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Radiation Laws

Planck's Distribution Law

The relation for the spectral **blackbody** emissive power $E_{b\lambda}$ was developed by Planck (1901). The relation is known as **Planck's distribution law**, and it is expressed as

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5 \; [\exp(C_2/\lambda T) - 1]} \qquad \left[\frac{W}{m^2 \cdot \mu m}\right]$$

where T is the absolute temperature of the surface, λ is the wavelength of the radiation emitted by the surface. Also

$$C_1 = 2\pi h c_0^2 = 3.742 \times 10^8 \qquad \left[\frac{W \cdot \mu m^4}{m^2}\right]$$

and

$$C_2 = \frac{h c_0}{k} = 1.439 \times 10^4 \qquad [\mu m \cdot K]$$

where $h = 6.625 \times 10^{-34}\ J \cdot s$ is **Planck's constant** and $c_0 = 2.998 \times 10^8\ m/s$ is the speed of light in a vacuum, and $k = 1.3805 \times 10^{-23}\ J/K$ is **Boltzmann's constant**. This relation is valid for a surface in a *vacuum* or a *transparent* gas.

Wien's Displacement Law

$$(\lambda T)_{\text{max power}} = 2897.8 \qquad [\mu m \cdot K]$$

Stefan-Boltzmann Law of Radiation

The integration of the spectral blackbody emissive power $E_{b\lambda}$ over the entire wavelength spectrum gives the *total* blackbody emissive power E_b:

$$E_b(T) = \int_0^\infty E_{b\lambda}(T)\, d\lambda = \sigma\, T^4 \qquad \left[\frac{W}{m^2}\right]$$

where $\sigma = 5.67 \times 10^{-8}\ W/(m^2 \cdot K^4)$ is the **Stefan-Boltzmann constant**. The Stefan-Boltzmann Law of Radiation gives the total radiation emitted by a blackbody at all wavelengths from $\lambda = 0$ to $\lambda = \infty$ at absolute temperature T.

Actual Radiation

Substances and surfaces of engineering interest have radiative characteristics which are different from the black-body radiation. Since E_b and $E_{b\lambda}$ are the maximum emissive powers for any given temperature, actual surfaces emit and absorb radiation less readily and they are called nonblack. The emissive power of a nonblack surface, at temperature T, radiating to a hemispherical surface above it is

$$E = \epsilon E_b = \epsilon\, \sigma\, T^4 \qquad \left[\frac{W}{m^2}\right]$$

where ϵ, called the **total hemispherical emittance**, is a function of the material, the condition of the surface, and the temperature of the surface.

Absorptivity, Reflectivity, Transmissivity

When radiant energy is incident on a surface, portions are absorbed, reflected, or transmitted through the material. From the first law of thermodynamics we get the relation:

$$\alpha + \rho + \tau = 1$$

where

$$\begin{array}{lcccl} \text{absorptivity :} & \alpha & = & \dfrac{\text{absorbed radiation}}{\text{incident radiation}}, & 0 \leq \alpha \leq 1 \\[2ex] \text{reflectivity :} & \rho & = & \dfrac{\text{reflected radiation}}{\text{incident radiation}}, & 0 \leq \rho \leq 1 \\[2ex] \text{transmissivity :} & \tau & = & \dfrac{\text{transmitted radiation}}{\text{incident radiation}}, & 0 \leq \tau \leq 1 \end{array}$$

Solids generaly do not transmit radiation unless the material is very thin. Metals absorb radiation within a fraction of a micrometer and electrical conductors within a fraction of a millimeter. Substances such a liquids and glass absorb most of the radiation within a millimeter. Solids and liquids therefore are generally assumed to be opaque with $\tau = 0$, therefore

$$\alpha + \rho = 1$$

This important relation allows one to determine both the absorptivity and reflectivity of an opaque surface by measuring either of these properties.

