UNIVERSITY OF WATERLOO

DEPARTMENT OF ELECTRICAL ENGINEERING

ECE 309 Thermodynamics and Heat Transfer

Final Examination Spring 1998

M.M. Yovanovich August 7, 1998 9:00 A.M.-12:00 Noon

NOTE:

1. Open book examination. You are permitted to use the text, lecture notes and material from the Website, a calculator and two crib sheets (8.5 x 11) both sides. All other material must be placed on the floor.
2. Clear systematic solutions are required. Process diagrams and sketches of equipment showing locations referred to in your analysis are both essential. Marks will not be assigned for problems that require unreasonable (in the opinion of your professor) effort for the marker to decipher.
3. Very briefly state the assumptions that are made in each problem.
4. Ask for clarification if any problem statement is unclear.
5. The weight for each problem is indicated. You must answer all problems.
6. Good luck in this exam.

Problem 1a. (10) A piston-cylinder system initially contains $0.5\,m^3$ of nitrogen gas at $400\,kPa$ and $27°C$. An electric heater inside the system is turned on and it is allowed to pass a current of $2\,A$ from a $120\,V$ line for $300\,s$. The nitrogen expands at constant pressure and there is heat loss of $2800\,J$ during the process.

(a) Determine the final temperature of the nitrogen.

Problem 1b. (10) Air at $300\,K$ and $200\,kPa$ is heated at constant pressure to $600\,K$. Determine the change in internal energy of air per unit mass using:

(b) the average value of c_v given in the tables of the text,
(c) the polynomial form:

$$C_p(T) = C_0 + C_1 T + C_2 T^2 + C_3 T^3, \quad \left[\frac{kJ}{kg \cdot K}\right]$$

with correlation coefficients:
$C_0 = 0.9703$,
$C_1 = 0.6790 \times 10^{-4}$,
$C_2 = 0.1658 \times 10^{-6}$,
$C_3 = -0.6786 \times 10^{-10}$

(d) report the percent difference.

Problem 2. (20) A rigid tank of volume V is initially empty (evacuated), ie $M_1 = 0$. It is filled with an ideal gas. The inlet conditions to the tank are controlled by a valve such that the inlet properties: T_{in}, P_{in} are constant during the *adiabatic* filling process.

(a) Obtain a relation for the final tank temperature T_2 when the tank pressure is P_2. The relation should show how T_2 is related to T_{in} and the specific heat ratio $k = c_P/c_v$.

(b) Obtain the relation for the entropy change S_{12} during the process.

Problem 3. (20) Consider the simple steam power plant which consists of a boiler, turbine, condenser and pump. The turbine and pump are assumed to be adiabatic and ideal. The following data are given for the power plant:

Location	Pressure	Temperature or Quality
Boiler exit, 1	$2.0\,MPa$	$300°C$
Turbine entrance, 2	$1.9\,MPa$	$290°C$
Turbine exit, 3	$15\,kPa$	$x = 0.9$
Condenser entrance, 3	$15\,kPa$	$x = 0.9$
Condenser exit, 4	$14\,kPa$	$45°C$
Pump entrance, 4	$14\,kPa$	$45°C$
Pump exit, 5	$--$	$--$

The pump work is found to be $w_P = 4kJ/kg$.

Provide a simple sketch of the power plant and use the location numbers given in the above table in your analysis.

Determine the following quantities per kilogram of steam flowing through each unit from the data provided above:
(a) Heat transfer loss q_{loss} in the line between the boiler and turbine.
(b) Turbine work w_T.
(c) Heat transfer in condenser q_C.
(d) Heat transfer in boiler q_B.
(e) Efficiency η of the power plant.

Problem 4a. (10) The food compartment of a refrigerator is maintained at $4°C$ by removing heat from it at the rate of $360kJ/min$. If the required power input to the refrigerator is $2kW$, determine:
(a) the coefficient of performance (COP) of the refrigerator which is defined as

$$COP_R = \frac{\text{desired output}}{\text{required input}} = \frac{Q_L}{W_{net,in}}$$

Conservation of energy for a cyclic device requires that $W_{net,in} = Q_H - Q_L$.

(b) the rate of heat transfer to the room which houses the refrigerator.

Problem 4b. (10) A heat pump is used to meet the heating requirements of a house and maintain it at $20\,^\circ C$. On a day when the outdoor temperature drops to $-2°C$, the house loses heat at the rate of $80,000\,kJ/h$. If the heat pump under these conditions has a COP of 2.5, determine:

(c) the power input to the heat pump,
(d) the rate at which heat is extracted from cold outdoor air.

The coefficient of performance for a heat pump is defined as

$$COP_{HP} = \frac{\text{desired output}}{\text{required input}} = \frac{Q_H}{W_{net,in}}$$

Problem 5. (20) A long stainless-steel wire of diameter $D = 1mm$ is submerged in an oil bath of temperature $T_f = 25^0C$. The wire has an electrical resistance per unit length of $R_e/L = \rho_e/A_c = 0.01\,\Omega/m$. The wire carries a current $i = 100\,A$. The convective heat transfer coefficient is estimated to be $h = 500\,W/m^2 \cdot K$. The wire thermophysical properties are:

$\rho = 8000\,kg/m^3$,
$C_P = 500\,J/kg \cdot K$,
$k = 20\,W/m \cdot K$.

(a) Compute the Biot number defined as $Bi = hD/(2k)$ for the wire and verify that the *lumped* capacitance model can be used.

(b) Compute the characteristic time t_c of the system.

