscellaneous

bot = abbreviation for bottom surface

down = abbreviation for facing downward

top = abbreviation for top surface

up = abbreviation for facing upward

vert = abbreviation for vertical orientation

INTRODUCTION

Natural convection heat (and mass) transfer from isothermal disks or finite plates of arbitrary shape (planform) in various orientations (inclinations), from vertical to horizontal, with respect to the gravity vector has been the subject of numerous experimental, theoretical and numerical studies.

Each study has been limited to one or perhaps two aspects of the possible cases which may occur. Some examples of cases which occur in the microelectronic industry are square heat sources which are cooled from one side with the heated side facing up or facing down, or in the vertical orientation.

If the square heat source is placed flush in a finite adiabatic surface which has adiabatic edges, and the heated side faces up, down or is vertical (see Fig. 1), the heat transfer rates will be quite different. If the insulated edges are sufficiently large, the heat source can be modeled as a disk (or finite plate) located in an *infinite* adiabatic plane (see Fig. 1) which is cooled by a stagnant fluid of *semi-infinite* extent. If the isothermal disk is cooled from both faces, the heat transfer rate will depend strongly on its inclination (horizontal to vertical orientation).

There are significant apparent discrepancies between the reported correlations of the theoretical, experimental or numerical results. The correlations which are based on several different characteristic body lengths are invariably limited to a narrow range of the Rayleigh number. If the correlations are based on experimental data or numerical results, they are limited to one value of the Prandtl number Pr or, at best, to a narrow range of Pr.

Also, the available numerous correlation equations are not applicable for many special industrial cases.

There is at present no single model which can predict the Nusselt number for all cases for all Prandtl numbers and all Rayleigh numbers in the laminar flow range.

AIMS OF THIS WORK

The aims of this work are to present upper and lower bounds on laminar natural convection from isothermal circular disks and finite plates of arbitrary shape which are cooled i) from one side only by a fluid of semi-infinite extent (see Fig. 1), and ii) from two sides by a fluid of infinite extent. The effect of plate inclination with respect to the gravity vector (see Fig. 1) and the effect of Prandtl and Rayleigh numbers will be incorporated into the development of the bounds.

The proposed correlation equations of Nusselt num-

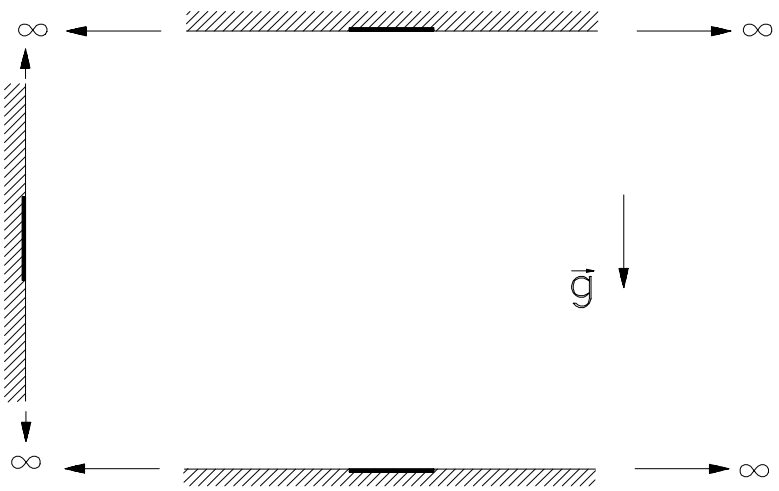

Figure 1: Schematic of Finite Isothermal Plates Embedded in Infinite Adiabatic Planes: Vertical and Horizontal Orientations.

ber versus Rayleigh number will be based on the notion that all laminar natural convection heat (or mass) transfer from single isothermal bodies can be simply, but accurately, modeled by the linear superposition of two asymptotes: i) the diffusive limit corresponding to molecular heat transfer which is independent of orientation and weakly dependent on shape if the square root of the active area is used to nondimensionalize the solution (Yovanovich, 1987a), and ii) the laminar boundary-layer asymptote (Yovanovich, 1987b, 1987c) which depends on body orientation and shape through the body-gravity function (Lee, Yovanovich and Jafarpur, 1991).

The use of the square root of the total active surface area, $\sqrt{A}$, as the characteristic body length in the definition of the Nusselt and Rayleigh numbers and the body-gravity function makes them relatively independent of body shape. The body gravity function does, however, depend on the body orientation or inclination.

It will be demonstrated that the laminar flow natural convection correlation equation:

$$Nu_{\sqrt{A}} = Nu^{\infty}_{\sqrt{A}} + F(Pr)\, G_{\sqrt{A}}\, Ra^{1/4}_{\sqrt{A}} \qquad (1)$$

proposed by Yovanovich, (1987b), with the *universal* Prandtl number function:

$$F(Pr) = \frac{0.670}{\left[1 + (0.5/Pr)^{9/16}\right]^{4/9}} \qquad (2)$$

recommended by Churchill and Churchill, (1975) which is applicable for all fluids and arbitrary body shapes; and the body-gravity function:

$$G_{\sqrt{A}} = \left[\frac{1}{A}\int\!\!\int_A \left(\frac{P(\theta)}{\sqrt{A}}\sin\theta\right)^{1/3} dA\right]^{3/4} \qquad (3)$$

recommended by Lee, Yovanovich and Jafarpur, (1991) for bodies of arbitrary shape and orientation can be used to develop the upper and lower bounds for all disks and finite plates of arbitrary planform. The geometric parameter $P(\theta)$ is the local perimeter of the body and $\sin\theta$ is the local tangential component of the unit gravity vector, and A is the total active heat transfer area of the body.

The body-gravity function, $G_{\sqrt{A}}$, defined above can be applied to axisymmetric and two-dimensional bodies of arbitrary shape and orientation. It can also be applied to plates of arbitrary planform which are inclined with respect to the gravity vector (see Fig. 2). It cannot be used for horizontal plates with the active surface facing up or down because $\sin\theta = 0$. The body-gravity function for numerous two- and three-dimensional body shapes and orientations have been presented (Lee, Yovanovich and Jafarpur, 1991).

It will be shown that all correlation equations based on experiment, theory and numerical simulation lie between the upper and lower bounds to be presented in this work. It will also be shown that all data for air cooling of many three-dimensional bodies obtained over a wide range of $Ra_{\sqrt{A}}$ lie between the proposed bounds.

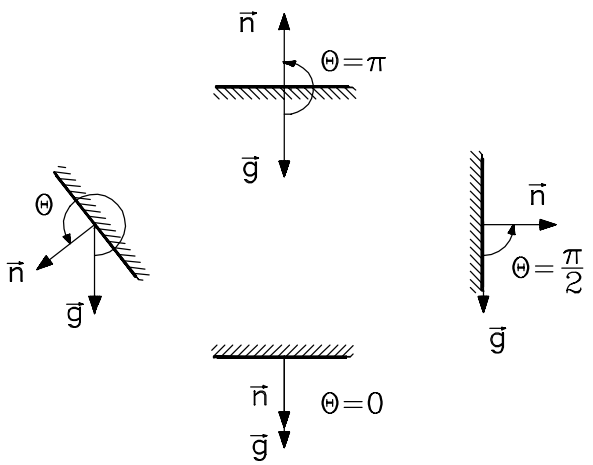

Figure 2: Schematic of Finite Isothermal Plates with Adiabatic Back Surface: Vertical, Horizontal and Inclined Orientations.

REVIEW OF MODELS AND CORRELATIONS

Heat Transfer Correlations

Fishenden and Saunders (1930) presented the first review and summary of the prior experimental results of natural convection cooling of isothermal square plates and a circular disk by air. The objectives of the studies were to examine the effects of shape and orientation on the area-mean Nusselt number. The horizontal square plates and circular disk were cooled by means of heat transfer from one side which faced either up or down, and the results were compared with the cooling results for the square plates and circular disk in the vertical orientation.

The results of the Fishenden and Saunders (1930) review are presented in Table 1. The effects of orientation (hot surface facing up or down) are given on columns 4 and 5. The cooling from the plates or disk facing up is superior to the vertical orientation, and the cooling from the plates or disk facing down is inferior to the cooling of the same surfaces in the vertical orientation.

The results of the different studies vary significantly for many reasons: thickness of the plates and disks, edge effects, radiation and the high surface temperatures relative to the ambient air temperature, for example. The ratio of the results appearing in columns 4 and 5 (i.e. relative cooling performance of the upward-facing orientation to the downward-facing orientation) is given in column 6. The results of Griffith and Davis, Langmuir and Kirpitacheff are within $\pm 10\%$ of 2; while the Rosin result is significantly lower.

