UNIVERSITY OF WATERLOO
DEPARTMENT OF MECHANICAL ENGINEERING
ME 353 HEAT TRANSFER 1

October 25, 1991 **Time: 4:30-6:30 p.m.**

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Two-hour Closed Book Mid-term Examination. You are allowed one crib sheet (both sides) and the solutions to partial differential equations. All questions are of equal value. State clearly all assumptions made and label all sketches and thermal circuits clearly.

PROBLEM 1

[10]

A hollow spherical shell $r_1 \leq r \leq r_2$ whose thermal conductivity k is constant is subjected to a steady, uniform heat flux q_i'' at its inner boundary r_1 while its outer boundary is convectively cooled by a fluid whose temperature is T_f. The convection coefficient at the outer surface h_0 is considered to be constant and uniform over the entire boundary. The outer heat transfer coefficient h_0 accounts for the radiation heat transfer from the outer surface to the surroundings, i.e., h_{rad} as well as the natural convection heat transfer from the outer surface to the surrounding stagnant fluid, i.e., h_{conv}. Therefore, $h_0 = h_{rad} + h_{conv}$.

a) By means of heat balances over appropriate differential control volumes derive the governing differential equation and the two boundary conditions.

b) Obtain the solution (temperature distribution within the spherical shell), and show that the dimensionless temperature defined as

$$\phi(r) = \frac{T(r) - T_f}{q'' r_1 / k}$$

is a function of the Biot number $Bi = h_0 r_2 / k$ and the radii ratio r_1/r_2.

c) By means of the thermal resistance concept obtain the solution for $\phi(r)$ directly.

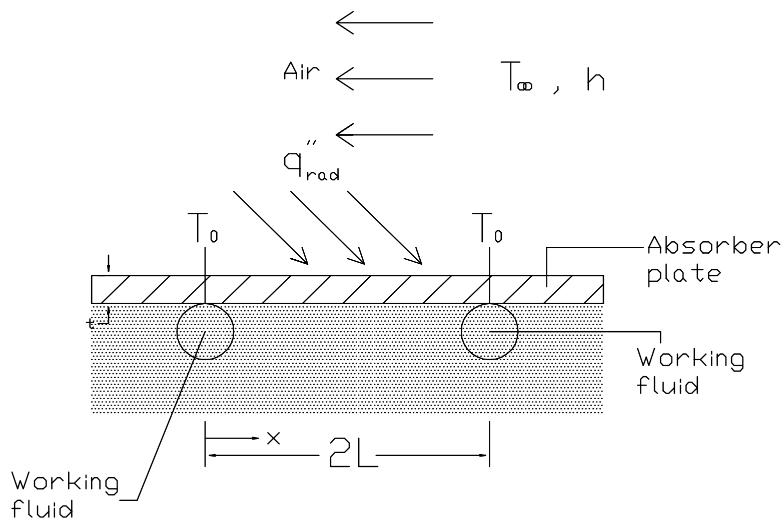
d) Determine the heat transfer rate per unit area at the outer boundary using three methods:

i)	Fourier's rate equation	$\mathbf{q}'' = -k \ \nabla T$
ii)	Newton's cooling law	$\mathbf{q}'' = h \ (T_s - T_f)$
iii)	Overall energy balance	

PROBLEM 2

[10]

Copper tubing is joined to a solar collector plate of thickness t, and the working fluid maintains the temperature of the plate above the tubes at T_o. There is a uniform net radiation heat flux q''_{rad} to the top surface of the plate, while the bottom surface is well insulated. The top surface is also exposed to a fluid at T_∞ that provides a uniform convection coefficient h.



Figure

(a) Derive by means of a heat balance over an appropriate differential control volume the governing differential equation for the temperature distribution $T(x)$ in the plate.

(b) Obtain a solution to the differential equation for appropriate boundary conditions.

PROBLEM 3

[10]

A pipeline, used for the transport of crude oil, is buried in the earth such that its centerline is a distance of 1.5 m below the surface. The pipe has an outer diameter of 0.5 m and is insulated with a layer of cellular glass 100 mm thick.

(a) Determine the shape factor per unit length of the pipe insulation system and the thermal resistance of the soil given the thermal conductivity of soil, $k_{soil} = 0.52 W/mK$.

(b) Estimate the heat loss per unit length of pipe under conditions for which heat oil at $120°C$ flows through the pipe and the surface of the earth is at a temperature of $0°C$. Let the thermal conductivity of the insulation be 0.069 W/mK and neglect the temperature drop across the pipe wall.

Included in the exam package was Table 4.1 of Conduction Shape Factors from Incropera and DeWitt, 3rd ed.