UNIVERSITY OF WATERLOO

DEPARTMENT OF ELECTRICAL ENGINEERING
ECE 309 Thermodynamics and Heat Transfer

Final Examination August 6, 1999
M.M. Yovanovich 9:00 A.M.-12:00 Noon

NOTE:

1. Open book examination. You are permitted to use the text, lecture notes and material from the Web site, a calculator and two crib sheets (8.5 x 11) both sides. All other material must be placed on the floor.

2. Clear systematic solutions are required. Process diagrams and sketches of equipment showing locations referred to in your analysis are both essential. Marks will not be assigned for problems that require unreasonable (in the opinion of your professor) effort for the marker to decipher.

3. Very briefly state the assumptions that are made in each problem.

4. Ask for clarification if any problem statement is unclear.

5. The weight for each problem is indicated. You must answer all problems.

6. Good luck in this exam.

Problem 1 (20)

Two kilograms of H_2O are placed in a piston-cylinder system. Initially the H_2O is at $T_1 = 280°C$ in the saturated vapor state. During an *isentropic* cooling process, the pressure of the system is reduced to $P_2 = 1.002\,MPa$. In the next process, where work is done on the system at constant temperature and constant pressure, the *mixture* becomes saturated liquid.

(i) Sketch the two cooling processes on a $T - s$ diagram. Clearly label the states 1, 2 and 3.

(ii) Find the quality x_2 of the mixture at state 2.

(iii) Calculate the change in entropy $S_2 - S_3$ between states 2 and 3.

(iv) Derive relations for the work W_{23} and the heat Q_{23} in terms of the system properties.

(v) Calculate W_{23} and Q_{23}.

Problem 2 (20)

A thermoelectric device cools a small refrigerator and discards heat to its surroundings at temperature $T_H = 298\ K$. The device is designed to deliver a maximum electrical power output $\dot{W}_{\max}$ of 100 W. The heat load $\dot{Q}_{\text{refrig}}$ on the refrigerator (i.e. the heat leakage through the refrigerator walls which must be removed by the thermoelectric device) is 350 W.

(i) Apply the First and Second Laws of Thermodynamics (FLOT and SLOT) to an appropriate control volume to obtain a relationship between the minimum temperature $T_{C,min}$ which can be maintained in the refrigerator and the parameters $T_H, \dot{Q}_{\text{refrig}}$ and $\dot{W}_{\max}$.

(ii) Calculate the $T_{C,min}$ for the given conditions.

Problem 3 (20)

A central power plant, whether the energy source is nuclear or fossil fuel, is a heat engine operating between the temperatures of the reactor or furnace and the surroundings, usually a body of water such as a lake or a river. Consider a modern nuclear plant generating $750,000\,kW$ for which the reactor temperature is $586\,K$ and a lake temperature of $293\,K$.

(i) What is the *maximum* possible thermal efficiency of the plant?

(ii) What is the *minimum* amount of heat that must be discarded into the lake?

(iii) If the actual thermal efficiency of the plant is 60% *of the maximum thermal efficiency*, how much heat must be discarded into the lake?

(iv) What will be the temperature rise of the lake water if the water intake to the plant is $165\,m^3/s$? Assume the water heat capacity is $4180\,J/(kg \cdot K)$ and the water mass density is $1000\,kg/m^3$.

Problem 4 (20)

A $4-\mathrm{m}^3$ tank is insulated and evacuated. It is connected to a $4-$MPa, $600°C$ steam line by means of a valve. The valve is opened and steam enters the tank. The filling process continues until the pressure in the tank is equal to the steam line pressure.

Use the First and Second Laws formulated for a control volume, i.e. FLOT/CV and SLOT/CV, to

(i) Find the final temperature T_f in the tank.

(ii) Find the final mass m_f of the steam in the tank.

(iii) Compute the entropy production in the system.

Problem 5 (20)

A long brass tube of inner and outer radii r_1, r_2 respectively has the following thermophysical properties: $\rho = 8500\, kg/m^3$, $c_p = 0.385\, kJ/(kg \cdot K)$, $k = 110\, W/(m \cdot K)$.

A hot fluid at temperature $T_{f1} = 390\, K$ flows through the tube and there is heat transfer to the inner tube wall through a film coeffcient estimated to be $h_1 = 100\, W/(m^2 \cdot K)$.

The heat is transferred through the tube wall and then dissipated into the environment which consists of the surrounding air which is at the temperature $T_\infty = 300\, K$ and to the walls, ceiling and floor of the room which is at the temperature $T_{\text{surr}} = 300\, K$.

The natural convection heat transfer coeffcient is estimated to be $h_{\text{conv}} = 10\, W/(m^2 \cdot K)$, and the radiation film coeffcient is estimated to be $h_{\text{rad}} = 6.5 W/(m^2 \cdot K)$. The inner and outer tube radii are: $r_1 = 6.35\, mm$ and $r_2 = 6.60\, mm$.

(i) Sketch the equivalent thermal circuit labeling all thermal nodes and thermal resistors.

(ii) Calculate all thermal resistors and the overall resistance per unit length of the tube.

(iii) Calculate the heat transfer rate from the system per unit length of the tube.

(iv) Calculate the heat transfer rate if the outer surface of the tube is covered with insulation whose thickness is $t = 1.68\, mm$ and its thermal conductivity is $k_{\text{insul}} = 0.06\, W/(m \cdot K)$. Assume that the convection and radiation heat transfer coefficients remain unchanged.

