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Overview of Convective Heat Transfer

Convection heat transfer is defined by **Newton's Law of Cooling**

$$\dot{Q}_{\text{conv}} = h\,A\,(T_s - T_f)$$

where A is the total convection area, T_s is the surface temperature, and T_f is the fluid temperature. The heat transfer coefficient h has units: $[W/m^2 \cdot K]$. This complex parameter depends on many geometric, thermal and fluid mechanics parameters. The following is an brief overview of convective heat transfer.

In general the magnitude of the heat transfer coefficient depends on:

Geometry

- Plate, Circular Cylinder, Sphere, Spheroids, Other Shapes
- Size, Aspect Ratio (thin or thick), Orientation (vertical or horizontal)

Type of Flow

- Forced, Natural, Mixed (combination of forced and natural)
- Laminar, Turbulent, Transitional
- Developing, Fully-Developed
- Internal, External, Enclosure

Boundary Conditions

- Isothermal, Isoflux

Type of Fluid

- Viscous Oil, Water, Gases (Air), Liquid Metals

Dimensionless Groups

Convective heat transfer (forced or natural) is characterized by several dimensionless groups discussed below.

The type of fluid is characterized by the dimensionless group called the **Prandtl number** which is defined as

$$\mathbf{Pr} = \frac{\text{molecular diffusion of momentum}}{\text{molecular diffusion of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

with the fluid properties: C_p, k, ρ, μ, $\nu = \mu/\rho$, $\alpha = k/(\rho\, C_p)$. The units of α and ν are: $[m^2/s]$. The Prandtl number lies in the range: $0 < Pr < \infty$.

The type of flow is characterized by two dimensionless groups called the **Reynolds number** denoted Re and the **Grashof number** denoted Gr.

Forced Convection

The Reynolds number is defined as:

$$\mathbf{Re} = \frac{\text{inertia force}}{\text{viscous force}} = \frac{\text{(velocity) (length scale)}}{\text{viscosity}} = \frac{\rho\, U \mathcal{L}}{\mu} = \frac{U\mathcal{L}}{\nu}$$

where U is some reference velocity scale and $\mathcal{L}$ is some length scale of the system. Laminar and turbulent forced flows are defined with respect to the magnitude of the Reynolds number. The fluid properties are the mass density ρ, and the dynamic viscosity μ and the kinematic viscosity ν. Some mechanical device such as a fan or pump is required to *move* the fluid over the heated surface.

Natural Convection

The Grashof number is defined as:

$$\mathbf{Gr} = \frac{\text{buoyancy force}}{\text{viscous force}} = \frac{g\,\beta\,\rho^2\,(T_s - T_f)\mathcal{L}^3}{\mu^2} = \frac{g\,\beta\,(T_s - T_f)\mathcal{L}^3}{\nu^2}$$

where β is the isobaric compressibility of the fluid and g is the local acceleration of gravity. The characteristic length scale denoted as $\mathcal{L}$ depends on the flow and geometry of the system. Laminar and turbulent natural convective flows are defined with respect to the magnitude of the Grashof number.

The Rayleigh number appears in natural convection correlation equations. It is defined as:

$$\mathbf{Ra} = Gr\,Pr = \frac{g\,\beta\,(T_s - T_f)\mathcal{L}^3}{\alpha\,\nu}$$

The heat transfer coefficient appears in the dimensionless groups called the **Nusselt number** and the **Stanton number** respectively. The definitions are:

Nusselt number

$$\mathbf{Nu} = \frac{\text{convection heat transfer}}{\text{conduction heat transfer}} = \frac{h\mathcal{L}}{k_f}$$

where k_f is the fluid thermal conductivity usually evaluated at the film temperature. The characteristic length scale denoted as $\mathcal{L}$ depends on the type of flow and the geometry of the system.

Stanton number

$$\mathbf{St} = \frac{h}{U\,\rho\,C_p} = \frac{Nu}{Re\,Pr}$$

Correlation Equations

The convective heat transfer correlations for **forced convection** are usually expressed in the form:

$$Nu = f(Re, Pr) \qquad \text{or} \qquad Nu = C_w\,Re^m\,Pr^n$$

where the coefficient C_w depends on the geometry, the boundary condition and whether the flow is laminar or turbulent, and the two exponents: m and n depend on whether the flow is laminar or turbulent. If the Stanton number is used, the correlation is of the form:

$$St = f(Re, Pr)$$

The convective heat transfer correlations for **natural convection** are usually expressed in the form:

$$Nu = f(Gr, Pr) \qquad \text{or} \qquad Nu = f(Ra, Pr)$$

or as

$$Nu = C_w\,Gr^m\,Pr^n \qquad \text{or} \qquad Nu = C_w\,Ra^m\,Pr^n$$

where the coefficient C_w depends on the geometry, the boundary condition and whether the flow is laminar or turbulent, and the two exponents: m and n depend on whether the flow is laminar or turbulent.

Correlations for Small Reynolds and Grashof Numbers

When the Reynolds number falls below $Re = 100$, it is necessary to include another term in the correlation to account for molecular diffusion:

$$Nu = [Nu^p_{\text{diffusion}} + (C_w \, Re^m \, Pr^n)^p]^{1/p}$$

where p is an empirical correlation parameter which depends on the geometry and the type of flow. When the Grashof number falls below $Gr = 10^4$, it is necessary to include another term in the correlation to account for molecular diffusion:

$$Nu = [Nu^p_{\text{diffusion}} + (C_w \, Gr^m \, Pr^n)^p]^{1/p}$$

where p is an empirical correlation parameter which depends on the geometry and the type of flow.

The *diffusion* term is related to pure conduction from the heated surface into the surrounding substance.