(c) Find the value of the steady-state temperature T_{ss}.

(d) From the time the current is applied, how long does it take for the wire to reach a temperature which is within 1^0C of the steady-state value?

Some Equations and Relationships

1. $E_1 + W_{12} + Q_{12} = E_2$
2. $e = u + \frac{1}{2}\bar{V}^2 + gz + \cdots$
3. $h \equiv u + Pv \qquad u = \frac{U}{M} \qquad v = \frac{V}{M}$
4. $v = (1-x)v_f + xv_g \qquad v_{fg} = v_g - v_f$
5. $\left(\frac{dE}{dt}\right)_{CV} = \dot{Q} + \dot{W} + \sum_{in}(e+Pv)\dot{M} - \sum_{out}(e+Pv)\dot{M}$
6. $\left(\frac{dM}{dt}\right)_{CV} = \sum_{in}\dot{M} - \sum_{out}\dot{M}$
7. $PV = MRT$
8. $c_v \equiv \left(\frac{\partial u}{\partial T}\right)_V \qquad c_p \equiv \left(\frac{\partial h}{\partial T}\right)_P \qquad c_p = c_v + R$

Ideal Gas Relations
Variable Specific Heats

$u_2 - u_1 = \int_1^2 c_v \, dT$

$h_2 - h_1 = \int_1^2 c_P \, dT$

$s_2 - s_1 = \int_1^2 c_v \, \frac{dT}{T} + R \, \ln \frac{v_2}{v_1}$

$s_2 - s_1 = \int_1^2 c_P \, \frac{dT}{T} + R \, \ln \frac{P_2}{P_1}$

Incompressible Fluid or Solid Relations
$v =$ constant and $c_P = c_v = c$.
Variable Specific Heats

$u_2 - u_1 = \int_1^2 c \, dT$

$h_2 - h_1 = \int_1^2 c \, dT + v(P_2 - P_1)$

$s_2 - s_1 = \int_1^2 c \, \frac{dT}{T}$

Control Volume Forms of the General Conservation Equations

Continuity Equation

$$\frac{\partial}{\partial t}\left(\int_{CV} \rho dV\right) = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m}$$

First Law of Thermodynamics

$$\frac{\partial}{\partial t}\left(\int_{CV} e\rho dV\right) = \sum_{\text{in}} \int_{\text{A}_{\text{in}}} (e + Pv)\ d\dot{m} - \sum_{\text{out}} \int_{\text{A}_{\text{out}}} (e + Pv)\ d\dot{m} + \dot{Q}_{CV} + \dot{W}_{CV}$$

Second Law of Thermodynamics

$$\frac{\partial}{\partial t}\left(\int_{CV} s\rho dV\right) - \sum_{\text{in}} \int_{\text{A}_{\text{in}}} s\ d\dot{m} + \sum_{\text{out}} \int_{\text{A}_{\text{out}}} s\ d\dot{m} - \sum_{i} \left(\frac{\dot{Q}_i}{T_i}\right)_{CV} = \mathcal{P}_{\text{s}} \geq 0$$

Heat Transfer Relations

Conduction, Convection and Radiation Laws

Fourier Law of Conduction
$\vec{q} = \frac{\dot{Q}}{A} = -k\ \nabla T$
Newton Law of Cooling
$q = \frac{\dot{Q}}{A} = h(T_{\text{wall}} - T_{\text{fluid}})$
Stefan-Boltzmann Law of Radiation for Black Bodies
$\dot{Q} = \sigma A_1 (T_1^4 - T_2^4)$

Thermal Resistances

Thermal resistance is generally defined as $R \equiv (T_1 - T_2)/\dot{Q}$. The units are K/W.

Conduction Resistances

Plane wall: $R = \dfrac{L}{kA}$

Cylindrical shell: $R = \dfrac{\ln(b/a)}{2\pi Lk}$

Spherical shell: $R = (1/a - 1/b)/(4\pi k)$

Fins: $R = 1/(\sqrt{hPkA}\ \tanh(mL))$ and $m = \sqrt{hP/kA}$

Fluid or Film Resistance

$R = 1/(hA)$

Radiation Resistances

Grey Surface Resistance: $R = \dfrac{(1-\epsilon)}{A\epsilon}$

Spatial Resistance: $R = \dfrac{1}{A_1 F_{12}} = \dfrac{1}{A_2 F_{21}}$

Notes: Units of radiation resistances are $1/m^2$. F_{12} is the view factor between two surfaces: A_1 and A_2 is dimensionless and its range is $0 \leq F_{12} \leq 1$. The surface emissivity ϵ is a complex radiation parameter which is determined experimentally for real (grey) surfaces. It is dimensionless and its range is $0 \leq \epsilon \leq 1$. Smooth, highly polished metals such as aluminum have values as low as $\epsilon \approx 0.01 - 0.1$. Very rough, oxidized surfaces have values as high as $\epsilon \approx 0.8 - 0.95$. Black bodies are ideal bodies for which $\epsilon = 1$.

The total radiation resistance of a two surface enclosure which is bounded by two isothermal, grey surfaces is given by:

$$R_{\text{total}} = \frac{(1-\epsilon_1)}{A_1\epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{(1-\epsilon_2)}{A_2\epsilon_2}$$

The radiation heat transfer rate between the two surfaces is given by

$$\dot{Q} = \frac{(e_{b1} - e_{b2})}{R_{\text{total}}} = \frac{\sigma(T_1^4 - T_2^4)}{R_{\text{total}}}$$