UNIVERSITY OF WATERLOO
DEPARTMENT OF MECHANICAL ENGINEERING
ME 353 HEAT TRANSFER 1

October 26, 1990 **Time: 7-9 p.m.**
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Two-hour Closed Book Mid-term Examination. You are allowed one crib sheet (both sides) and the solutions to partial differential equations. All questions are of equal value. State clearly all assumptions made and label all sketches and thermal circuits clearly.

PROBLEM 1

[10]

Consider the compound pipe length L shown in Figure 1 where a, b, c, k_1, k_2, k_3, h_1, h_2, T_{f1}, T_{f2} ($T_{f1} > T_{f2}$) are known quantities. The outer heat transfer coefficient h_2 accounts for the radiation heat transfer from the outer surface to the surroundings, i.e., h_{rad} as well as the natrual convection heat transfer from the outer surface to the surrounding stagnant fluid, i.e., h_{conv}. Therefore, $h_2 = h_{rad} + h_{conv}$. Assume that the fluid temperature T_{f2} and the surrounding temperature T_{sur} are equal. Steady-state conditions prevail, there are no heat sources present and <u>all</u> interfaces are assumed to be *perfect*.

a) Write the thermal resistance expressions for the three solid components and the two film components.

b) For the <u>lower</u> bound on the total resistance of the system (parallel isotherms model), depict the thermal circuit of the system labelling <u>all</u> component resistances. Write the expression for R_l in symbolic form in terms of the component resistances.

c) For the <u>upper</u> bound on the total resistance of the system (parallel adiabats model),depict the thermal circuit, labelling <u>all</u> components. Write the expression for R_u in symbolic form in terms of the component resistances.

d) Compute R_l, R_u and $R_{approx.} = (R_l + R_u)/2$ for the following conditions:

a = 5 cm
b = 15 cm
c = 20 cm
L= 1 m
$k_1 = 17$ W/mK $\quad h_1 = 120$ W/m^2K
$k_2 = 90$ W/mK $\quad h_2 = 11.5$ W/m^2K
$k_3 = 2$ W/mK

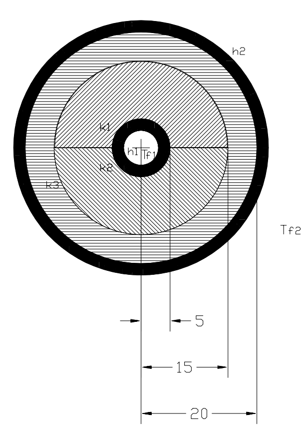

Figure 1

PROBLEM 2

[10]

The slab shown in Figure 2 has uniformly distributed heat sources of constant magnitude q [W/m^3], thermal conductivity k[W/mK] and has thickness D. The left-hand face is separated by a vacuum from a high temperature source which supplies energy at the net rate of q_0'' [W/m^2] to the left-hand face by radiation. The slab is in contact with stationary fluid whose undisturbed temperature is T_f [K] and the surface coefficient of heat transfer is known to be h [W/m^2 K]. The heat transfer through the slab is steady-state.

a) By means of a heat balance on a differential control volume within the slab derive the governing differential equation; and by means of heat balances on differential control volumes at the boundaries derive the appropriate boundary conditions.

b) Find the temperature distribution within the slab as a function of x.

c) Determine the heat transfer rate per unit area at the right-hand face (x = D) using three methods:

i)	Fourier's rate equation	$\mathbf{q}'' = -k\nabla T$
ii)	Newton's cooling law	$\mathbf{q}'' = h(T_s - T_f)$
iii)	Overall energy balance	

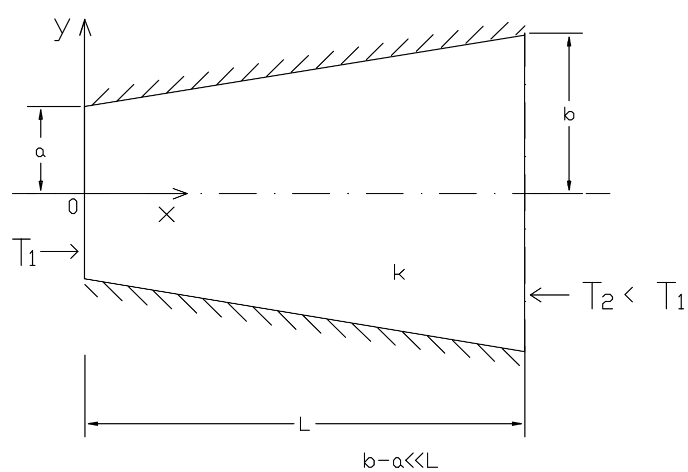

Figure 2

PROBLEM 3

[10]

Consider the trapezodial-shaped conductor shown in Figure 3 in which there is no heat transfer in the z-direction. The top and bottom surfaces are *perfectly* insulated while the left-hand face is maintained at a uniform temperature T_1 and the right-hand face is held at another uniform temperature T_2 where $T_1 > T_2$. The dimensions of the conductor are shown in Figure 3. The thermal conductivity is k; there are not heat sources present, and heat transfer is steady.

a) Beginning with the resistance of a differential control volume, obtain the expression for the steady-state conduction shape factor of the conductor per unit width.

b) Determine the steady-state temperature distribution $T(x)$ using Fourier's law of conduction at an arbitrary plane.

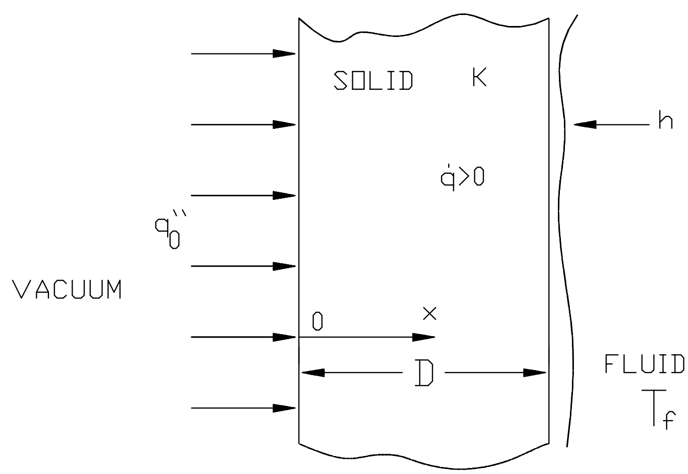

Figure 3