OHMIC HEATING

Consider a long copper wire of diameter D, thermal conductivity k through which a steady current i is flowing. We wish to establish a relationship between the distributed heat source strength S and the current density J and the resistivity of the wire. All electrical and thermal properties are assumed to be constants.

The electrical resistance of the copper wire of length L is given by the following relationship (which comes from the application of Ohm's Law):

$$R_e = \rho_e \frac{L}{A_c}$$

where ρ_e is the electrical resistivity of the wire and A_c is the cross-section area. The units of resistivity are $\Omega - cm$.

The power dissipated within the wire (assumed to be uniform over the cross-section and over the length) is given by

$$P = i^2 R_e = i^2 \rho_e \frac{L}{A_c}$$

The distributed heat source strength S is defined as the total power dissipated divided by the total volume $V = A_c L$:

$$S = \frac{P}{V} = i^2 \rho_e \frac{1}{A_c^2}$$

The units of S are W/m^3.

Introducing the current density $J = i/A_c$ leads to the following relationship between S and the electrical parameters: current density and electrical resistivity:

$$S = J^2 \rho_e$$

The units of current density are frequently quoted as A/cm^2; however in the SI system the units are A/m^2.

The above relationship is valid for all *constant* cross-section wires or thin strips.