UNIVERSITY OF WATERLOO
Department of Mechanical Engineering

ME 303 Advanced Engineering Mathematics

Project #1

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Project due date is Friday, June 4 at 9:30 AM

The following linear, second-order, nonhomogeneous partial diffferential equation (**PDE**) was derived using circular cylinder coordinates:

$$\frac{1}{r}\frac{\partial}{dr}\left(r\frac{\partial T}{\partial r}\right) + \frac{S}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, \qquad t > 0, \qquad 0 \leq r \leq a$$

The dependent variable $T(r,t)$ is the temperature whose unit is K. The independent space and time variables are: r, t whose units are m and s respectively. The other thermophysical parameters appearing in the above *parabolic type* **PDE** are: S which represents the uniformly distributed volumetric heat sources; k which is the thermal conductivity, and α which is the thermal diffusivity. These thermophysical parameters are assumed to be constants. The units of S and k are W/m^3 and $W/m \cdot K$ respectively.

The two boundary conditions **BCs** and the initial condition **IC** are:

$$\frac{\partial T(0,t)}{\partial r} = 0, \qquad T(a,t) = 0, \qquad T(r,0) = 0$$

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1) What are the units of α?

2) Obtain the nondimensional form of the given **PDE**, the **BCs** and **IC**. Use the following dimensionless parameters:

$$\phi(\rho,\tau) = \frac{T(r,t)}{T_r}, \qquad \tau = \frac{t}{t_r}, \qquad \rho = \frac{r}{a}$$

where T_r is some arbitrary reference temperature, t_r is some arbitrary reference time, and a is the radius of the solid circular cylinder.

3) Show that when $T_r = Sa^2/k$ and $t_r = a^2/\alpha$ the nondimensional form of the **PDE** becomes:

$$\frac{1}{\rho}\frac{\partial}{d\rho}\left(\rho\frac{\partial\phi}{\partial\rho}\right)+1=\frac{\partial\phi}{\partial\tau}, \qquad \tau > 0, \qquad 0 \leq \rho \leq 1$$

4) Consider the *steady-state* case of the nondimensional **PDE** when $\phi(\rho)$. Obtain its solution for the homogeneous boundary conditions specified above.

5) What is the nondimensional $\phi(0)$ and dimensional $T(0)$ temperature along the axis of the cylinder?

6) The heat transfer rate *per unit length* of the cylinder can be obtained by application of Fourier's Law of Conduction at the cylinder boundary:

$$Q = -k2\pi a\frac{\partial T(r)}{\partial r}, \qquad r = a$$

(a) Nondimensionalize the heat transfer rate and call it $Q^{\star}$.

(b) Determine Q from the *nondimensional* solution.