SUMMARY OF CONVECTION CORRELATION EQUATIONS

ME 353 Heat Transfer 1
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The attached material is a summary of some of the important results for forced and natural convection heat transfer from isothermal or isoflux surfaces. Correlation equations for local and area-average heat transfer for external and internal flow are given.

Many empirical and analytic correlation equations have been developed for the local and area-average values of the Nusselt number for limited ranges of the forced and buoyancy-induced flow parameters: Reynolds, Peclet, Grashof and Rayleigh numbers; for laminar or turbulent flows; and for various fluids which are characterized by the Prandtl number.

One should consult the course text for the definitions of the various dependent and independent parameters and the basis for the fluid properties evaluation for the particular correlation equation.

This summary does not cover the numerous forced and buoyancy-induced internal flows through and within complex configurations. One should consult the course text or the several handbooks which deal with these topics.

1. Laminar and Turbulent Forced External Flow

<u>Definitions of Local and Area-average Values</u>

$$h_x = \frac{-k \dfrac{\partial T(x, y=0)}{\partial y}}{(T_w - T_\infty)}, \qquad \overline{h} = \frac{1}{L}\int_0^L h(x)\, dx = 2h(x=L)$$

$$Nu_x = \frac{h_x x}{k}, \qquad Re_x = \frac{U_\infty x}{\nu}$$

$$\overline{Nu}_L = \frac{\bar{h} L}{k}, \qquad Re_L = \frac{U_\infty L}{\nu}$$

Flat Plate, Laminar Boundary Layer Flow

Correlation	Limits	Conditions
$\delta = 5xRe_x^{-1/2}$	$100 < Re_x < 500,000$	Local
$C_{f,x} = 0.664Re_x^{-1/2}$	$100 < Re_x < 500,000$	Local
$\overline{C}_{f,x} = 1.328Re_x^{-1/2}$	$100 < Re_x < 500,000$	Area-Average
$\Delta = \delta Pr^{-1/3}$	$100 < Re_x < 500,000$	Local
$\frac{Nu_x}{Re_x^{1/2}} = 0.3387Pr^{1/3}$	$100 < Re_x < 500,000$	Local, UWT, $Pr \to \infty$
$\frac{Nu_x}{Re_x^{1/2}} = 0.564Pr^{1/2}$	$100 < Re_x < 500,000$	Local, UWT, $Pr \to 0$
$\frac{Nu_x}{Re_x^{1/2}} = \frac{0.3387Pr^{1/3}}{\left[1+(0.0468/Pr)^{2/3}\right]^{1/4}}$	$100 < Re_x < 500,000$	Local, UWT, $0 < Pr < \infty$
$\frac{Nu_x}{Re_x^{1/2}} = 0.4637Pr^{1/3}$	$100 < Re_x < 500,000$	Local, UWF, $Pr \to \infty$
$\frac{Nu_x}{Re_x^{1/2}} = 0.886Pr^{1/2}$	$100 < Re_x < 500,000$	Local, UWF, $Pr \to 0$
$\frac{Nu_x}{Re_x^{1/2}} = \frac{0.4637Pr^{1/3}}{\left[1+(0.0205/Pr)^{2/3}\right]^{1/4}}$	$100 < Re_x < 500,000$	Local, UWF, $0 < Pr < \infty$
$\frac{\overline{Nu}_L}{Re_L^{1/2}} = \frac{0.6774Pr^{1/3}}{\left[1+(0.0468/Pr)^{2/3}\right]^{1/4}}$	$100 < Re_L < 500,000$	Average, UWT, $0 < Pr < \infty$
$\frac{\overline{Nu}_L}{Re_L^{1/2}} = \frac{0.9274Pr^{1/3}}{\left[1+(0.0205/Pr)^{2/3}\right]^{1/4}}$	$100 < Re_L < 500,000$	Average, UWF, $0 < Pr < \infty$

Flat Plate, Turbulent Boundary Layer Flow

Correlation	Limits	Conditions
$\delta = 0.37xRe_x^{-1/5}$	$5 \times 10^5 < Re_x < 10^8$	Local
$C_{f,x} = 0.0592Re_x^{-1/5}$	$5 \times 10^5 < Re_x < 10^8$	Local
$\overline{C}_{f,L} = 0.074Re_L^{-1/5} - 1742Re_L^{-1}$	$Re_{x,c} = 5 \times 10^5$	Mixed-Average
$\Delta = \delta Pr^{-1/3}$	$100 < Re_x < 500,000$	Local
$Nu_x = 0.0296Re_x^{4/5}Pr^{1/3}$	$100 < Re_x < 500,000$	Local, UWT, $0.6 < Pr < 60$
$\overline{Nu}_L = \left(0.037Re_L^{4/5} - 871\right) Pr^{1/3}$	$100 < Re_x < 500,000$	Average, UWT, $0.6 < Pr < 60$

