ME 353 M.M. Yovanovich

Week 3

Lecture 1
ME 353 Web Site: Summary of week 2 topics are on the Web.
Contact resistance: $R_c = 1/(h_c A)$; see Table 3.1 for typical values of $R''_{t,c}$; how to get h_c, contact conductance, from Table 3.1; $h_c = 1/R''_{t,c}\,[W/m^2 K]$.
Thermal resistance R of composite systems.
Overall conductance U for cylindrical systems. This parameter has units of $[W/m^2 \cdot K]$.
Definitions of U_i and U_o and relationships.

$$Q_{sys} = U_i A_i (T_{f1} - T_{f2}) = U_o A_o (T_{f1} - T_{f2}) = (T_{f1} - T_{f2})/R_{total}$$

where A_i and A_o are the inner and outer surface areas and $A_o > A_i$. Since $U_i A_i = U_o A_o$, then $U_i > U_o$. Overall conductances are frequently used in heat exchanger design. It is recommended that the total resistance be used in analysis because there is no confusion when it is implemented.

A cylindrical shell $a \le r \le b$ with temperature dependent thermal conductivity, $k(T)$, in the temperature range $T_2 \le T \le T_1$. Separate the variables T and r, and integrate between appropriate limits:

$$\int_{r=a}^{r=b} \frac{Q\,dr}{2\pi L\,r} = \int_{T=T_1}^{T=T_2} -k_0(1+\beta T)\,dT$$

where k_0 is the value of k at some reference temperature and β is thermal conductivity temperature coefficient. This is an empirical (experimental) relation. Complete integrations to get

$$\frac{Q}{2\pi L} \ln\frac{b}{a} = k_0 \left(1 + \frac{\beta}{2}(T_1+T_2)\right)(T_1 - T_2) = k_{ave}(T_1 - T_2)$$

where $k_{ave} = k_0 \left(1 + \frac{\beta}{2}(T_1+T_2)\right)$ is the value of the thermal conductivity at the average temperature of the cylindrical shell $(T_1+T_2)/2$.
Thermal resistance of the cylindrical shell is

$$R_{cyl} = \frac{T_1 - T_2}{Q} = \frac{1}{2\pi L\,k_{ave}} \ln\frac{b}{a}$$

A similar analysis can be applied to the plane wall and the spherical shell.

Lecture 2

Full solution of 1D Laplace equation in cylindrical shell with boundary conditions of the third kind (Robin conditions) at inner and outer boundaries. Found $T = T(r, a, b, k, h_1, h_2, T_{f1}, T_{f2})$ and system heat transfer rate Q_{sys}. The total resistance of the system consists of three resistances (2 film resistances and 1 solid resistance) in series. Discussion of the physical interpretations of the dimensionless temperature excess and the system heat flow rate.
The full solution can be applied to a plane wall and a spherical shell.

Lecture 3

Solutions of one-dimensional Poisson equation in plane wall, long solid circular cylinder and solid sphere; all systems are convectively cooled;
see ME 353 Web Site for general equation and general solution with parameter n where $n = 0$ for plane wall, $n = 1$ for cylinder, and $n = 2$ for sphere;
surface temperature rise: $\boxed{T_s - T_f = \mathcal{P}b/((n+1)h)}$;
solid temperature rise: $\boxed{T_{max} - T_s = \mathcal{P}b^2/(2(n+1)k)}$;
Biot number for all cases: $\boxed{Bi = hb/(k)}$;
$\boxed{\Delta T_{\text{solid}}/\Delta T_{\text{film}} = Bi/2}$ for all cases;
examples: nuclear fuel element with cladding; ohmic heating in wire with insulation.
