UNIVERSITY OF WATERLOO
DEPARTMENT OF MECHANICAL ENGINEERING
ME 353 HEAT TRANSFER 1

December 8, 1997 **Time: 2-5 P.M.**
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Three-hour Closed Book Final Examination. Two crib sheets (both sides) are permitted. Calculator is allowed. All questions are of equal value. State clearly and justify all assumptions made and label all sketches and thermal circuits clearly. Allocate your time wisely and do not spend disproportionate time on any single problem. Good luck.

PROBLEM 1

[20]

Consider a long, hollow cylinder of thermal conductivity k with inner and outer radii of r_i and r_o, respectively. The outer surface experiences a uniform incident heat flux q_o while at the inner surface there is convective heat transfer through a uniform heat transfer coefficient h_i into a fluid whose temperature is T_{fi}. Let the unknown temperatures at the inner and outer surfaces be denoted as T_1 and T_2 respectively.

(a) Sketch the equivalent thermal circuit for the system, showing clearly all thermal nodes, thermal resistors and the heat transfer rate Q through the system.

(b) Obtain the relation between the inner surface temperature T_1 and the given system parameters using thermal resistances.

(c) Obtain the relation between the outer surface temperature T_2 and the given system parameters using thermal resistances.

(d) Beginning with the appropriate form of the heat diffusion equation, derive an expression for the temperature distribution, $T(r)$, in terms of r_i, r_o, k, h_i, T_{fi}, and q_o.

(e) Compute the surface temperatures T_1, T_2 for the thermophysical parameters: $r_i = 10\ mm, r_o = 14\ mm, k = 15\ W/m \cdot K, h_i = 40\ W/m^2 \cdot K, T_{fi} = 300\ K, q_o = 2000\ W/m^2$. Justify the observed small difference between the calculated values.

PROBLEM 2

[20]

When steel plates are *thinned* by a rolling process, periodic reheating is required. Plain carbon steel plate 8 cm thick, initially at 440^0C, is reheated to a minimum centerline temperature of 520^0C in a furnace maintained at 600^0C. The combined convective and radiative heat transfer coefficient is estimated to be 200 $W/m^2 \cdot K$ and it is uniform over both surfaces of the plate. The thermal conductivity is $k = 40\ W/m \cdot K$ and the thermal diffusivity is $\alpha = 8.0 \times 10^{-6}\ m^2/s$.

(a) Sketch the system, showing clearly the expected temperature distribution.

(b) Find the time in seconds for the heating process using the relationships given below.

(c) Use the lumped capacitance model to estimate the time for the heating process.

(d) Determine the percent difference between the heating times found in parts (b) and (c). Comment on the observed difference in the calculated times.

The first term approximation for the heating or cooling of a plate is given below.

$$\frac{\theta}{\theta_i} = A_1 e^{-\delta_1^2 Fo} \cos(\delta_1 \zeta)$$

where $\zeta = x/L, Fo = \alpha t/L^2$ and

$$A_1 = \frac{2 \sin \delta_1}{\delta_1 + \sin \delta_1 \cos \delta_1}, \qquad \delta_1 = \frac{\pi/2}{\left[1 + \left(\frac{\pi/2}{\sqrt{Bi}}\right)^n\right]^{1/n}}$$

with $Bi = hL/k$ and $n = 2.139$.

PROBLEM 3

[20]

Water flows with a bulk temperature of $T_b = 30^0C$ at a mean velocity of $U = 1\ m/s$ in a tube with an inner diameter of $D = 2.5\ cm$. The surface temperature is maintained at $T_s = 50^0C$. The water properties are: $\mu = 0.798 \times 10^{-3}\ [kg/m \cdot s]\,, \nu = 0.801 \times 10^{-6}\ [m^2/s]\,, k = 0.615\ [W/m \cdot K]\,, Pr = 5.42$.
The dynamic viscosity of the water evaluated at the surface temperature is $\mu_s = 0.547 \times 10^{-3}\ [kg/m \cdot s]$.

(a) Compute the Reynolds number, Re_D, and determine whether the flow is laminar or turbulent.

(b) Compute the Nusselt number, Nu_D, and the heat transfer coefficient, h. Use the recommended correlation equation of Petukov:

$$Nu_D = \frac{(f/8)\, Re_D Pr}{1.07 + 12.7\sqrt{(f/8)}\,(Pr^{2/3} - 1)} \left(\frac{\mu}{\mu_s}\right)^n$$

where the friction factor is given by

$$f = (1.82 \log_{10} Re_D - 1.64)^{-2}$$

and the coefficient $n = 0.11$ for a heating process.

(c) Compute and compare the heat transfer coefficient using the older, simpler correlation equation of Dittus-Boelter:

$$Nu_D = 0.023 Re_D^{0.8} Pr^n$$

where $n = 0.4$ for a heating process.

(d) Compute the heat transfer rate to the water given the relation:

$$Q = \dot{m} c_p (T_s - T_{bi}) \left[1 - \exp\left(-(hPL)/(\dot{m} c_p)\right)\right]$$

for the inlet temperature $T_{bi} = 20^0C$, a tube length $L = 3\ m$, and the heat transfer coefficient found in part (b).