Cross Flow Over Circular Cylinders

Correlation	Limits	Conditions
$\overline{Nu}_D = S_D^\star + \dfrac{0.6 Re_D^{1/2} Pr^{1/3}}{\left[1+\left(\dfrac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \left[1+\left(\dfrac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$	$100 < Re_D < 10^7$	Average, UWT, $0 < Pr < \infty$
$S_D^\star = \frac{4}{\pi}\left(\dfrac{1+0.869(L/D)^{0.76}}{0.5+L/D}\right)$	$0 \le L/D \le 8$	$Re_D \to 0$
$S_D^\star = \dfrac{4}{\sqrt{\pi}}\left(\dfrac{1}{\sqrt{1+0.5D/L}}\right)\dfrac{1}{\ln(2L/D)}$	$L/D \ge 8$	$Re_D \to 0$, Asymptote

Flow Over Spheres

Correlation	Limits	Conditions
$C_D = 0.4 + \dfrac{24}{Re_D} + \dfrac{6}{1+\sqrt{Re_D}}$	$0 \le Re_D \le 2 \times 10^5$	Total Drag, $\pm 10\%$
$\overline{Nu}_D = 2 + \dfrac{0.6 Re_D^{1/2} Pr^{1/3}}{\left[1+\left(\dfrac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \left[1+\left(\dfrac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$	$100 < Re_D < 10^7$	Average, UWT, $0 < Pr < \infty$

2. External Flow Over Isothermal Oblate and Prolate Spheroids

The following *universal* correlation equation:

$$\overline{Nu}_{\sqrt{A}} = \overline{Nu}_{\sqrt{A}}^{0} + \left[0.15\left(\frac{P}{\sqrt{A}}\right)^{1/2} Re_{\sqrt{A}}^{1/2} + 0.35 Re_{\sqrt{A}}^{0.566}\right] Pr^{1/3}$$

was developed by Yovanovich (1988) from two accurate correlation equations proposed by Yuge (1960) for air cooling of isothermal spheres, and the correlation equations developed by several researchers for convection heat and mass transfer from isothermal oblate and prolate spheroids.

In the above correlation equation the Nusselt and Reynolds numbers are both based on the length scale $\mathcal{L} = \sqrt{A}$. The diffusive limit $\overline{Nu}_{\sqrt{A}}^{0}$ corresponding to $Re \to 0$ is the dimensionless shape factor $S_{\sqrt{A}}^\star$.

Yovanovich (1988) *blended* the two Yuge equations and introduced the parameter $P/\sqrt{A}$ which accounts for the *blockage* of the body as the fluid flows around it. Also the Yuge correlation equations which were developed for air were extended to account for large Prandtl number fluids, i.e. $Pr > 0.7$. The correlation equation is valid for the wide Reynolds number range: $0 \leq Re_{\sqrt{A}} < 2 \times 10^5$.

The general correlation equation is in very good agreement with numerous analytical and experimental correlation equations over various ranges of the Reynolds number for $Pr = 0.7$.

It is also in good agreement with the empirical correlation equation of Pasternak and Gauvin (1960) which was developed from 20 different convex body shapes to account for both body shape and orientation. The body length scale which they proposed was based on the ratio of the total surface area of the body divided by the maximum projected area of the body perpendicular to the air flow. They achieved good correlation of their heat and mass transfer data with a single power-law equation which was converted to the body scale length $\mathcal{L} = \sqrt{A}$

$$\overline{Nu}_{\sqrt{A}} = 0.914 Re_{\sqrt{A}}^{0.514} Pr^{1/3} \qquad 886 \leq Re_{\sqrt{A}} \leq 8860$$

This equation correlated data for spheres, finite circular cylinders with axes parallel and perpendicular to the flow, prisms, cubes in various orientations, and hemispheres positioned with the flat section at the rear. The turbulence intensity was reported to be in the range: 9 to 10 % in all their experiments. The single equation correlated all data with a *deviation* of only ±15 % in the specified range.

The general correlation equation agrees with the Pasternak-Gauvin correlation equation within the given Reynolds number range to within 15 %. Therefore, the general equation of Yovanovich can be used for arbitrary convex isothermal bodies over a much wider range of the Reynolds number.