Some Equations and Relationships

1. $E_1 + W_{12} + Q_{12} = E_2$
2. $e = u + \frac{1}{2}\bar{V}^2 + gz + \cdots$
3. $h \equiv u + Pv \qquad u = \frac{U}{M} \qquad v = \frac{V}{M}$
4. $v = (1-x)v_f + xv_g \qquad v_{fg} = v_g - v_f$
5. $\left(\frac{dE}{dt}\right)_{CV} = \dot{Q} + \dot{W} + \sum_{in}(e + Pv)\dot{M} - \sum_{out}(e + Pv)\dot{M}$
6. $\left(\frac{dM}{dt}\right)_{CV} = \sum_{in}\dot{M} - \sum_{out}\dot{M}$
7. $PV = MRT$
8. $c_v \equiv \left(\frac{\partial u}{\partial T}\right)_V \qquad c_p \equiv \left(\frac{\partial h}{\partial T}\right)_P \qquad c_p = c_v + R$

Ideal Gas Relations
Variable Specific Heats

$u_2 - u_1 = \int_1^2 c_v \, dT$

$h_2 - h_1 = \int_1^2 c_P \, dT$

$s_2 - s_1 = \int_1^2 c_v \, \frac{dT}{T} + R \, \ln \frac{v_2}{v_1}$

$s_2 - s_1 = \int_1^2 c_P \, \frac{dT}{T} - R \, \ln \frac{P_2}{P_1}$

Incompressible Fluid or Solid Relations
$v =$ constant and $c_P = c_v = c$.
Variable Specific Heats

$u_2 - u_1 = \int_1^2 c \, dT$

$h_2 - h_1 = \int_1^2 c \, dT + v(P_2 - P_1)$

$s_2 - s_1 = \int_1^2 c \, \frac{dT}{T}$

Control Volume Forms of the General Conservation Equations

Continuity Equation

$$\frac{\partial}{\partial t}\left(\int_{CV} \rho dV\right) = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m}$$

First Law of Thermodynamics

$$\frac{\partial}{\partial t}\left(\int_{CV} e\rho dV\right) = \sum_{\text{in}} \int_{\mathbf{A}_{\text{in}}} (e+Pv)\, d\dot{m} - \sum_{\text{out}} \int_{\mathbf{A}_{\text{out}}} (e+Pv)\, d\dot{m} + \dot{Q}_{CV} + \dot{W}_{CV}$$

Second Law of Thermodynamics

$$\frac{\partial}{\partial t}\left(\int_{CV} s\rho dV\right) - \sum_{\text{in}} \int_{\mathbf{A}_{\text{in}}} s\, d\dot{m} + \sum_{\text{out}} \int_{\mathbf{A}_{\text{out}}} s\, d\dot{m} - \sum_{i} \left(\frac{\dot{Q}_i}{T_i}\right)_{CV} = \dot{\mathcal{P}}_{\mathrm{S}} \geq 0$$

Heat Transfer Relations

Conduction, Convection and Radiation Laws

Fourier Law of Conduction
$\vec{q} = \frac{\dot{Q}}{A} = -k\ \nabla T$
Newton Law of Cooling
$q = \frac{\dot{Q}}{A} = h(T_{\text{wall}} - T_{\text{fluid}})$
Stefan-Boltzmann Law of Radiation for Black Bodies
$\dot{Q} = \sigma A_1 (T_1^4 - T_2^4)$

Thermal Resistances

Thermal resistance is generally defined as $R \equiv (T_1 - T_2)/\dot{Q}$. The units are K/W.

Conduction Resistances

Plane wall: $R = \dfrac{L}{kA}$

Cylindrical shell: $R = \dfrac{\ln(b/a)}{2\pi L k}$

Spherical shell: $R = (1/a - 1/b)/(4\pi k)$

Fins: $R = 1/(\sqrt{hPkA}\ \tanh(mL))$ and $m = \sqrt{hP/kA}$

Fluid or Film Resistance

$R = 1/(hA)$

Radiation Resistances

Grey Surface Resistance: $R = \dfrac{(1-\epsilon)}{A\epsilon}$

Spatial Resistance: $R = \dfrac{1}{A_1 F_{12}} = \dfrac{1}{A_2 F_{21}}$

Notes: Units of radiation resistances are $1/m^2$. F_{12} is the view factor between two surfaces: A_1 and A_2 is dimensionless and its range is $0 \leq F_{12} \leq 1$. The surface emissivity ϵ is a complex radiation parameter which is determined experimentally for real (grey) surfaces. It is dimensionless and its range is $0 \leq \epsilon \leq 1$. Smooth, highly polished metals such as aluminum have values as low as $\epsilon \approx 0.01 - 0.1$. Very rough, oxidized surfaces have values as high as $\epsilon \approx 0.8 - 0.95$. Black bodies are ideal bodies for which $\epsilon = 1$.

The total radiation resistance of a two surface enclosure which is bounded by two isothermal, grey surfaces is given by:

$$R_{\text{total}} = \frac{(1-\epsilon_1)}{A_1\epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{(1-\epsilon_2)}{A_2\epsilon_2}$$

The radiation heat transfer rate between the two surfaces is given by

$$\dot{Q} = \frac{(e_{b1} - e_{b2})}{R_{\text{total}}} = \frac{\sigma(T_1^4 - T_2^4)}{R_{\text{total}}}$$