PROBLEM 4

[20]

A system consists of two concentric isothermal spheres of inner and outer diameters: $D_i = 70\ mm, D_o = 80\ mm$ respectively. The annular gap formed by the spheres is filled with dry air at one atmosphere pressure. The inner sphere contains electronic devices which dissipate heat which passes across the air gap between the spheres whose temperatures are measured to be $T_i = 400\ K$ and $T_o = 300\ K$ respectively. The emissivities of the inner and outer spherical surfaces are estimated to be $\epsilon_i = 0.9, \epsilon_o = 0.1$ respectively.

The thermophysical properties of dry air at one atmosphere and $T_{\text{film}} = 350K$ are:
$\rho = 0.9950\ [kg/m^3], c_p = 1.009\ [kJ/kg \cdot K], \mu = 2.08 \times 10^{-5}\ [N \cdot s/m^2], \nu = 2.092 \times 10^{-5}\ [m^2/s], \alpha = 2.99 \times 10^{-5}\ [m^2/s], k_f = 0.030\ [W/m \cdot K], Pr = 0.700$

(a) Determine the conductive heat transfer rate when convective and radiative heat transfer rates are neglected.

(b) Determine the convective heat transfer rate using the correlation equation given below.

(c) Determine the radiative heat transfer rate.

(d) Show the radiative circuit labelling all radiative nodes and radiative resistors.

(e) Show the thermal resistance circuit of the system labelling clearly all thermal nodes, thermal resistances, and heat transfer rates.

The appropriate correlation equation is

$$Nu_{D_i} = 0.74 \left[\frac{Pr}{0.861 + Pr}\right]^{1/4} \left[1 + (D_i/D_o)^{7/5}\right]^{-5/4} Ra_{D_i}^{1/4}$$

where the Rayleigh number is defined as

$$Ra_{D_i} = \frac{g\beta(T_i - T_o)D_i^3}{\alpha\nu}$$

PROBLEM 5

[20]

A diffuse, gray radiation shield of 60 mm diameter and emissivities of $\epsilon_{2,i} = 0.01$ and $\epsilon_{2,o} = 0.1$ on the inner and outer surfaces, respectively, is concentric with a long tube transporting a hot process fluid. The tube surface is black with a diameter of 20 mm. The region interior to the shield is evacuated. The exterior surface of the shield is exposed to a large room whose walls are at 17^0C and experiences convection with air at 27^0C and a convection heat transfer coefficient of 10 $W/m^2 \cdot K$.

(a) Sketch and label clearly the radiation network inside the shield, and the thermal resistance circuit outside the shield. The shield is very thin and has high thermal conductivity, therefore its thermal resistance can be neglected.

(b) Determine the operating temperature for the inner tube if the shield temperature is maintained at 42^0C.

Some Heat Transfer Relations

Conduction, Convection and Radiation

Fourier Law of Conduction
$Q = -k\ A\ \nabla T$
Newton Law of Cooling
$Q = h\ A(T_{\text{wall}} - T_{\text{fluid}})$
Stefan-Boltzmann Law of Radiation for Black Bodies
$Q = \sigma A_1(T_1^4 - T_2^4)$

Thermal Resistances

Thermal resistance is generally defined as $R \equiv (T_1 - T_2)/Q$. The units are K/W.
Conduction Resistances

Plane wall: $R = \dfrac{L}{kA}$

Cylindrical shell: $R = \dfrac{\ln(b/a)}{2\pi Lk}$

Spherical shell: $R = (1/a - 1/b)/(4\pi k)$

Fins: $R = 1/\left[\sqrt{hPkA}\ \tanh(mL)\right], \qquad m = \sqrt{\frac{hP}{kA}}$

Fluid or Film Resistance

$R = 1/(hA)$

Radiation Resistances

Gray Surface Resistance: $R = \dfrac{(1-\epsilon)}{A\epsilon}$

Spatial Resistance: $R = \dfrac{1}{A_1 F_{12}} = \dfrac{1}{A_2 F_{21}}$

Notes: Units of radiation resistances are $1/m^2$. F_{12} is the view factor between two surfaces: A_1 and A_2 is dimensionless and its range is $0 \leq F_{12} \leq 1$. The surface emissivity ϵ is a complex radiation parameter which is determined experimentally for real (grey) surfaces. It is dimensionless and its range is $0 \leq \epsilon \leq 1$. Smooth, highly polished metals such as aluminum have values as low as $\epsilon \approx 0.01 - 0.1$. Very rough, oxidized surfaces have values as high as $\epsilon \approx 0.8 - 0.95$. Black bodies are ideal bodies for which $\epsilon = 1$.

The total radiation resistance of a two surface enclosure which is bounded by two gray, diffuse, isothermal surfaces is given by:

$$R_{\text{total}} = \frac{(1-\epsilon_1)}{A_1 \epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{(1-\epsilon_2)}{A_2 \epsilon_2}$$

The radiative heat transfer rate between the two surfaces is given by

$$Q = \frac{(E_{b1} - E_{b2})}{R_{\text{radiation, total}}} = \frac{\sigma(T_1^4 - T_2^4)}{R_{\text{radiation, total}}}$$