3. Laminar Forced Internal Flow

Definitions and Notation

Reynolds number: $$Re_D = \frac{U D_h}{\nu}$$

Laminar Flow: $$Re_D < 2300$$

Hydraulic Diameter: $$D_h = 4\frac{A}{P} = \frac{\text{Cross Section Area}}{\text{Wetted Perimeter}}$$

Dimensionless axial distance: $$x^\star = \frac{x}{D_h Pe} = \frac{x}{D_h Re Pr} = \frac{\pi}{4}\frac{1}{Gz}$$

Local, Isothermal Wall, Nusselt number: $$Nu_{x,UWT} = \frac{q_w(x)}{[T_w - T_m(x)]}\frac{D_h}{k}$$

Mean-value, Isothermal Wall, Nusselt number: $$Nu_{m,UWT} = \frac{q_w}{[T_w - T_m(x)]}\frac{D_h}{k}$$

Local, Isoflux Wall, Nusselt number: $$Nu_{x,UWF} = \frac{q_w}{[T_w(x) - T_m(x)]}\frac{D_h}{k}$$

Mean-value, Isoflux Wall, Nusselt number: $$Nu_{m,UWF} = \frac{q_w}{[T_w(x) - T_m(x)]}\frac{D_h}{k}$$

Local Nusselt Number for Thermally Developing Flow

Churchill and Ozoe (1973) propose the following correlation equations for the local Nusselt number for the developing thermal field for the UWT and UWF cases:

$$\frac{Nu_{x,UWT} + 1.7}{5.357} = \left[1 + \left(\frac{388}{\pi} x^{\star}\right)^{-8/9}\right]^{3/8} \qquad \pm 5\ \%$$

$$\frac{Nu_{x,UWF} + 1}{5.364} = \left[1 + \left(\frac{220}{\pi} x^{\star}\right)^{-10/9}\right]^{3/10} \qquad \pm 5\ \%$$

They developed these expressions based on asymptotic solutions valid for small and large values of $x^{\star}$.

Area-Average Nusselt Number for Fully Developed Hydraulic, Thermally Developing Flow

The following approximations of Shah (1975) for fully developed hydraulic flow and thermally developing in an isothermal (UWT) or an isoflux (UWF) circular pipe are based on the analytic solutions of the Graetz-type problems. Expressions for area-mean $Nu_{m,UWT}$, $Nu_{m,UWF}$ versus the local dimensionless position $x^{\star} = (x/D_h)/(RePr)$ are given below. The approximations are quite accurate over the entire range: $0.005 < x^{\star} \leq 1$. The maximum difference with respect to accurate analytic results is less than 4.4%.

For very small values $x^{\star} < 0.005$ the approximate expressions approach the Leveque asymptotes which were obtained by the method of similarity transformation. For large values $x^{\star} \geq 0.25$, the approximations go to the fully-developed hydraulic and thermal solutions: $Nu_{UWT} = 3.656$ and $Nu_{UWF} = 4.354$ which were obtained by the method of separation of variables which leads to a differential equation of the Sturm-Liouville type. The solution is presented as an infinite series expansion of eigenfunctions and corresponding eigenvalues.

$$Nu_{m,UWT} = \begin{cases} \dfrac{1.615}{(x^{\star})^{1/3}} - 0.2, & 0.005 < x^{\star} < 0.03 \\ 3.656 + \dfrac{0.0499}{x^{\star}}, & x^{\star} \geq 0.03 \end{cases}$$

$$Nu_{m,UWF} = \begin{cases} \dfrac{1.953}{(x^{\star})^{1/3}}, & x^{\star} \leq 0.03 \\ 4.354 + \dfrac{0.0722}{x^{\star}}, & x^{\star} > 0.03 \end{cases}$$

The circular cylinder results may be used to find approximate values for isothermal and isoflux tubes having other cross-sections (e.g. square or triangular pipes) by the use of the hydraulic diameter in the Nusselt and Reynolds numbers.