Fishenden and Saunders (1930) recommended the following relative cooling ratios:

$$\frac{Nu_{up}}{Nu_{vert}} = 1.30 \qquad \frac{Nu_{down}}{Nu_{vert}} = 0.65 \qquad (4)$$

The recommendations yield the following empirical result which relates the relative cooling performance of square plates or circular disks facing upward or downward:

$$\frac{Nu_{up}}{Nu_{down}} = 2 \qquad (5)$$

which is very close to the mean value of the first three entries of column 6.

By means of an optical method Saunders and Fishenden (1935) determined the area-mean heat flux at the top and bottom surfaces of an isothermal aluminum rectangular plate. They reported that the ratio of the mean value of the heat flux over the top surface to the mean value of the heat flux over the bottom surface to be 1.6, which is lower than the value given in 1930.

Jakob (1949) examined the air data of Griffiths and Davis (1922) and reported: $Nu_{up}/Nu_{vert} = 1.28$, $Nu_{down}/Nu_{vert} = 0.64$ and $Nu_{down}/Nu_{up} \approx 0.5$ in close agreement with Fishenden and Saunders (1930).

Fishenden and Saunders (1950) reviewed the earlier results on natural convection from isothermal finite plates and circular disks in the vertical, horizontal facing upward and downward orientations which they summarized as

$$Nu_{\mathcal{L}} = C Ra_{\mathcal{L}}^{1/4} \qquad Nu_{\mathcal{L}} = C' Ra_{\mathcal{L}}^{0.33} \qquad (6)$$

Table 1. Results of Fishenden and Saunders (1930) Review

Author(s)	Geometry	Dimensions, mm	$\frac{Nu_{up}}{Nu_{vert}}$	$\frac{Nu_{down}}{Nu_{vert}}$	$\frac{Nu_{up}}{Nu_{down}}$
Griffith and Davis	Square Plate	122×122	1.27	0.67	1.90
Langmuir	Circular Disk	18.42	1.10	0.50	2.20
Kirpitacheff	Square Plate	61×61	1.55	0.80	1.94
Rosin	—	—	1.12	0.91	1.23
Nusselt and Hencky	Square Plate	45.7×45.7	1.27	—	—

for laminar and turbulent convection heat transfer into air respectively.

They recommended a correlation equation for gases and liquids in natural convection for $Ra_{\mathcal{L}} > 10^5$, and another one for gases in turbulent flow. Their reported coefficients C and C' given in Table 2 are based on the plate side dimension as the characteristic length.

Table 2. Coefficients C and $C^{'}$ recommended by Fishenden and Saunders (1950)

Geometry	C	$C^{'}$	$C/C^{'}$
Vertical Plate ($\mathcal{L}$ = Height)	0.56	0.12	4.67
Horizontal Plate, Facing Up ($\mathcal{L}$ = Side)	0.54	0.14	3.85
Horizontal Plate, Facing Down ($\mathcal{L}$ = Side)	0.25	—	—

Bosworth (1952) also reviewed the results of the earlier investigations which he summarized in the two correlation equations

$$Nu_{\mathcal{L}} = C Ra_{\mathcal{L}}^{1/4} \qquad Nu_{\mathcal{L}} = C' Ra_{\mathcal{L}}^{1/3} \qquad (7)$$

for laminar and turbulent natural convection heat transfer into air, respectively.

The coefficients C and C' are given in Table 3 for the plates in the vertical, face up, and face down orientations. The characteristic length $\mathcal{L}$ is the plate height in the vertical orientation and the plate side in the horizontal orientation.

Table 3. Coefficients C and $C^{'}$ recommended by Bosworth (1952)

Geometry	C	$C^{'}$	$C/C^{'}$
Vertical Plate	0.55	0.13	4.23
Horizontal Plate, Facing Up	0.71	0.17	4.18
Horizontal Plate, Facing Down	0.35	0.08	4.38

Examination of the Bosworth results leads to the ratios: $C_{up}/C_{vert} = 1.29$, $C_{down}/C_{vert} = 0.64$, and $C_{up}/C_{down} = 2.02$ in agreement with Fishenden and Saunders (1930) and Jakob (1949).

The coefficients recommended for turbulent conditions lead to the following ratios: $C'_{up}/C'_{vert} = 1.31$,

$C'_{down}/C'_{vert} = 0.62$, and $C'_{up}/C'_{down} = 2.11$ which are remarkably similar to the ratios for the laminar flow condition.

With the exception of the values given for the vertical plate, the coefficients of Fishenden and Saunders (1950) for the horizontal plates facing up and down are significantly different from those given by Bosworth, Jakob and those recommended by Fishenden and Saunders (1930).

McAdams (1954) recommended the form of the correlation equations of Fishenden and Saunders (1950), with altered correlation coefficients: $C_{vert} = 0.59$, $C_{up} = 0.54$ and $C_{down} = 0.27$. The recommended characteristic length is the side dimension. The coefficient for the vertical plate is approximately 7.3% larger than the one recommended by Bosworth. The coefficients for the horizontal surfaces facing up and down respectively are approximately $23-24\%$ smaller than those of Bosworth; but the ratio $C_{up}/C_{down} = 2$ is preserved.

Mikheyev (1968) recommended for gases and liquids $Pr > 0.7$ the correlation equation:

$$Nu_{\mathcal{L}} = C Ra_{\mathcal{L}}^{n} \tag{8}$$

with $C = 1.18, n = 1/8$ for $1 \times 10^{-3} \leq Ra_{\mathcal{L}} \leq 5 \times 10^{2}$, and $C = 0.54, n = 1/4$ for $5 \times 10^{2} \leq Ra_{\mathcal{L}} \leq 2 \times 10^{7}$. The same correlation equations can be used to estimate heat transfer from horizontal plates with the characteristic length set equal to the smaller side dimension of the plate, and for surfaces facing upward the heat transfer coefficient must be increased by 30%, and decreased by 30% if the surface faces downward. Thus, according to Mikheyev, $Nu_{up}/Nu_{vert} = 1.30$, $Nu_{down}/Nu_{vert} = 0.70$, and $Nu_{up}/Nu_{down} = 1.86$. The basis of Mikheyev's correlation is not given.

Al-Arabi and El-Riedy (1976) present a review of natural convection heat transfer from horizontal plates through 1975. They reported local and area-mean heat transfer coefficients over the top surface for air cooling of six circular plates, eight square plates and five rectangular plates with aspect ratios from 1.67 to 4. Their laminar heat transfer results for $Pr = 0.70$ and all circular disks, and all square and rectangular plates are represented to within $\pm 14\%$ by

$$Nu_{\mathcal{L}} = 0.70 Ra_{\mathcal{L}}^{1/4} \qquad 2 \times 10^{5} \leq Ra_{\mathcal{L}} \leq 4 \times 10^{7} \tag{9}$$

with $\mathcal{L}$ set to the diameter of the circular disk and the smaller side dimension of the rectangular plate, or the side of the square plates.

Equation (9) is close to the correlation equation recommended by Bosworth (1952) and gives heat-transfer values approximately 30% higher than the values given by the correlation equation recommended by Fishenden and Saunders (1950) and McAdams (1954).

Al-Arabi and El-Riedy also commented on the fact that the correlation equation of Fishenden and Saunders (1950) and, therefore, of McAdams for horizontal plates facing upward gives values of the area-mean Nusselt number approximately 4% less than from the same plates in the vertical orientation. This contradicts the findings of Mikheyev which are in good agreement with the results of Al-Arabi and El-Riedy.

They also reported that the circular plate results are well represented (within a maximum deviation of 14%) by the correlation equation developed for the square plates. They attribute this finding to the fact that natural convection heat transfer from an isothermal surface facing upward under laminar flow is dominated by the edge effect, and, therefore, the square and circular plates should produce similar results.

Churchill (1983) reviewed the available literature on natural convection from isothermal and isoflux bodies in a large quiescent medium. For downward facing finite plates he recommended

$$Nu_{\mathcal{L}} = G \ [Ra_{\mathcal{L}} \ \psi \ (Pr)]^{1/5} \tag{10}$$

with $\mathcal{L} = A/P$, the parameter $G = 0.60$ and the Prandtl number function $\psi(Pr) = (1 + (0.492)^{9/16})^{-16/9}$. The above correlation for downward facing plates is reported to be applicable in the laminar range $10^{3} \leq Ra_{\mathcal{L}}\psi(Pr) \leq 10^{10}$.

For laminar flow over upward facing plates he proposed

$$Nu_{\mathcal{L}} = \frac{0.766 \ Ra_{\mathcal{L}}^{1/5}}{\left[1 + (0.322/Pr)^{11/20}\right]^{4/11}} \tag{11}$$

Churchill observed that Eq. (11) was well below (approximately $40 - 50\%$), but parallel to, the mass transfer data of Goldstein et al. (1973) for the range $1 \leq Ra_{\mathcal{L}}\psi(Pr) \leq 10^{5}$. He did not account for the discrepancy between the correlation equation and the data.