Most elementary gases such as hydrogen, oxygen and nitrogen (and mixtures of these such as air) have $\tau \approx 1$, and therefore $\rho = 0$ and $\alpha = 0$. For this reason radiation through air is generally estimated using the relationships for radiation through a vacuum. Gases with a more complex structure, such as water vapor and carbon dioxide, generally absorb and emit radiation as well as transmit radiation.

Specular and Diffuse Reflections

The reflection of radiation from a solid surface may be of a specular or diffuse nature. Specular reflection occurs at a surface which is very smooth and clean, such as a mirror, and an image of the radiation source is projected. The optical laws apply and the angle of reflection is equal to the angle of incidence. Diffuse reflection occurs when the surface is rough and dirty, and there is no preferential direction of reflection. No real surface is perfectly specular or diffuse, however, it is often useful to approximate surfaces as specular or diffuse.

Kirchhoff's Law

A useful relation between emissivity and absorptivity of any opaque surface can be developed directly from thermodynamic considerations. **Kirchhof's Law** states that for any surface in *thermodynamic equilibrium* with its surroundings, the monochromatic emissivity equals the monochromatic absorptivity. Therefore,

$$\epsilon_\lambda = \alpha_\lambda$$

If the surface is gray or the incident radiation is from a black surface at the same temperature, then the relation of the total values is also true

$$\epsilon = \alpha$$

Gray Surface

If a surface behaves such that ϵ_λ can be considered to be independent of the wavelength λ and it is equal to the total hemispherical emissivity ϵ, it is then said to be a gray surface.

Radiation View Factor

When radiation leaves a black convex surface whose area is A_1 at absolute temperature T_1, a certain fraction F_{12} will be absorbed by a second convex surface whose area is A_2 at absolute temperature $T_2 < T_1$. The **radiation**

view factor is given by the following relation:

$$A_1\,F_{12} = A_2\,F_{21} = \int\int_{A_1}\int\int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi r^2}\,dA_1\,dA_2$$

where r is the radial distance between the centroids of the arbitrary differential areas dA_1 and dA_2 which are located in the surfaces A_1 and A_2 respectively. The angle β_1 is subtended by the radius r and the outward-directed normal to the differential area dA_1 at its centroid. The angle β_2 is defined in a similar manner. In the previous relation F_{21} is the fraction of radiation which leaves surface A_2 and is intercepted by surface A_1. The view factors are dimensionless radiation parameters where

$$0 \le F_{12} \le 1 \qquad \text{and} \qquad 0 \le F_{21} \le 1$$

Reciprocity Relation

The relation

$$A_1\,F_{12} = A_2\,F_{21}$$

is called the reciprocity relation which is valid for any two convex surfaces. The view factors for a simple system consisting of two isothermal surfaces the view factors have the relations:

$$F_{11} + F_{12} = 1 \qquad \text{and} \qquad F_{21} + F_{22} = 1$$

If the surfaces are convex or flat $F_{11} = F_{22} = 0$. If the surfaces are concave, then $0 < F_{11} < 1$ and $0 < F_{22} < 1$.

The radiation view factors depend on the geometry of the surfaces and their spatial relation to each other. The calculations of the view factors by the above relation are usually difficult to do. There are simple techniques available to determine the view factors for certain axisymmetric systems and some infinitely long two-dimensional systems.

Radiant Exchange Between Two Isothermal Black Surfaces

The net radiant exchange between two isothermal black surfaces is given by

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_{12}}$$

where $E_{b1} = \sigma\,T_1^4$ and $E_{b2} = \sigma\,T_2^4$ are the blackbody radiative *nodes* and $R_{12} = 1/A_1\,F_{12} = 1/A_2\,F_{21} = R_{21}$ is the spatial radiative resistance between the two surfaces. The units of R_{12} and R_{21} are m^{-2}.

The previous relation can be expressed in the following form:

$$\dot{Q}_{12} = \sigma\, A_1\, F_{12}\left[T_1^4 - T_2^4\right]$$

which clearly reveals the non-linear nature of radiative heat exchange. The radiative exchange between two isothermal black surfaces can be represented by a simple thermal circuit consisting of two nodes: E_{b1}, E_{b2} separated by a single radiative resistance R_{12}. The throughput is $\dot{Q}_{12}$.