4. Laminar and Turbulent Natural External Flow

Definitions of Local and Area-average Values

$$^{\dagger}h_x = \frac{-k\dfrac{\partial T(x,y=0)}{\partial y}}{(T_w - T_\infty)}, \qquad \bar{h} = \tfrac{4}{3}h(x=L)$$

$$Nu_x = \frac{h_x x}{k}, \qquad \overline{Nu}_L = \frac{\bar{h}L}{k}$$

$$^{\dagger}Gr_x = \frac{g\beta\,(T_w - T_\infty)\,x^3}{\mu^2}, \qquad Ra_x = \frac{g\beta\,(T_w - T_\infty)\,x^3}{\alpha\nu}, \qquad Ra_x = Gr_x Pr$$

$$^{\dagger}Gr_L = \frac{g\beta\,(T_w - T_\infty)\,L^3}{\mu^2}, \qquad Ra_L = \frac{g\beta\,(T_w - T_\infty)\,L^3}{\alpha\nu}, \qquad Ra_L = Gr_L Pr$$

†For UWF cases, use the midpoint temperature difference:

$$(T_w(x = L/2) - T_\infty)$$

Flat Plate, Buoyancy-Induced Laminar Boundary Layer Flow

$\dfrac{Nu_x}{Ra_x^{1/4}} = 0.5027$	$10^4 < Gr_x < 10^9$	Local, UWT, $Pr \to \infty$
$\dfrac{Nu_x}{Ra_x^{1/4}} = 0.6004 Pr^{1/4}$	$10^4 < Gr_x < 10^9$	Local, UWT, $Pr \to 0$
$\dfrac{Nu_x}{Ra_x^{1/4}} = \dfrac{0.5027}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}}$	$10^4 < Gr_x < 10^9$	Local, UWT, $0 < Pr < \infty$
$\dfrac{Nu_x}{Ra_x^{1/4}} = 0.5627$	$10^4 < Gr_x^\star < 10^9$	Local, UWF, $Pr \to \infty$
$\dfrac{Nu_x}{Ra_x^{1/4}} = 0.6922 Pr^{1/4}$	$10^4 < Gr_x^\star < 10^9$	Local, UWF, $Pr \to 0$
$\dfrac{Nu_x}{Ra_x^{1/4}} = \dfrac{0.5627}{\left[1 + (0.437/Pr)^{9/16}\right]^{4/9}}$	$10^4 < Gr_x^\star < 10^9$	Local, UWF, $0 < Pr < \infty$
$\dfrac{\overline{Nu}_L}{Ra_L^{1/4}} = \dfrac{0.6703}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}}$	$10^4 < Gr_L < 10^9$	Average, UWT, $0 < Pr < \infty$
$\dfrac{\overline{Nu}_L}{Ra_L^{1/4}} = \dfrac{0.7503}{\left[1 + (0.437/Pr)^{9/16}\right]^{4/9}}$	$10^4 < Gr_L^\star < 10^9$	Average, UWF, $0 < Pr < \infty$

Flat Plate, Buoyancy-Induced Turbulent Boundary Layer Flow

$\overline{Nu}_L = \dfrac{0.150 Ra_L^{1/3}}{\left[1 + (0.492/Pr)^{9/16}\right]^{16/27}}$	$10^9 < Gr_L < 10^{12}$	Average, UWT, $0 < Pr < \infty$
$\overline{Nu}_L = \left\{0.825 + \dfrac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2$	$10^{-1} < Ra_L < 10^{12}$	Average, UWT, $0 < Pr < \infty$

Long Horizontal Isothermal Circular Cylinders, Laminar and Turbulent Flow

$$\overline{Nu}_D = \left[0.60 + \frac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right]^2 \qquad 0 < Pr < \infty, \qquad 10^{-5} < Ra_D \leq 10^{12}$$

Finite Horizontal Isothermal Circular Cylinders, Laminar Flow

$$\overline{Nu}_D = S_D^{\star} + \frac{0.518 Ra_D^{1/4}}{[1 + (0.559/Pr)^{9/16}]^{4/9}} \qquad 0 < Pr < \infty, \qquad 0 < Ra_D \leq 10^9$$

Isothermal Spheres, Laminar Flow

$$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}, \qquad 0 < Pr < \infty, \qquad 0 \leq Ra_D < 10^{11}$$

5. General Correlation Equation for Arbitrary Isothermal Convex Bodies

$$Nu_{\mathcal{L}} = Nu_{\mathcal{L}}^0 + F(Pr) G_{\mathcal{L}} Ra_{\mathcal{L}}^{1/4}, \qquad 0 < Pr < \infty \qquad 0 \leq Ra_{\mathcal{L}} \leq 10^{11}$$

where the characteristic body length is $\mathcal{L} = \sqrt{A}$ and A is the total active or wetted surface area. The *universal* Prandtl number function valid for all isothermal convex bodies is given by:

$$F(Pr) = \frac{0.670}{[1 + (0.5/Pr)^{9/16}]^{4/9}}$$

which for air ($Pr = 0.71$) has the value $F(Pr = 0.71) = 0.513$. The diffusive limit $Nu_{\mathcal{L}}^0$ or shape factor $S_{\mathcal{L}}^{\star}$ with $\mathcal{L} = \sqrt{A}$ is a weak function of body shape and its aspect ratio. For example, its range is $3.20 \leq S^{\star}_{\sqrt{A}} < 7.55$ for a solid circular cylinder whose length-to-diameter ratio varies from 0 (a circular disk) to 100 (very long cylinder). For long axisymmetric bodies (e.g. circular cylinder and long square cuboid) the shape factor can be accurately approximated by $S_{\sqrt{A}} = 4\sqrt{L/D}/\ln(2L/D)$ where D is the diameter of the circular cylinder and it is equal to the geometric-mean of the diameters of the inscribed and circumscribed circular cylinders respectively, and L is the length.

The body-gravity function $G_{\mathcal{L}}$ accounts for the buoyancy-induced flow over the convex body. It is a relatively weak function of the body shape and its orientation with respect to the gravity vector when $\mathcal{L} = \sqrt{A}$ is used and the complex convex body (e.g. a cuboid) has dimensions (H, W, L) which do not go to zero $H \neq 0$ in the direction parallel to the gravity vector (e.g. a horizontal rectangular plate), and the other dimensions (W, L) which are perpendicular to the gravity vector do not go to ∞. Otherwise the body-gravity function will lie in the narrow range $0.9 < G_{\sqrt{A}} < 1.1$. For example the body-gravity function for a sphere, horizontal cube, and a short circular cylinder $L/D = 1$ with active sides and ends in the horizontal, inclined at $45-$ degrees and vertical orientations have the empirical values: $G_{\sqrt{A}} = 1.023, 0.951, 1.019, 1.004, 0.967$ respectively. These empirical values are within 3 % of the theoretical values. There are many other complex convex bodies which have body-gravity functions close to unity.

The body-gravity function for a horizontal cuboid (H, W, L) where the $H-$side is parallel to g and the other two sides (W, L) are perpendicular to g may be used to estimate $G_{\sqrt{A}}$ for convex bodies which have surfaces which are parallel and perpendicular to g. The body-gravity function for cuboids is given by:

$$G_{\sqrt{A}} = 2^{1/8}\left[\frac{0.625L^{4/3}W + H(L+W)^{4/3}}{(HW + HL + LW)^{7/6}}\right]^{3/4}$$

The above analytic-empirical relationship reduces to several important special cases.

<u>Horizontal Cube, All Surfaces Active:</u> $H = W = L = 1$

$$G_{\sqrt{A}} = 0.984$$

<u>Horizontal Rectangular Plates, Both Sides Active:</u> $H = 0, L \geq W$

$$G_{\sqrt{A}} = 0.7665(L/W)^{1/8}, \qquad \frac{L}{W} \geq 1$$

<u>Horizontal Square Plates, Both Sides Active:</u> $H = 0, L = W$

$$G_{\sqrt{A}} = 0.7665$$

Vertical Rectangular Plates, Both Sides Active: $L = 0$

$$G_{\sqrt{A}} = 2^{1/8}(W/H)^{1/8}, \qquad 0 < \frac{W}{H} < \infty$$

If the vertical plate has *one side active*, omit the factor $2^{1/8}$.

Vertical Square Plate, Both Sides Active: $H = W, L = 0$

$$G_{\sqrt{A}} = 2^{1/8} = 1.0905$$

If the vertical square plate has *one side active*, omit the factor $2^{1/8}$ and $G_{\sqrt{A}} = 1$.

Long Horizontal Square Prisms with Active Ends: $H = W << L$

$$G_{\sqrt{A}} = 0.856 \left(\frac{L}{H}\right)^{1/8}, \qquad \frac{L}{H} > 10$$

Long Vertical Square Prisms with Active Ends: $L = W, 0 \leq H/W < \infty$

$$G_{\sqrt{A}} = 2^{1/4}\frac{(0.250 + H/W)^{3/4}}{(0.500 + H/W)^{7/8}}$$

Heated Horizontal Rectangular Plates Facing Upward or Downward: $H = 0, L/W \geq 1$

$$G_{\sqrt{A}} = 2^{1/8}\left(\frac{L}{W}\right)^{1/8} \qquad \text{facing upward}$$

$$G_{\sqrt{A}} = \frac{2^{1/8}}{2}\left(\frac{L}{W}\right)^{1/8} \qquad \text{facing downward}$$

Other Body Shapes

The body-gravity function for other body shapes such as the finite circular cylinder with active sides and ends in the horizontal and vertical orientations can be accurately approximated by using the results for the square prism with active ends in the horizontal or vertical orientations.