The upward and downward facing plate correlations of Churchill for $Pr = 0.71$ lead to $Nu_{up}/Nu_{down} \approx 1.33$ which is $40 - 50\%$ too small.

Gunaji, Pederson, and Leslie (1990) reported a numerical solution for the case of a horizontal, isothermal circular disk located in an infinite adiabatic plane in contact with a fluid ($Pr = 0.71$) of semi-infinite extent for Grashof numbers based on the disk radius between 0.5×10^{4} and 10^{6} which they reported lie within the laminar range. They presented a least-squares fit of the area-average Nusselt number based on disk radius ($\mathcal{L} = a$) for five values of the Grashof number

$$Nu_{\mathcal{L}} = 0.61 \ Gr_{\mathcal{L}}^{0.209} \qquad Pr = 0.71 \tag{12}$$

A comparison of the local Nusselt number with boundary-layer solutions shows close agreement over the outer disk regions where the laminar flow solutions are applicable.

Mass Transfer Correlations

Wragg and Loomba (1970) examined natural convection ionic mass transfer at horizontal upward facing circular disk electrodes (five diameters ranging from 0.1 to 2.0 cm) in spatially unrestricted convection. They presented

$$Sh_{\mathcal{L}} = 0.72\,(Sc\,Gr_{\mathcal{L}})^{0.25} \qquad 3\times10^4 < Sc\,Gr_{\mathcal{L}} < 2.5\times10^7 \tag{13}$$

and

$$Sh_{\mathcal{L}} = 0.18\,(Sc\,Gr_{\mathcal{L}})^{0.33} \qquad 2.5\times10^7 < Sc\,Gr_{\mathcal{L}} < 10^{12} \tag{14}$$

with $\mathcal{L} = d$ and $Sc \approx 2300$. They reported that their mass transfer correlation equations compared favourably with the heat transfer correlation equations of Bosworth (1952); but, they were considerably higher than those of Fishenden and Saunders (1950).

Goldstein, Sparrow and Jones (1973) reported natural convection mass transfer correlation equations corresponding to $Sc \approx 2.5$ for circular, square and 7:1 rectangular plane surfaces. The dimensionless results by analogy correspond to natural convection heat transfer from horizontal isothermal heated surfaces facing upward.

The $Nu_{\mathcal{L}} - Ra_{\mathcal{L}}$ data with the characteristic length $\mathcal{L} = A/P$ where A is the plan area and P is the perimeter of the surface fall in the range $1 < Ra_{\mathcal{L}} < 10^4$, and are correlated well by

$$Nu_{\mathcal{L}} = 0.96\,Ra_{\mathcal{L}}^{1/6} \qquad Ra_{\mathcal{L}} < 200 \tag{15}$$

and

$$Nu_{\mathcal{L}} = 0.59\,Ra_{\mathcal{L}}^{1/4} \qquad Ra_{\mathcal{L}} > 200 \tag{16}$$

Equations (15) and (16) were obtained after setting the Rayleigh number exponents to the values of 1/6 and 1/4, respectively, in place of the least-squares fit values of 0.169 and 0.256.

The authors further compared their data with correlations of the form $Nu_{\mathcal{L}} = CRa_{\mathcal{L}}^n$ obtained from heat and mass transfer data, as well as with analytical and numerical results. The correlations were developed for fluids having Prandtl or Schmidt number values 0.7, 2.5, 2300, and ∞. The characteristic length $\mathcal{L}$ was equal to the such conventional lengths as diameter for the circular plate, and side dimension for the square plate, and smaller side for the rectangular plate. The exponent of the Rayleigh number ranged from 1/8 to 1/4; and the correlation coefficient ranged from 0.54 to 1.90. The lowest value of $C = 0.54$ with $n = 1/4$ was reported by Fishenden and Saunders (1950) and the highest value of $C = 1.90$ with $n = 1/6$ is from their own work.

They reported that their data for the circular and square plates lie approximately 20% above the Mikheyev correlation, and significantly above the Fishenden and Saunders correlation recommended by McAdams and many present heat transfer texts.

The analytic results based on integral and perturbation methods give a $\frac{1}{5}$-power on the Rayleigh number and the correlations fall much lower than the data. The numerical results obtained for $Pr = 0.7, 10$ and creeping flow also fall much below the mass transfer data.

They concluded that their data supported the correlations of Bosworth and Mikheyev.

Lloyd and Moran (1973) obtained a mass transfer correlation equation for $Sc \approx 2300$, for circular disks, squares, rectangles ($2 \leq$ length/width ≤ 10), and several right-triangles as

$$Nu_{\mathcal{L}} = 0.54\,Ra_{\mathcal{L}}^{1/4} \qquad 2.2\times10^4 \leq Ra_{\mathcal{L}} \leq 8\times10^6 \tag{17}$$

with $\mathcal{L} = A/P$.

The second Goldstein et al. correlation and the Lloyd and Moran correlation and the corresponding data are in acceptable agreement in a small interval of Rayleigh number; the difference is partially due to the difference of the Schmidt number in the respective experiments. The exponent on the Rayleigh number was found to be 1/4 in both sets of measurements.

Goldstein and Lau (1983) studied laminar natural convection from a horizontal plate by numerical methods and by experiment for Rayleigh numbers from 10 to 10^4. The plate was isothermal on one side (one side active) while the opposite side was adiabatic; and the plate was situated in an *infinite* fluid medium.

The paper presents an excellent, extensive review of the heat and mass transfer correlations of the form $Nu_{\mathcal{L}} = CRa_{\mathcal{L}}^n$ obtained by many researchers who used either experimental techniques or one of several analytic techniques to obtain solutions for different cases: geometry, Prandtl (or Schmidt) number, active surface facing upward or downward, without and with adiabatic extensions.

The results of the analytic solutions for circular and square plates and "infinite" strips facing upward or downward gave the same value of the exponent on the Rayleigh number, i.e $n = 1/5$; however, the coefficient C for upward facing plates ranged from 0.560 to 0.688. The finite difference solution of Goldstein and Lau for an "infinite" strip for $40 < Ra_{\mathcal{L}} < 8 \times 10^3$ gave $C = 0.621$

when the exponent was set to the value $n = 1/5$ in place of the least-square value of 0.203. For the downward facing plates the coefficient C ranged from 0.44 to 0.543.

The experimental results for upward facing plates for a wide range of Rayleigh number $7.8 \leq Ra \leq 8 \times 10^6$ gave correlations with the Rayleigh number exponent $1/8 \leq n \leq 1/4$ and the corresponding coefficients $0.38 \leq C \leq 0.96$. The lowest value of C was reported by Fishenden and Saunders (1950).

The experimental results for downward facing plates for a wide range of Rayleigh number $10 \leq Ra \leq 1.25 \times 10^8$ gave correlations with $0.089 \leq n \leq 1/5$ and $0.31 \leq C \leq 1.27$.

Theoretical Heat and Mass Transfer Results

Horizontal Surfaces Facing Upward

Goldstein and Lau (1983) summarized the heat transfer ($Pr = 0.72$) and the mass transfer ($Sc = 2.5$) results obtained by means of integral, perturbation and similarity methods for a single isothermal (isopotential) circular disk, square plate or semi-infinite plate with the active surface facing upward.

The area-mean Nusselt or Sherwood numbers were found to be correlated by $Nu_{\mathcal{L}} = C Ra_{\mathcal{L}}^{1/5}$ with the coefficient C ranging between 0.560 to 0.688 for $\mathcal{L} = A/P$. The lowest and highest values of C were reported for the two mass transfer studies. The lowest values $C = 0.578$ and $C = 0.603$ were given for similarity solutions for the circular disk and the square plate respectively. The reported values of the coefficient for the semi-infinite plate were in the tight range: $0.620 - 0.646$. No limits on the range of $Ra_{\mathcal{L}}$ were given.

Goldstein and Lau presented the correlation equation: $Nu_{\mathcal{L}} = 0.621 Ra_{\mathcal{L}}^{1/5}$ of their numerical heat transfer results from an infinite strip for $Pr = 0.71$ and $40 < Ra_{\mathcal{L}} < 8 \times 10^3$ which is in good agreement with the theoretical results for $Pr = 0.72$.

Other theoretical results published since 1983 (Gebhart, Jaluria, Mahajan and Sammakia, 1988) support the earlier work; in particular the $\frac{1}{5}-$power on the Rayleigh number.

