ME 303 M.M. Yovanovich

Week 1

Lecture 1
Information:
Instructor: M.M. Yovanovich, CPH 3375C X3588, E3-2133A, X6181 or X4586
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Teaching Assistants:

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Course Text: *Applied Differential Equations*, Third Edition, 1981, Murray, R. Spiegel, Prentice-Hall, Inc.

Reference Texts:

1) *Differential Equations and Boundary Value Problems: Computing and Modeling*, 1996, C.H. Edwards and David E. Penney, Prentice-Hall, Inc.
2) *Elementary Applied Partial Differential Equations With Fourier Series and Boundary Value Problems*, Third Edition (1998) by Richard Haberman
3) *Applied Partial Differential Equations*, 1990, Donald, M. Trim, PWS-Kent Publishing Company.
4) *Partial Differential Equations for Scientists and Engineers*, 1982, Stanley J. Farlow, Dover Publications.
5) *An Introduction to Partial Differential Equations for Science Students*, 1970, G. Stephenson, Longman Group Ltd.
6) *Fourier Series and Integrals of Boundary Value Problems*, 1982, J. Ray Hanna, John Wiley and Sons.
7) *Theory and Problems of Partial Differential Equations*, 1986, Paul Duchateau and David W. Zachmann, Shaum's Outline Series, McGraw-Hill.
8) *Differential Equations with Maple V*, 1994, Martha L. Abell and James P. Braselton, AP Professional.
9) *Differential Equations with Maple*, 1996, K.R. Coombes, B.R. Hunt, R.L.

Lipsman, J.E. Osborn and G.J. Stuck, John Wiley & Sons, Inc.
10) *Applied Partial Differential Equation*, 1998, J. David Logan, Springer. Text contains Maple solutions.
11) *Mathematical Handbook of Formulas and Tables*, 1968, Murray R. Spiegel, Schaum's Outline Series, McGraw-Hill. This is an important and useful text.
12) *Theory Problems of Fourier Series with Applications to Boundary Value Problems*, 1974, Murray R. Spiegel, Schaum's Outline Series, McGraw-Hill.
13) *Theory and Problems of Laplace Transforms*, 1965, Murray R. Spiegel, Schaum's Outline Series, McGraw-Hill.
14) *Theory and Problems of Acoustics*, William W. Seto, 1971, Schaum's Outline Series, McGraw-Hill.
15) *Theory and Problems Mechanical Vibrations*, 1964, William W. Seto, Schaum's Outline Series, McGraw-Hill.

Students are requested to sign in.

Makeup Lectures at 8:30 AM
Tuesday, May 11
Thursday, May 27
Thursday, June 24

Week 9: June 28 - July 2: ME 303 lectures cancelled. Dates and times of the lectures will be arranged later.

Midterm and Final Examination Dates and Hours are Tentative
Midterm Examination: Monday, June 7, 4:30-6:30 PM
Final Examination: Wednesday, August 4, 9:00-12:00 Noon

Project 1: Deadline to be determined in Week 5
Project 2: Deadline to be determined in Week 10

Final Grade Components and Marks:

Project 1: 10 points
Project 2: 10 points
Mid Term Examination: 30 points
Final Examination: 50 points

ME 303 website: http://www.mhtl.uwaterloo.ca/courses/me303/me303.html/

Course lectures and course material will be placed at the web site each week.

Read Chapter 12: Sections: 1, 2, 3
Work on Problems:
page 547: A Exercises (a), (d)
page 548: B Exercises: 4, 6, 7
page 548: C Exercises: 5, 6
Solutions will be posted at web site at appropriate dates.

Lecture 2
Overview of mathematical methods applied to physical problems - see the ME 303 web site.

Notation and Terminology

• scalar: $T(x, y, z, t)$ or in vector notation $T(\mathbf{r}, t)$ depends on position (radius) vector $\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$ and time t with unit vectors in cartesian coordinates defined as: $\mathbf{i}, \mathbf{j}, \mathbf{k}$

• vector: $\mathbf{V} = \mathbf{i}u + \mathbf{j}v + \mathbf{k}w$ where, for example, $\mathbf{V}$ is a velocity vector and u, v, w are the velocity components corresponding to the $x-, y-, z-$ coordinates, respectively.

• partial derivatives and subscript notation: let $u = u(x, y, z, t)$, then the first partial derivatives are defined as $\partial u/\partial x = u_x$, $\partial u/\partial y = u_y$, $\partial u/\partial z = u_z$, $\partial u/\partial t = u_t$, and the second partial derivatives are defined as $\partial^2 u/\partial x^2 = u_{xx}$, $\partial^2 u/\partial y^2 = u_{yy}$, $\partial^2 u/\partial z^2 = u_{zz}$, $\partial^2 u/\partial t^2 = u_{tt}$

• higher order partial derivatives are defined as $\partial^3 u/\partial x^3 = u_{xxx}$ and $\partial^4 u/\partial x^4 = u_{xxxx}$, for example.

