**ECE 309 M.M. Yovanovich**

# Forced Convection Calculation

A long circular wire of diameter $D$ is maintained at temperature $T_s$ is cooled by an air stream at temperature $T_f$. The velocity of the air is $U = 5\,m/s$.

- Determine the Reynolds number
- Determine the heat transfer coefficient
- Compute the heat loss rate per unit length of wire

Ignore radiation loss from the surface of the wire into the surroundings. Use the air properties given in Table $C \cdot 5b$ of Reynolds and Perkins.

Consider the following correlation equation developed by Churchill and Berstein (1977) which is valid for forced convection heat transfer from long isothermal circular cylinder in cross flow. The correlation is valid for all fluids and for $Re\,Pr > 0.2$. The fluid properties are based on the film temperature $T_{\text{film}} = (T_s + T_f)/2$. The correlation is also valid for high Reynolds when the flow is turbulent.

$$Nu_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{28,200}\right)^{5/8}\right]^{4/5}$$

The constant 0.3 is the diffusive limit for long cylinders when $Re_D$ is very small. The square bracketted term accounts for turbulent flow effects.

The alternate approach is to use the general empirical correlation equation of the form:

$$Nu_D = C_w\,Re_D^m\,Pr^n$$

where $C_w$ and $m$ are correlation coeffcients which depend on the Reynolds number as given in the table below. For gases and other high Prandtl number fluids the index on the Prandtl number is $n = 1/3$.

| Range of Re | $C_w$ | $m$ |
|---|---|---|
| $0.4 - 4$ | 0.989 | 0.330 |
| $4 - 40$ | 0.911 | 0.385 |
| $40 - 4000$ | 0.683 | 0.466 |
| $4000 - 40,000$ | 0.193 | 0.618 |
| $40,000 - 400,000$ | 0.027 | 0.805 |

## Calculations

**Computation the film temperature**

$$T_{\text{film}} = \frac{T_s + T_f}{2} + 273 = \frac{127 + 27}{2} + 273 = 350$$

**Dry air properties at 350 K**

| property, units | | |
|---|---|---|
| $c_P,\ [J/kg \cdot K]$ | = | 1.0090 |
| $\mu,\ [kg/m \cdot s]$ | = | $2.075 \times 10^{-5}$ |
| $\nu,\ [m^2/s]$ | = | $2.076 \times 10^{-5}$ |
| $k_f,\ [W/m \cdot K]$ | = | 0.03003 |
| $Pr$ | = | 0.697 |

**Computation of Reynolds number**

$$Re_D = \frac{UD}{\nu} = \frac{5 \times 2 \times 10^{-3}}{2.076 \times 10^{-5}} = 481.7$$

The fluid flow is laminar.

**Computation of Nusselt number**
¿From the given table for $Re_D = 481.7$, $C_w = 0.683$ and $m = 0.466$.

$$Nu_D = 0.683 \times (481.7)^{0.466} \times (0.697)^{1/3} = 10.77$$

For $Re_D = 481.7$ and $Pr = 0.697$, the Churchill-Berstein correlation gives $Nu_D = 11.54$ which is approximately 7.1 % larger than the previous value. This is acceptable agreement.

**Calculation of h**

The heat transfer coefficient $h$ is

$$h = \frac{k_f \, Nu_D}{D} = \frac{0.3003 \times 10.77}{2 \times 10^{-3}} = 161.76 \, \left[\frac{W}{m^2 \cdot K}\right]$$

based the correlation coefficients given in the table. The value is $h = 173.29 \, [W/m^2 \cdot K]$ using the Churchill-Berstein correlation equation.

**Computation of Heat Transfer Rate Per Unit Length**

The heat transfer rate per unit length of the heated wire is

$$\dot{Q} = h \, A \, (T_s - T_f) = h \, \pi \, D \, (T_s - T_f) = 161.76 \times \pi \times 2 \times 10^{-3} \times (127 - 27) = 101.63 \, [W/m]$$

according to the correlation coefficients given in the table. The value computed according to the Churchill-Berstein correlation is $\dot{Q} = 108.88 \, [W/m]$.