Robinson and Liburdy (1987) present a correlation equation for air cooling of an isothermal circular disk with extensions facing upward. The area-mean Nusselt was obtained by means of the method used by Zakeralah and Ackeroyd (1979); it was given as:

$$Nu_{\mathcal{L}} = 0.982\ Gr_{\mathcal{L}}^{1/5} \tag{18}$$

which is valid in the range $10^4 \leq Gr_{\mathcal{L}} \leq 10^6$ and the characteristic length in the correlation equation and the Grashof number are the disk diameter and radius respectively.

The above correlation equation is approximately 14 % above the one developed by Gunaji et al. (1990).

A recent paper by Lewandowski (1991) reviews the previously published theoretical and experimental investigations on heat transfer from isothermal plates of finite dimensions. He comments on the discrepancies and apparent disagreements between the results for vertical, inclined and horizontal plates facing upward. The results for the vertical plates differ by ± 20 %; for inclined plates the results differ by ± 45 %; and for the horizontal case the discrepancies are ± 50 %.

He presents a simplified, quasi-analytical, solution for isothermal vertical, inclined and horizontal (facing upward) finite rectangular plates with the back side insulated. The proposed solution leads to a system of four strongly-coupled equations which must be solved numerically.

Lewandowski compares his complex theoretical model against air data which he obtained for a rectangular plate ($0.1\ m, 0.06\ m$), and for which he had developed the following correlation equations: $Nu_{\mathcal{L}} = 0.612\ Ra_{\mathcal{L}}^{1/4}$ and $Nu_{\mathcal{L}} = 0.766\ Ra_{\mathcal{L}}^{1/5}$ for the vertical and horizontal orientations respectively. He did not provide limits on the Rayleigh number for either of the two correlation equations. He further reports that 95 % of the data fall within ± 20 % of the theoretical predictions.

He did not compare his theoretical solution against other data, or other theoretical models or other correlation equations.

Horizontal Surfaces Facing Downward

The results of several theoretical studies for isothermal (isopotential) circular disk, square plate and infinite strip facing downward were summarized by Golstein and Lau (1983). Integral methods were used to obtain solutions for heat transfer ($Pr = 0.7$) and mass transfer ($Sc = 2.5$).

The area-mean Nusselt and Sherwood numbers were correlated by $Nu_{\mathcal{L}} = C Ra_{\mathcal{L}}^{1/5}$ where the coefficient C was found to lie in the range $0.44 - 0.527$. The highest value of C was observed for $Sc = 2.5$ and the arage value of the $Pr = 0.71$ results is $C = 0.475$.

Goldstein and Lau presented a correlation of their numerical results for an infinite strip, with $Pr = 0.71$ and $40 < Ra_{\mathcal{L}} < 8 \times 10^3$ having the $\frac{1}{5}-$power and $C = 0.524$.

Summary of Review of Theoretical and Experimental Results

It is apparent that the results of the experimental and

theoretical investigations of heat and mass transfer from isothermal (isopotential) circular disks, rectangular and square plates, and plates of other planforms are numerous and diverse.

The reported correlation equations, generally of the form $Nu_{\mathcal{L}} = C\ Ra_{\mathcal{L}}^{m}$, differ in the coefficient C and the exponent m because of many factors. Some of the factors are: type of fluid, shape of plate, inclination of plate, effect of adiabatic edges and their extent relative to the dimensions of the plate, supports and connectors, characteristic length chosen to define the Nusselt and Rayleigh numbers, range of Rayleigh number, etc.

All previous studies have ignored the contribution and importance of the diffusive limit to the area-mean Nusselt (Sherwood) number for Rayleigh numbers which lie in the range $1 \leq Ra_{\mathcal{L}} \leq 10^{6}$. Therefore their reported correlation equations implicitly account for this effect primarily through the correlation coefficient C and to some degree by the exponent of the Rayleigh number. The exponent of the Rayleigh number is influenced to a large extent by the range of the Rayleigh number which varies for a fixed shape and size by the choice of the characteristic length (Yovanovich, 1987a, 1987b).

The use of the characteristic length:

$$\mathcal{L} = \frac{\text{Active Area}}{\text{Maximum Perimeter}} = \frac{A}{P_{max}} \qquad (19)$$

has been shown in several investigations (Goldstein et al., 1973; Lloyd and Moran, 1974; Weber et al., 1984; Sahraoui et al., 1990) to be quite effective in reducing experimental data obtained for heat and mass from isothermal (isopotential) upward facing finite plates of various planforms. This characteristic length when applied to the diffusive limit has been shown by Yovanovich (1987a, 1987b) to be inferior to the characteristic length: $\mathcal{L} = \sqrt{A}$ (A is the active surface area) which minimizes the effect of shape (Chow and Yovanovich, 1982).

There is general agreement in the air cooling heat transfer data obtained for isothermal finite plates with one side active in the vertical orientation, or facing upward or facing downward that the ratio of the area-mean Nusselt number is approximately 30 % greater for the horizontal upward-facing case relative to the vertical orientation due to edge effects; and that the ratio of the area-mean Nusselt number for the horizontal upward-facing plate is approximately two times greater than the downward-facing plate due to the different flow conditions.

One exception to this observation is the result reported by Fishenden and Saunders (1950) which does not agree with their own summary of 1930, as well as the results presented by Jacob (1949), Bosworth (1952), Mikheyev (1968) and Al-Arabi and El-Riedy (1976).

The second exception is found in the correlation equations of Churchill (1983); however, Churchill noted that his correlation equation was approximately $40 - 50$ % below the mass transfer data of Goldstein et al. (1973).

It will therefore be accepted as an empirical fact that the body-gravity function for the downward-facing square plate or circular disk is 50 % as large as the body-gravity function for the upward-facing square plate or circular disk.

The second accepted empirical fact is that the body-gravity function, $G_{\sqrt{A}}$, for the upward-facing square plate or circular disk is approximately 30 % greater than the body-gravity function for the vertical orientation.

The numerical results given by Gunaji et al. (1990) will be accepted as data applicable for the development of a lower bound corresponding to an air cooled, upward-facing, circular isothermal disk with one side active and embedded in an adiabatic plane in contact with a quiescent fluid of semi-infinite extent.

FULL SPACE AND HALF-SPACE DIFFUSIVE AND BOUNDARY-LAYER LIMITS

The development of correlation equations begins with definitions of full- and half-spaces for the diffusive limit and the laminar boundary-layer asymptote. The circular disk and square plate of equal surface areas (either one or two sides active) are interchangeably used to establish bounds on the diffusive limit $Nu_{\sqrt{A}}^{\infty}$ and the body-gravity function $G_{\sqrt{A}}$ corresponding to the laminar boundary-layer asymptote.

Diffusive Limit $Ra_{\sqrt{A}} \to 0$

The solutions for steady-state conduction from an isothermal circular disk (one or two sides active) in full-space are used to establish two values of the diffusive limit $Nu_{\sqrt{A}}^{\infty}$. The solution for two sides active will provide a lower bound value, and the solution for a circular disk with one side isothermal and the other side adiabatic, but the disk is located in full-space, will provide an upper bound value.

The thermal resistance of an isothermal circular disk of area $2\pi a^2$ in full-space is $R = 1/(8ka)$ (Yovanovich, 1987a); and the resistance of an isothermal disk of area πa^2 in contact with a half-space is therefore $R = 1/(4ka)$ where k is the thermal conductivity of the full- or half-space. The dimensionless thermal resistance defined as $R^{\star}_{\sqrt{A}} = k\sqrt{A}R$ has the values $\sqrt{2\pi}/8$ and $\sqrt{\pi}/4$ for the full-space and half-space models respectively.

It has been demonstrated (Yovanovich, 1987a) that the results for the circular disk are within 1% of the numerical results obtained for plates of square, hexagon, semi-circular, triangular, etc. planforms provided their total surface areas are equal and they have similar aspect ratios (Chow and Yovanovich, 1982). If the aspect ratios are nominally different, the results are different by approximately $2-3$ %.

If one side of the circular disk is isothermal and the other side is adiabatic; but the disk is located in full-space, the active area is πa^2 and the resistance has been found to be $R = 1/(2\pi ka)$, (Lemczyk, 1991), which has been verified numerically (Yovanovich and Teertstra, 1991, and Teertstra, 1992). This case gives for the dimensionless resistance $R^{\star}_{\sqrt{A}} = 1/\sqrt{2\pi}$.

The three models described above provide two bounds on the resistance which yield two bounds on the diffusive limits. The relationship between the dimensionless resistance and the diffusive limit Nusselt number is $Nu^{\infty}_{\sqrt{A}} = 1/R^{\star}_{\sqrt{A}}$, (Yovanovich, 1987a), leading to the upper and lower bounds

$$\text{Upper Bound:} \quad Nu^{\infty}_{\sqrt{A}} = 2\sqrt{\pi} = 3.545 \tag{20}$$

$$\text{Lower Bound:} \quad Nu^{\infty}_{\sqrt{A}} = \frac{4}{\sqrt{\pi}} = 2.257 \tag{21}$$

Although the above bounds for the difffusive limit have been developed for the circular disk, they should give very close estimates for plates of arbitrary shape provided their aspect ratios are close to unity.

