UNIVERSITY OF WATERLOO
DEPARTMENT OF MECHANICAL ENGINEERING
ME 353 HEAT TRANSFER 1

October 22, 1996 **Time: 5-7 PM**
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Two-hour Closed Book Mid-term Examination. You are allowed one crib sheet (both sides). All questions are of equal value. State clearly all assumptions made and label all sketches and thermal circuits clearly.

PROBLEM 1

[10]

A composite wall consists of two materials of thermal conductivities k_1 and k_2 respectively. The walls are in mechanical contact. The left wall of thickness L_1 has incident heat flux of magnitude q_i over its left boundary. The right wall of thickness L_2 looses heat by convection and radiation to the surrounding fluid at temperature T_∞ and the surroundings at uniform temperature T_{sur}. The fluid and surrounding temperatures are equal. The convection heat transfer coefficient is h_{conv} and the equivalent radiation heat transfer coeffcient h_{rad} are uniform over the entire boundary. The contact conductance h_c at the common interface is uniform. The heat transfer through the composite wall is steady.

For this problem let the temperature at the left face of the composite wall be denoted T_1, and the temperatures at the interface be denoted T_2 and T_2' where $T_2 > T_2'$. Let the temperature at the right face of the composite wall be denoted T_3.

For this problem denote the thermal resistances as: $R_1, R_2, R_c, R_{conv}, R_{rad}$.

(a) Sketch the system labeling clearly all geometric and thermophysical parameters and temperatures.

(b) Sketch the equivalent thermal circuit showing all thermal resistances and temperature nodes.

(c) Obtain the expression for the total thermal resistance of the system.

(d) Obtain expressions for $T_1 - T_\infty$, $T_2 - T_\infty$, and $T_3 - T_\infty$.

(e) Use local coordinates $0 \leq x \leq L_1$ and the resistance method to obtain the relationship for the dimensionless temperature distribution $\dfrac{T_1 - T(x)}{T_1 - T_\infty}$ within the first wall.

(f) Use local coordinates $0 \leq x \leq L_2$ and the resistance method to obtain the relationship for the dimensionless temperature distribution $\dfrac{T(x) - T_3}{T_1 - T_\infty}$ within the second wall.

PROBLEM 2

[10]

A *thin* electrical heater is inserted between a long circular rod of radius r_1 and a concentric tube having radii r_1 and r_2. The thermal conductivities of the rod and tube are k_1 and k_2 respectively. The outer surface of the tube is subjected to convection cooling by a fluid whose temperature is T_f and the heat transfer coefficient is h. The thermal contact resistance is negligible because a high conductivity thermal grease is placed between the heater surfaces and the rod and tube surfaces.

(a) Sketch the system showing the radii, thermal conductivities, film coefficient, and fluid temperature.

(b) Provide a thermal circuit showing the thermal resistances, the temperature nodes, and the direction of the heat flow rate.

(c) Calculate the electrical power per unit length of the cylinders (W/m) that is required to maintain the outer surface of the tube at 5 ^0C given the following system parameters: $r_1 = 20\ mm, r_2 = 40\ mm, k_1 = 0.15\ W/m \cdot K, k_2 = 1.5\ W/m \cdot K, h = 50\ W/m^2 \cdot K, T_f = -15\ ^0C$.

(d) What is the temperature at the center-line of the rod? Use an energy balance at the rod boundary to support your answer.

PROBLEM 3

[10]

A Pentium CPU produces so much heat $Q_{Pentium} = 15\ W$ that it overheats and frequently fails without the addition of a heat sink which is frequently placed in *mechanical* contact with its case surface.
A thermal design engineer proposes using a heat sink having the following characteristics: base dimensions are 40 mm by 40 mm, base thickness is 3 mm with 50 circular pin fins of length $L_p = 50\ mm$ and diameter $D_p = 1.5\ mm$. The fins are integral with the base and the thermal conductivity of the base and fins is $k = 100\ W/m \cdot K$. The heat transfer coefficient over the bare base surface and the sides and ends of the pin fins is estimated to be $h = 120\ W/m^2 \cdot K$. To reduce the thermal contact resistance between the Pentium case and the heat sink base, a thin layer of thermal grease is placed at the interface. It is estimated that the corresponding contact conductance is $h_c = 500\ W/m^2 \cdot K$.

The Pentium is designed to operate in an environment where the air temperature is $T_f = 35\ ^0C$.

(a) Compute the fin parameters: $Bi = (hD_p)/2k$ and mL_p before you begin your analysis. What do you conclude from the magnitude of these two parameters?

(b) Your solution must show the thermal circuit, with all thermal resistances and temperature nodes labelled. Express the total thermal resistance of the system symbollically showing the component resistances. Neglect convection from the *edges* of the heat sink base because the heat transfer coefficient is much smaller at this surface.

(c) If the maximum case temperature is limited to $T_{case} = 95\ ^0C$, compute the heat transfer rate $Q_{Heat\ Sink}$ through the heat sink. Is this a good design?