Radiant Exchange Between Two Isothermal Gray Surfaces

The net radiant exchange between two isothermal gray surfaces whose areas are A_1, A_2 respectively and whose emissivities are ϵ_1, ϵ_2 respectively is given by

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_{\text{total}}}$$

where the total radiative resistance now consists of three resistances in series:

$$R_{\text{total}} = R_{s1} + R_{12} + R_{s2}$$

The gray surface resistances are given by:

$$R_{s1} = \frac{1-\epsilon_1}{A_1\,\epsilon_1} \qquad \text{and} \qquad R_{s2} = \frac{1-\epsilon_2}{A_2\,\epsilon_2}$$

with $0 \leq \epsilon_1 \leq 1$ and $0 \leq \epsilon_2 \leq 1$. When $\epsilon_1 = 1$ and $\epsilon_2 = 1$ the gray surface relation goes to the blackbody relation given above. The equivalent thermal circuit consists of two blackbody nodes E_{b1}, E_{b2} and two internal nodes denoted J_1 and J_2 which are called the *radiosity*. The units of radiosity J are identical to the units of E_b. The surface resistance R_{s1} connects the two nodes E_{b1} and J_1, and the surface resistance R_{s2} connects the two nodes E_{b2} and J_2. The two radiosity nodes are connected by the spatial resistance R_{12}. The throughput is the net radiant exchange $\dot{Q}_{12}$.

The general gray surface radiant exchange expression gives the following expressions for two infinite parallel planes, two long concentric circular cylinders, and two concentric spheres.

Two Infinite Parallel Planes

In this case $A_1 = A_2 = A$ and $F_{12} = 1$. Therefore we get

$$\dot{Q}_{12} = \frac{A\,(E_{b1} - E_{b2})}{\dfrac{1}{\epsilon_1} + \dfrac{1}{\epsilon_2} - 1}$$

Long Concentric Gray Circular Cylinders

In this case the inner isothermal cylinder at absolute temperature T_1 and whose area is $A_1 = 2\,\pi\, r_1\, L$ is placed inside the outer cylinder at absolute temperature $T_2 < T_1$ and whose area is $A_2 = 2\,\pi\, r_2\, L$. Both cylinder have the same length $L >> r_2 > r_1$. The emissivities of the two cylinders are ϵ_1 and ϵ_2 respectively.

The net radiative exchange is given by

$$\dot{Q}_{12} = \frac{2\,\pi\, r_1\, L\, (E_{b1} - E_{b2})}{\dfrac{1}{\epsilon_1} + \dfrac{r_1}{r_2}\left(\dfrac{1}{\epsilon_2} - 1\right)}$$

Concentric Gray Spheres

In this case the inner isothermal sphere at absolute temperature T_1 and whose area is $A_1 = 4\,\pi\, r_1^2$ is placed inside the outer sphere at absolute temperature $T_2 < T_1$ and whose area is $A_2 = 4\,\pi\, r_2^2$. The emissivities of the two spheres are ϵ_1 and ϵ_2 respectively.

The net radiative exchange is given by

$$\dot{Q}_{12} = \frac{4\,\pi\, r_1^2\, (E_{b1} - E_{b2})}{\dfrac{1}{\epsilon_1} + \dfrac{r_1^2}{r_2^2}\left(\dfrac{1}{\epsilon_2} - 1\right)}$$

The expression for two gray spheres gives a simple relation when the outer sphere is much larger than the inner sphere, i.e. $r_2 >> r_1$. For this case the previous expression goes to

$$\dot{Q}_{12} = \epsilon_1\, 4\,\pi\, r_1^2\, (E_{b1} - E_{b2})$$

which is independent of the emissivity and surface area of the larger sphere. The emissivity and the surface area of the smaller sphere control the radiative exchange between the two surfaces.