Body-Gravity Functions for Boundary-Layer Asymptote

Lower Bound on Body-Gravity Function $G_{\sqrt{A}}$

A lower bound of the body-gravity function can be developed for the case of a horizontal, isothermal, circular disk which is located in an infinite adiabatic plane in contact with a semi-infinite fluid (case of a horizontal disk, one side active in contact with a half-space; see Fig. 2). The numerical results of Gunaji et al. (1990) are used to develop this lower bound.

The numerical results for $Pr = 0.71$ are first converted to $Nu_{\sqrt{A}}$ and $Ra_{\sqrt{A}}$. Following the procedure established by Yovanovich, 1987b, one subtracts $Nu^{\infty}_{\sqrt{A}} = 2.257$ from the reported values of $Nu_{\sqrt{A}}$ for the corresponding Rayleigh numbers to obtain a set of values of $F(Pr)G_{\sqrt{A}}$. The arithmetic average of the last four points ($Gr > 10^4$) gives a value of 0.382. Putting $Pr = 0.71$ in $F(Pr)$ of Eq. (2) gives a value of 0.513 for the Prandtl number function which is used to extract from the above result $G_{\sqrt{A}} = 0.742$ for the first estimate of a lower bound of the body-gravity function for the case described above.

Upper Bound on Body-Gravity Function $G_{\sqrt{A}}$

An upper bound of the body-gravity function will be established for vertical isothermal rectangular plates of height H and width W which have one or two sides active. For either case, since the unit gravity vector is parallel to the surface (see Fig. 2), $\sin\theta$ in Eq. (3) is independent of position on the surface. If two sides are active, then $A = 2HW$ and $P(\sin\theta) = 2W$; therefore $P\sin\theta/\sqrt{A} = \sqrt{2W/H}$. If one side only is active, omit the factor 2. The body-gravity function is therefore:

$$G_{\sqrt{A}} = \left(2\frac{W}{H}\right)^{1/8} \tag{22}$$

We see from the above result that a plate with two sides active has a body-gravity function which is $2^{1/8}$ times greater for all aspect ratios $0 < H/W < \infty$. For square plates $H/W = 1$, $G\sqrt{A} = 1$ for one side active and $G_{\sqrt{A}} = 1.091$ for two sides active. For air cooling, $F(Pr) = 0.513$, therefore $F(Pr)\ G_{\sqrt{A}} = 0.513$ or 0.561 for one or two sides active respectively. The corresponding values for a vertical circular disk are $G_{\sqrt{A}} = 1.021$ and $G_{\sqrt{A}} = 1.113$ for one or two sides active respectively (Lee, Yovanovich and Jafarpur, 1991). The difference between the square plate and the circular disk is approximately 2 %.

Body-Gravity Function for Horizontal Disk or Square Plates: Two Sides Active

The body-gravity function for thin horizontal circular disks and square plates with both sides active in a fluid of infinite extent can be developed from the expression (Lee, Yovanovich and Jafarpur, 1991)

$$G_{\sqrt{A}} = \left[G^{4/3}_{top}\ \tilde{A}^{7/6}_{top} + G^{4/3}_{bot}\ \tilde{A}^{7/6}_{bot}\right]^{3/4} \tag{23}$$

which is based on the assumption that the fluid flows over the bottom surface from the center to the edge, then it flows over the top surface from the edge to the center. The bottom and top surfaces are said to be in the *series flow arrangement*.

Putting the area fractions: $\tilde{A}_{top} = \tilde{A}_{bot} = 1/2$ and using the empirical fact that $G_{bot} = \frac{1}{2}G_{top}$ into the preceding equation gives

$$G^{hor}_{\sqrt{A}} = \left(\frac{1}{2}\right)^{7/8}\left[1 + \left(\frac{1}{2}\right)^{4/3}\right]^{3/4} G_{top} = 0.7006\ G_{top} \tag{24}$$

If we use the theoretical values of the body-gravity function derived for the vertical square plate and the vertical circular disk (one or two sides active) we estimate that $0.701 \leq G^{hor}_{\sqrt{A}} \leq 0.780$ based on the extreme limits set by the vertical square with one side active and the vertical circular disk with two sides active; or we have the tight range $0.771 \leq G^{hor}_{\sqrt{A}} \leq 0.780$ established by the vertical square and the vertical circular disk values respectively for two sides active.

The empirical correlation equations of Goldstein et al. (1973), Lloyd and Moran (1974), Al-Arabi and El-Riedy (1976) which are valid for circular disks and finite plates of various planforms facing upward with one side active in contact with a fluid of *infinite* extent have been reduced to the form of Eq. (1) and by the method proposed by Yovanovich (1987b) with Eq. (2) and $Nu^{\infty}_{\sqrt{A}} = 2\sqrt{\pi}$ the following semi-empirical values of the body-gravity function are derived respectively: $1.094, 1.018 \pm 5\%, 1.121 \pm 14\ \%$.

The empirical results are consistent and fall in a range covered by the empirical uncertainty reported by Al-Arabi and El-Riedy. The empirical and theoretical results are in acceptable agreement considering the many parameters which can influence the experimental results.

The numerical results of Gunaji et al. (1990) which are valid for an upward-facing circular disk embedded in an infinite adiabatic plane in contact with a fluid ($Pr = 0.70$) of semi-infinite extent were reduced by means of Eqs. (1) and (2) with $Nu^{\infty}_{\sqrt{A}} = 2.257$. The empirical value of the body-gravity function based on the five reported numerical values is 0.744 and it rises slightly to the value 0.752 if the first numerical value is excluded.

As expected this empirical value of the body-gravity function is lower than the theoretical-empirical value given above, because the case investigated by Gunaji et al. (1990) is a lower bound for upward-facing circular disks and square plates or other planforms having aspect ratios close to unity.

UPPER AND LOWER BOUND CORRELATION EQUATIONS FOR AIR COOLING

The results of the previous sections will now be used to develop upper and lower bound correlation equations which are dependent on the diffusive limit and the body-gravity functions, Eq. (1).

The upper and lower bounds will be established for air ($Pr = 0.71$) cooling, first, then the results will be generalized for all values of Pr.

Upper Bound Correlation Equation

The recommended upper bound correlation equation for air cooling is

$$Nu_{\sqrt{A}} = 3.545 + 0.571\ Ra^{1/4}_{\sqrt{A}} \tag{25}$$

which is established for an upward-facing, isothermal, circular disk in full-space with its lower face adiabatic. It consists of the upper bound value of the diffusive limit and $G_{\sqrt{A}} = 1.113$ which is well within the empirical values and experimental uncertainty reported by several investigators.

Lower Bound Correlation Equation

The recommended lower bound correlation equation for air cooling is

$$Nu_{\sqrt{A}} = 2.257 + 0.382\ Ra^{1/4}_{\sqrt{A}} \tag{26}$$

which is developed from the numerical results for an upward-facing, isothermal, circular disk imbedded in an *infinite* adiabatic plane. It consists of the lower bound value of the diffusive limit and $G_{\sqrt{A}} = 0.745$ which is approximately two-thirds of the upper bound value consistent with empirical observations.

All natural convection heat transfer air data for inclined disks and inclined finite plates of arbitrary shape and aspect ratio should fall between the proposed bounds.

These correlation equations are also proposed as bounds for natural convection heat transfer into air from isothermal three-dimensional complex convex bodies in various orientations. This will be demonstrated in a following section.

GENERAL UPPER AND LOWER BOUND CORRELATION EQUATIONS FOR ALL PRANDTL

The above upper and lower bound correlation equations for isothermal disks and finite plates of arbitrary shape and aspect ratio can be generalized to be valid for all fluids. Introducing the Prandtl number function, Eq. (2) into the above equations leads to the following general correlation equations.

Upper Bound For $0 \leq Pr < \infty$

$$Nu_{\sqrt{A}} = 2\sqrt{\pi} + 2^{1/8}\ F(Pr)\ Ra^{1/4}_{\sqrt{A}} \tag{27}$$

and

Lower Bound For $0 \leq Pr < \infty$

$$Nu_{\sqrt{A}} = \frac{4}{\sqrt{\pi}} + \frac{1}{\pi^{1/4}} F(Pr)\, Ra_{\sqrt{A}}^{1/4} \qquad (28)$$

COMPARISON OF BOUNDS WITH AIR DATA FROM HORIZONTAL AND VERTICAL BODIES

The assertion that all data for natural convection heat transfer from isothermal three-dimensional bodies must lie between the bounds established in this investigation will now be demonstrated.

