ECE 309 M.M. Yovanovich

Specific Heats

Constant Volume Specific Heat

The specific internal energy is a function of temperature and specific volume:

$$u = u(T, v)$$

The differential change in the internal energy is given by:

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

Since u, v and T are all properties, the partial derivative is also a property and it is called the **constant-volume specific heat** defined as

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

The specific internal energy can now be written as

$$du = c_v \, dT$$

Integrating between the states T_1 and T_2 we have

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v \, dT = \bar{c}_v \, (T_2 - T_1)$$

where $\bar{c}_v$ is the mean value of the constant-volume specific heat over the temperature range: $T_1 \le T \le T_2$:

$$\bar{c}_v = \frac{1}{(T_2 - T_1)} \int_{T_1}^{T_2} c_v(T) \, dT$$

Constant Pressure Specific Heat

The specific enthalpy is a function of temperature and pressure:

$$h = h(T, P)$$

The differential change in the enthalpy is given by:

$$dh = \left(\frac{\partial h}{\partial T}\right)_P dT + \left(\frac{\partial h}{\partial P}\right)_T dP$$

For constant pressure $dP = 0$. The partial derivative is also a property and it is called the **constant-pressure specific heat** defined as

$$c_P = \left(\frac{\partial h}{\partial T}\right)_P$$

The specific enthalpy can now be written as

$$dh = c_P \, dT$$

Integrating between the states T_1 and T_2 we have

$$h_2 - h_1 = \int_{T_1}^{T_2} c_P \, dT = \bar{c}_P \, (T_2 - T_1)$$

where $\bar{c}_P$ is the mean value of the constant-pressure specific heat over the temperature range: $T_1 \leq T \leq T_2$:

$$\bar{c}_P = \frac{1}{(T_2 - T_1)} \int_{T_1}^{T_2} c_P(T) \, dT$$