• See Spiegel's Mathematical Handbook: 22.28-22.34 for vector operator material.

• del operator: $\nabla = \mathbf{i}\ \partial/\partial x + \mathbf{j}\ \partial/\partial y + \mathbf{k}\ \partial/\partial z$

• gradient of a scalar: grad $T = \nabla T = \mathbf{i}\ \partial T/\partial x + \mathbf{j}\ \partial T/\partial y + \mathbf{k}\ \partial T/\partial z$.

• divergence of a vector: div $\mathbf{V} = \nabla \cdot \mathbf{V} = \dfrac{\partial}{\partial x}u + \dfrac{\partial}{\partial y}v + \dfrac{\partial}{\partial z}w$ with velocity components: u, v, w.

• Laplacian operator: $\nabla^2 = \nabla \cdot \nabla = \dfrac{\partial}{\partial x}\left(\dfrac{\partial}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(\dfrac{\partial}{\partial y}\right) + \dfrac{\partial}{\partial z}\left(\dfrac{\partial}{\partial z}\right) =$

$$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

• curl of a vector: curl $\mathbf{V} = \nabla \times \mathbf{V} =$
$(\mathbf{i}\ \partial/\partial x + \mathbf{j}\ \partial/\partial y + \mathbf{k}\ \partial/\partial z) \times (\mathbf{i}\ u + \mathbf{j}\ v + \mathbf{k}\ w) =$
$\mathbf{i}\ (\partial w/\partial y - \partial v/\partial z) + \mathbf{j}\ (\partial u/\partial z - \partial w/\partial x) + \mathbf{k}\ (\partial v/\partial x - \partial u/\partial y)$

• $\mathbf{V} \cdot \nabla = (\mathbf{i}\ u + \mathbf{j}\ v + \mathbf{k}\ w) \cdot \nabla = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$

• $\mathbf{q} \cdot \mathbf{n} = q_n$ where $\mathbf{n}$ is the outward-directed normal at a point on the surface and q_n is the normal component of the vector $\mathbf{q}$

• Substantial derivative. Appears in fluid flow. $\dfrac{D}{Dt} = \dfrac{\partial}{\partial t} + \vec{V} \cdot \nabla$.

• Biharmonic operator. $\nabla^4 = \nabla^2(\nabla^2)$. See Handbook of Mathematics. 22.34.

• Divergence theorem (Gauss's theorem): $\iiint_V \nabla \cdot \mathbf{A}\ dV = \iint_S \mathbf{A} \cdot \mathbf{n}\ dS$ relates a volume integral of the divergence of a vector $\mathbf{A}$ throughout the volume V to a surface integral of the vector component normal $\mathbf{A}\cdot\mathbf{n}$ to the bounding surface S.

Lecture 3

Some examples of linear second-order partial differential equations (PDEs) of physics and engineering:

• Laplace equation: $\nabla^2 U = 0$; homogeneous PDE

• Poisson equation: $\nabla^2 U = -G(x, y, z)/k$; nonhomogeneous PDE

• Diffusion equation: $\nabla^2 T = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}$; homogeneous PDE

• Diffusion equation with source term: $\nabla^2 T = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t} - G(x, y, z)/k$; nonhomogeneous PDE

• Wave equation: $c^2\nabla^2 u = \dfrac{\partial^2 u}{\partial t^2}$; homogeneous PDE

• Wave equation with external forces: $c^2\nabla^2 u = \dfrac{\partial^2 u}{\partial t^2} + g + k_1 u + k_2\dfrac{\partial u}{\partial t}$; nonhomogeneous PDE

• Helmholtz equation: $\nabla^2 U + \lambda^2 U = 0$; homogeneous PDE.

• Biharmonic equation: $\nabla^4 U = 0$.

• Biharmonic wave equation: $\nabla^4 U = -\frac{1}{c^2}\frac{\partial^2 U}{\partial t^2}$.

• All coefficients in the above partial differential equations are positive, physical parameters.

• Boundary Conditions (BCs) and Initial Conditions (ICs).
All PDEs require boundary conditions:
6 for 3-D problems, 4 for 2-D problems, and 2 for 1-D problems,
and initial conditions: 2 for the wave equation, and 1 for the diffusion equation.

First and second order ordinary differential equations in cartesian, circular cylinder (polar) and spherical coordinates. See the Maple solutions on ME 303 web site.