The air data of Hassani, 1987 obtained from many isothermal three-dimensional bodies having very different aspect ratios and orientations will be used in the comparison. The data were obtained over seven decades of the Rayleigh number: $10 < Ra_{\sqrt{A}} < 10^8$.

The comparison between the proposed upper and lower bound correlation equations are shown in Figures 3 through 6 for a range of body shapes.

In Fig. 3 the data for horizontal *thin*: i) circular disk, ii) square disk, iii) oblate spheroid and iv) a vertical "applecore" are shown to lie between the upper and lower bound correlation equations.

The aspect ratio (height-to-breadth) is 0.10 for the first three bodies. The fourth body consists of two spherical caps connected by a small diameter solid circular cylinder. Its effective aspect ratio is is approximately 0.1.

The diffusive limit for these bodies lies in the narrow range: $3.34 < Nu^{\infty}_{\sqrt{A}} < 3.54$.

Clearly all four bodies have similar trends with increasing Rayleigh number. Their heat transfer characteristics are very similar: their Nusselt numbers lie midway between the proposed bounds at the lowest value of $Ra_{\sqrt{A}}$, and as $Ra_{\sqrt{A}}$ increases the values of $Nu_{\sqrt{A}}$ approach the lower bound as $Ra_{\sqrt{A}} \rightarrow 10^8$. The observed $Nu_{\sqrt{A}} - Ra_{\sqrt{A}}$ trends are anticipated in the light of the results of the review of previous experimental, numerical and analytical work which are frequently restricted to narrow ranges of $Ra_{\sqrt{A}}$.

Therefore, development of correlation equations of data found in any one or two decades of $Ra_{\sqrt{A}}$ for any one of the body shapes would require different coefficients and exponents on $Ra_{\sqrt{A}}$.

In Fig. 4 the data from *tall* vertical: i) circular disk, ii) square disk, iii) prolate spheroid, iv) bisphere, and v) circular cylinder with hemispherical ends are also seen to lie between the proposed upper and lower bound correlation equations.

The aspect ratios (height-to-breadth) for these bodies are 2 or greater. The first two body shapes are the *thin* circular disk and *thin* square disk oriented such that the thickness is perpendicular to the gravity vector. This makes the body appear to have a very large aspect ratio.

The diffusive limit for these bodies lies in the small range: $3.34 < Nu^{\infty}_{\sqrt{A}} < 3.57$.

As in the previous comparison, all five bodies behave in a similar manner with respect to the Rayleigh number. Their heat transfer characteristics are similar; i.e., their Nusselt numbers lie between the proposed bounds at the lowest value of $Ra_{\sqrt{A}}$, and as $Ra_{\sqrt{A}}$ increases the values of $Nu_{\sqrt{A}}$ approach the upper bound as $Ra_{\sqrt{A}} \rightarrow 10^8$.

As discussed above the development of correlation equations of the data found in any one or two decades of $Ra_{\sqrt{A}}$ for any one of the body shapes would require different coefficients and exponents on $Ra_{\sqrt{A}}$. The developed correlation equations would obviously differ significantly from those developed from the data shown in Fig. 4.

All data shown in Figs. 3 and 4 are presented again in Fig. 5 along with the proposed upper and lower bound correlation equations to show more clearly the $Nu_{\sqrt{A}} - Ra_{\sqrt{A}}$ trends of the two sets of data. At the lowest values of $Ra_{\sqrt{A}}$ the two sets of data lie between the bounds and they are in close agreement, independent of the body shape and its orientation. This is due to the fact that the diffusive limit of the general equation governs natural convection heat transfer at the lowest values of the Rayleigh number.

We also observe that at the highest values of $Ra_{\sqrt{A}}$ the two sets of data separate and approach the upper and lower bounds respectively.

Other data from isothermal spheres and cubes in various orientations (Hassani, 1987 and Chamberlain, 1985) have also been compared with the proposed bounds. The observations made in Figs. 3 and 4 are fully confirmed. The $Nu_{\sqrt{A}} - Ra_{\sqrt{A}}$ trends for spheres and cubes are identical to those observed in Fig. 4, and therefore they will not be presented here.

All data including spheres, cubes in several orientations, and a short vertical circular cylinder with flat ends are presented in Fig. 6. All data (approximately 1200 data points) lie between the proposed upper and lower bounds. Most of the data lie several percent below the upper bound curve and follow it closely over six decades of $Ra_{\sqrt{A}}$.

All previously published correlation equations after conversion to $Nu_{\sqrt{A}} - Ra_{\sqrt{A}}$ data sets are found to also lie between the proposed bounds. The published correlation equations are usually limited to one or two

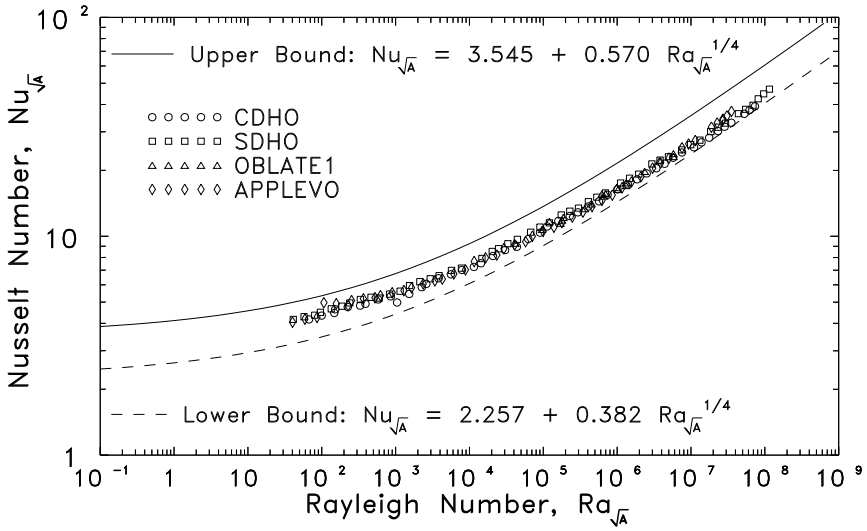

Figure 3: Comparison of Upper and Lower Bounds with Air Data of Hassani, 1987 for Thin Horizontal Bodies.

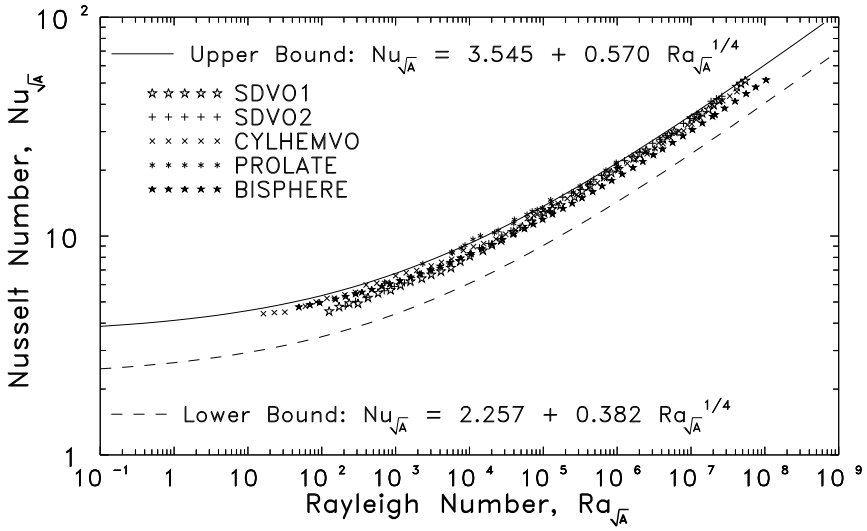

Figure 4: Comparison of Upper and Lower Bounds with Air Data of Hassani, 1987 for Vertical Bodies.

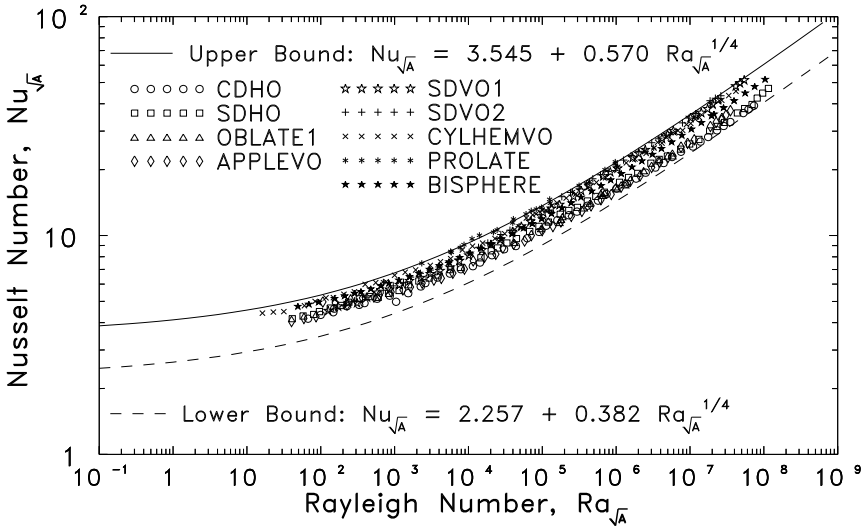

Figure 5: Comparison of Upper and Lower Bounds with Air Data for Hassani, 1987 for Horizontal and Vertical Bodies.

