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Radiation Laws

Planck's Distribution Law

The relation for the spectral **blackbody** emissive power $E_{b\lambda}$ was developed by Planck (1901). The relation is known as **Planck's distribution law**, and it is expressed as

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5 \;\; [\exp(C_2/\lambda T) - 1]} \qquad \left[\frac{W}{m^2 \cdot \mu m}\right]$$

where T is the absolute temperature of the surface, λ is the wavelength of the radiation emitted by the surface. Also

$$C_1 = 2\pi h c_0^2 = 3.742 \times 10^8 \qquad \left[\frac{W \cdot \mu m^4}{m^2}\right]$$

and

$$C_2 = \frac{h c_0}{k} = 1.439 \times 10^4 \qquad [\mu m \cdot K]$$

where $h = 6.625 \times 10^{-34}\ J \cdot s$ is **Planck's constant** and $c_0 = 2.998 \times 10^8\ m/s$ is the speed of light in a vacuum, and $k = 1.3805 \times 10^{-23}\ J/K$ is **Boltzmann's constant**. This relation is valid for a surface in a *vacuum* or a *transparent* gas.

Wien's Displacement Law

$$(\lambda T)_{\text{max power}} = 2897.8 \qquad [\mu m \cdot K]$$

Stefan-Boltzmann Law of Radiation

The integration of the spectral blackbody emissive power $E_{b\lambda}$ over the entire wavelength spectrum gives the *total* blackbody emissive power E_b:

$$E_b(T) = \int_0^\infty E_{b\lambda}(T)\, d\lambda = \sigma\, T^4 \qquad \left[\frac{W}{m^2}\right]$$

where $\sigma = 5.67 \times 10^{-8}\ W/(m^2 \cdot K^4)$ is the **Stefan-Boltzmann constant**. The Stefan-Boltzmann Law of Radiation gives the total radiation emitted by a blackbody at all wavelengths from $\lambda = 0$ to $\lambda = \infty$ at absolute temperature T.

Actual Radiation

Substances and surfaces of engineering interest have radiative characteristics which are different from the black-body radiation. Since E_b and $E_{b\lambda}$ are the maximum emissive powers for any given temperature, actual surfaces emit and absorb radiation less readily and they are called nonblack. The emissive power of a nonblack surface, at temperature T, radiating to a hemispherical surface above it is

$$E = \epsilon E_b = \epsilon\, \sigma\, T^4 \qquad \left[\frac{W}{m^2}\right]$$

where ϵ, called the **total hemispherical emittance**, is a function of the material, the condition of the surface, and the temperature of the surface.

Absorptivity, Reflectivity, Transmissivity

When radiant energy is incident on a surface, portions are absorbed, reflected, or transmitted through the material. From the first law of thermodynamics we get the relation:

$$\alpha + \rho + \tau = 1$$

where

$$\begin{aligned}
\text{absorptivity}: \quad \alpha &= \frac{\text{absorbed radiation}}{\text{incident radiation}}, \quad 0 \le \alpha \le 1 \\
\text{reflectivity}: \quad \rho &= \frac{\text{reflected radiation}}{\text{incident radiation}}, \quad 0 \le \rho \le 1 \\
\text{transmissivity}: \quad \tau &= \frac{\text{transmitted radiation}}{\text{incident radiation}}, \quad 0 \le \tau \le 1
\end{aligned}$$

Solids generaly do not transmit radiation unless the material is very thin. Metals absorb radiation within a fraction of a micrometer and electrical conductors within a fraction of a millimeter. Substances such a liquids and glass absorb most of the radiation within a millimeter. Solids and liquids therefore are generally assumed to be opaque with $\tau = 0$, therefore

$$\alpha + \rho = 1$$

This important relation allows one to determine both the absorptivity and reflectivity of an opaque surface by measuring either of these properties.

Most elementary gases such as hydrogen, oxygen and nitrogen (and mixtures of these such as air) have $\tau \approx 1$, and therefore $\rho = 0$ and $\alpha = 0$. For this reason radiation through air is generally estimated using the relationships for radiation through a vacuum. Gases with a more complex structure, such as water vapor and carbon dioxide, generally absorb and emit radiation as well as transmit radiation.

Specular and Diffuse Reflections

The reflection of radiation from a solid surface may be of a specular or diffuse nature. Specular reflection occurs at a surface which is very smooth and clean, such as a mirror, and an image of the radiation source is projected. The optical laws apply and the angle of reflection is equal to the angle of incidence. Diffuse reflection occurs when the surface is rough and dirty, and there is no preferential direction of reflection. No real surface is perfectly specular or diffuse, however, it is often useful to approximate surfaces as specular or diffuse.

Kirchhoff's Law

A useful relation between emissivity and absorptivity of any opaque surface can be developed directly from thermodynamic considerations. **Kirchhof's Law** states that for any surface in *thermodynamic equilibrium* with its surroundings, the monochromatic emissivity equals the monochromatic absorptivity. Therefore,

$$\epsilon_\lambda = \alpha_\lambda$$

If the surface is gray or the incident radiation is from a black surface at the same temperature, then the relation of the total values is also true

$$\epsilon = \alpha$$

Gray Surface

If a surface behaves such that ϵ_λ can be considered to be independent of the wavelength λ and it is equal to the total hemispherical emissivity ϵ, it is then said to be a gray surface.