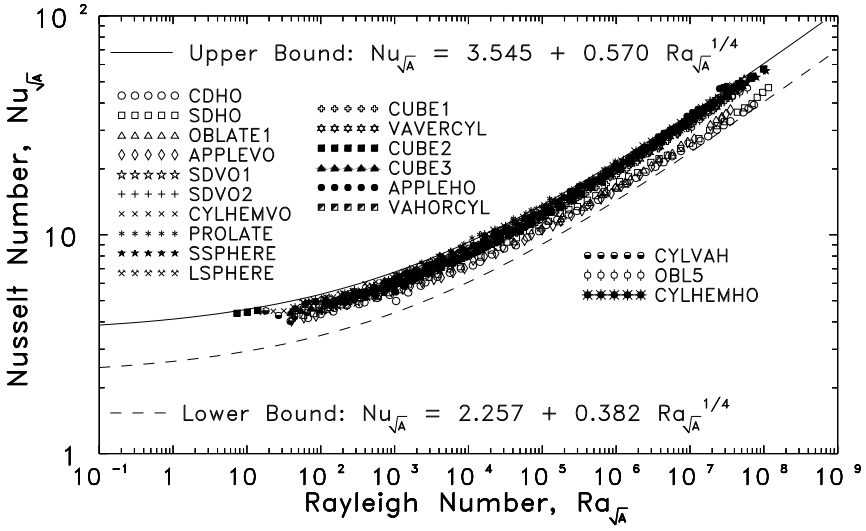

Figure 6: Comparison of Upper and Lower Bounds with Air Data of Hassani, 1987 for Spheres, Cubes and Circular Cylinders

decades of $Ra_{\sqrt{A}}$, and they have very different correlation coefficients and the Rayleigh number exponents are also different as discussed in the review section of this investigation.

SUMMARY AND CONCLUDING REMARKS

The proposed upper and lower bound correlation equations are developed from the general natural convection model (Yovanovich, 1987a, 1987b) which consists of the linear superposition of the diffusive and laminar boundary-layer limits. The characteristic body length is based on the square root of the total active heat transfer surface (Yovanovich, 1987a, 1987b).

The important geometric-flow parameter, the body-gravity function, was extended using empirical facts obtained from an extensive review of previously reported experimental, numerical and analytical results, to include natural convection from horizontal isothermal surfaces facing upward or downward.

The lower bound equation was developed from numerical data obtained for a horizontal, upward facing, isothermal circular disk imbedded in an *infinite* adiabatic plane. This corresponds to a circular disk with one side active which is cooled by a stagnant fluid of *semi-infinite* extent.

The upper bound equation was developed from analytical-numerical results for the shape factor for an isothermal, circular disk with the back surface insulated and the disk is placed in full-space. The body-gravity function was developed from the empirical observations of the natural convection cooling of disks and finite plates of arbitrary shape facing upward or placed in the vertical orientation.

All natural convection air data obtained from numerous body shapes having different aspect ratios and different orientations were observed to lie between the proposed bounds. The data are also found to follow two distinct trends depending on whether the body shapes are i) *thin* horizontal (low aspect ratio) or ii) they are *tall* vertical (high aspect ratio).

The proposed upper and lower bound correlation equations are valid over a wide range of the Rayleigh number and for all Prandtl numbers.

The proposed bounds resolve the apparent discrepancies between the previously published results and correlation equations which are restricted to one or two decades of the Rayleigh number because they do not include the diffusive limit which contributes significantly to the overall heat transfer rate, especially at lower values of the Rayleigh number.

The published correlation equations are also limited to one or two specific values of the Prandtl number whereas the proposed bounds include the effect of arbitrary Prandtl number.

ACKNOWLEDGMENTS

The authors acknowledge the support of the Natural Science and Engineering Research Council of Canada under grant A7445. K. Jafarpur wishes to acknowledge the financial support of Shiraz University, in Shiraz, Iran. MMY acknowledges the assistance of M.R. Sridhar in the preparation of the figures and the plots.

REFERENCES

Aihara, T., 1991, "Air Cooling Techniques By Natural Convection," *Advances In Cooling Techniques For Computers*, W. Aung, ed., Chapter 1, pp. 1-45.

Aihara, T., Yamada, Y., and Endo, S., 1972, "Free Convection Along the Downward-Facing Surface of a Heated Horizontal Plate," *Int. J. Heat Mass Transfer*, Vol. 15, pp. 2535-2549.

Al-Arabi, M., and El-Riedy, M.K., 1986, "Natural Convection Heat Transfer From Isothermal Horizontal Plates of Different Shapes," *Int. J. Heat Mass Transfer*, Vol. 16, pp. 1399-1404.

Al-Arabi, M., and Sakr, B., 1988, "Natural Convection Heat Transfer From Inclined Isothermal Plates," *Int. J. Heat Mass Transfer*, Vol. 31, No. 3, pp. 559-566.

Bahl, S., and Liburdy, J.A., 1989, "Measurement of the Three Dimensional Convective Heat Transfer Coefficient on Heated Disks," 1989 National Heat Transfer Conference, Philadelphia, Pennsylvania, Vol. 112, pp. 61-67.

Birkebak, R.C., and Abdulkadir, A., 1970, "Heat Transfer By Natural Convection From The Lower Side Of Finite Horizontal, Heated Surface," *Proceedings of Fourth Int. Heat Transfer Conference*, Heat Transfer 1970, V. Grigull and E. Hahne, eds., NC2.2, pp. 1-10.

Boberg, J.E., and Starrett, P.S., 1958, "Determination of Free Convection Heat Transfer Properties of Fluids," *Industrial And Engineering Chemistry*, Vol. 50, No. 5, pp. 807-810.

Bosworth, R.L.C., 1952, *Heat Transfer Phenomena*, John Wiley and Sons Inc., New York.

Chen, C.J., 1970, "Free Convection From a Two-Dimensional Finite Horizontal Plate," *J. of Heat Transfer*, Vol. 92, pp. 548-550.

Chow, Y.L., and Yovanovich, M.M., 1982, " The Shape Factor of the Capacitance of a Conductor," *Journal of Applied Physics*, Vol. 53, No. 12, pp. 8470-8475.

Clausing, A.M., and Berton, J.J., 1987, "An Experimental Investigation of Natural and Combined Convection From an Isothermal Horizontal Plate," *Proceedings of the ASME/JSME Thermal Engingeering Joint Conference*, P.J. Marto and I. Tanasawa, eds., ASME Publication, New York, Vol. 3, pp. 563-568.

Clausing, A.M., and Berton, J.J., 1989, "An Experimental Investigation of Natural Convection From an Isothermal Horizontal Plate," *J. Heat Transfer*, Vol. 111, pp. 904-908.

Clifton, J.V., and Chapman, J., 1969, "Natural-Convection on a Finite-Size Horizontal Plate," *Int. J. Heat Mass Transfer*, Vol. 12, pp. 1573-1584.

Fishenden, M., and Saunders, O.A., 1930, "The Calculation Of Convection Heat Transfer," *Engineering*, p. 177.

Fishenden, M., and Saunders, O.A., 1950, *An Introduction to Heat Transfer*, Oxford University Press, London, England.

Fujii, T., and Imura, H., 1972, Natural Convection Heat Transfer From a Plate With Arbitrary Inclination, *Int. J. Heat Mass Transfer*, Vol. 15, pp. 755-767.

Gebhart, B., 1971, *Heat Transfer*, 2nd ed., McGraw-Hill Book Company, New York.

Gill, W.N., Zeh, D.W., and Del Casal, E., 1965, "Free Convection on a Horizontal Plate," *ZAMP*, Vol. 16, pp. 539-541.

Goldstein, R.J., Sparrow, E.M., and Jones, D.C., 1973, "Natural Convection Mass Transfer Adjacent to Horizontal Plates," *Int. J. Heat Mass Transfer*, Vol. 16, pp. 1025-1035.

Goldstein, R.J., and Lau, K.E., 1983, "Laminar Natural Convection From a Horizontal Plate and the Influence of Plate-Edge Extensions," *J. Fluid Mechanics*, Vol. 129, pp. 55-75.

Gunaji, M.V., Pederson, R.J., and Leslie, I.H., 1990, "Numerical Study of Natural Convection Over a Finite Heated Disc," Fundamentals of Natural Convection *HTD*-Vol. 140, V.S. Arpaci and Y. Bayazitoglu, eds., ASME Publication, New York.

Hassan, K.E., and Mohamed, S.A., 1970, "Natural Convection From Isothermal Flat Surfaces," *Int. J. Heat Mass Transfer*, Vol. 13, pp. 1873-1886.

Hatfield, D.W., and Edwards, D.K., 1981, "Edge And Aspect Ratio Effects On Natural Convection From The Horizontal Heated Plate Facing Downwards," *Int. J. Heat Mass Transfer*, Vol. 24, No. 6, pp. 1019-1024.

Husar, R.B., and Sparrow, E.M., 1968, "Patterns Of Free Convection Flow Adjacent To Horizontal Heated Surfaces," *Int. J. Heat Mass Transfer*, Vol. 11, pp. 1206-1208.

Jakob, M., 1956, *Heat Transfer*, John Wiley and Sons Inc., New York.

Kraus, W., 1955, *Messung des Temperatur- und Geschwindigkeitsfeldes bei freier Konvektion*, Verlag G. Braun, Karlsruhe, Germany.

Kukulka, D.J., and Mollendorf, J.C., 1988, "An Experimental Study of Transient Transport Near a Heated Horizontal Surface," *Proceedings of the First World Conference on Experimental Heat Transfer*, Fluid Mechanics and Thermodynamics, Dubrovnik, Yugoslavia, R.K. Shah et al., eds., Elsevier Publishing, New York, pp. 185-192.

Levy, S., 1955, "Integral Methods In Natural Convection Flow," *J. Appl. Mech.*, Vol. 77, pp. 515-522.

Lewandowski, W.M., 1991, "Natural Convection Heat Transfer From Plates of Finite Dimensions," *Int. J. Heat Mass Transfer*, Vol. 34, No. 3, pp. 875-885.

Liburdy, J.A., Dorrah, R.L., and Bahl, S., 1987, "Experimental Investigation of Natural Convection From a Horizontal Disk," *Proceedings of the ASME/JSME Thermal Engingeering Joint Conference*, P.J. Marto and I. Tanasawa, eds., ASME Publication, New York, Vol. 3, pp. 605-611.

Lloyd, J.R., and Moran, W.R., 1974, "Natural Convection Adjacent to Horizontal Surface of Various Plan-

forms," *Journal of Heat Transfer*, Vol. 96, pp. 443-447.

McAdams, W.H., 1954, *Heat Transmission*, McGraw-Hill Book Company, New York.

Merkin, J.H., 1983, "Free Convection Above a Heated Horizontal Circular Disk," *ZAMP*, Vol. 34, pp. 596-608.

Merkin, J.H., 1985, "Free Convection Above a Uniformly Heat Circular Disk," *Int. J. Heat Mass Transfer*, Vol. 28, pp. 1157-1163.

Mikheyev, M., 1968, *Fundamentals Of Heat Transfer*, Peace Publishers, Moscow.

Miyamoto, M., Katoh, Y., Kurima, J., Kurihara, S., and Yamashita, K., 1985, "Free Convection Heat Transfer From Vertical And Horizontal Short Plates," *Int. J. Heat Mass Transfer*, Vol. 28, No. 9, pp. 1733-1745.

Patrick, M.A., Wragg, A.A., and Pargeter, D.M., 1977, "Mass Transfer By Free Convection During Electrolysis at Inclined Electrodes," *The Canadian J. of Chemical Engineering*, Vol. 55, pp. 432-438.

Pera, L., and Gebhart, B., 1973a, "Natural Convection Boundary Layer Flow Over Horizontal And Slightly Inclined Surfaces," *Int. J. Heat Mass Transfer*, Vol. 16, pp. 1131-1146.

Pera, L., and Gebhart, B., 1973b, "On The Stability Of Natural Convection Boundary Layer Flow Over Horizontal And Slightly Inclined Surfaces," *Int. J. Heat Mass Transfer*, Vol. 16, pp. 1147-1163.

Restrepo, F., and Glicksman, L.R., 1974, "The Effect Of Edge Conditions On Natural Convection From A Horizontal Plate," *Int. J. Heat Mass Transfer*, Vol. 17, pp. 135-142.

Rich, B.R., 1953, "An Investigation of Heat Transfer From an Inclined Flat Plate in Free Convection," *Int. J. Heat Mass Transfer*, Vol. 75, pp. 489-499.

Robinson, S.A., and Liburdy, J.A., 1987, "Prediction of the Natural Convection Heat Transfer From a Horizontal Heated Disk," *Journal of Heat Transfer*, Vol. 109, pp. 906-911.

Rotem, Z., 1970, "Free Convection From Heated, Horizontal Downward-Facing Plates," *ZAMP*, Vol. 21, pp. 472-475.

Rotem, Z., and Claassen, L., 1969a, "Natural Convection Above Unconfined Horizontal Surfaces," *J. Fluid Mech.*, Vol. 38, Part 1, pp. 173-192.

Rotem, Z., and Claassen, L., 1969b, "Free Convection Boundary-Layer Flow Over Horizontal Plates and Discs," *The Canadian J. of Chemical Engineering*, Vol. 47, pp. 461-468.

Sahraoui, M., Kaviany, M., and Marshall, H., 1990, "Natural Convection From Horizontal Disks and Rings," *Journal of Heat Transfer*, Vol. 112, pp. 110-116.

Saunders, O.A., Fishenden, M., and Mansion, H.D., 1935, "Some Measurements Of Convection By An Optical Method," *Engineering*, p. 483.

Singh, P. and Narayan, K.A., 1989, "Free Convection Heat Transfer Over a Horizontal Plate at Low Prandtl Number," *The Canadian J. of Chemical Engineering*, Vol. 67, pp. 507-512.

Singh, S.N., and Birkebak, R.C., 1969, "Laminar Free Convection From a Horizontal Infinite Strip Facing Downwards," *ZAMP*, Vol. 20, pp. 454-461.

Singh, S.N., Birkebak, R.C., and Drake Jr., R.M., 1969, "Laminar Free Convection Heat Transfer From Downward-Facing Horizontal Surfaces of Finite Dimensions," *Progress in Heat and Mass Transfer*, Vol. 2, pp. 87-98.

Stewartson, K., 1958, "On the Free Convection From a Horizontal Plate," *ZAMP*, Vol. 9a, pp. 276-282.

Suriano, F.J., and Yang, K.T., 1968, "Laminar Free Convection About Vertical and Horizontal Plates at Small and Moderate Grashof Numbers," *Int. J. Heat Mass Transfer*, Vol. 11, pp. 473-490.

Thomas, D.B., and Townsend, A.A., 1957, "Turbulent Convection Over A Heated Horizontal Surface," *J. Fluid Mech.*, Vol. 2, pp. 473-492.

Tsai, B.J., and Liburdy, J.A., 1986, "Numerical Prediction of Natural Convection From a Horizontal Disk," *Proceedings of the Eighth International Heat Transfer Conference*, C.L. Tien, eds., Hemisphere Publishing Corporation, New York, Vol. 2, pp. 441-446.

Wagner, C., 1956, "Discussion of: Integral Methods In

Natural Convection Flow by S. Levy," *J. Appl. Mech.*, pp. 320-321.

Weber, M.E., Astrauskas, P., and Petsalis, S., 1984, "Natural Convection Mass Transfer to Nonspherical Objects at High Rayleigh Number", *The Canadian Journal of Chemical Engineering*, Vol. 62, Feb. 1984, pp. 68-72.

Weise, R., 1935, "Wärmeübergang durch freie Konvektion an quadratischen Platten," *Forschung auf dem Gebiete des Ingenieurwesens*, 6.Bd., Heft 6, pp. 281-292.

Wragg, A.A., and Loomba, R.P., 1970, "Free Convection Flow Patterns At Horizontal Surfaces With Ionic Mass Transfer," *Int. J. Heat Mass Transfer*, Vol. 13, pp. 439-442.

Yousef, W.W., Tarasuk, J.D., and McKeen, W.J., 1982, "Free Convection Heat Transfer From Upward-Facing Isothermal Horizontal Surfaces," *J. of Heat Transfer*, Vol. 104, pp. 493-500.

Zakerullah, M.D., and Ackroyd, J.A.D., 1979, "Laminar Natural Convection Boundary Layers on Horizontal Disks," *Z. Agnew Math. Phy. [ZAMP]*, Vol. 30, pp. 427-435.