ME 353 M.M. Yovanovich

Week 2

Lecture 1

Solutions to problems are available in Engineering Photocopy Center.
First makeup lecture: Thursday, 8:30 AM, CPH 3385.
Discuss the ME 353 Website. Calendar, Assigments, Projects, Exams, Lectures, Notes, Maple, Calculators and Links.

Survey students regarding experience with Spread Sheets and Computer Algebra Systems: **Excel, QuattroPro, Lotus 1,2,3** and **Mathcad, Matlab, Maple, Mathematica, Macsyma**.

Continued discuss of radiation; Stefan-Boltzmann Law of Radiation; thermal circuit; definition of radiative conductance, h_r; $Q_{12} = h_r A_1 (T_1 - T_2)$;
general relation: $\boxed{h_r = \dfrac{\sigma(T_1^2 + T_2^2)(T_1 + T_2)}{A_1 R_{rad}}}$;
special case: $A_2 >> A_1, F_{12} = 1$; $\boxed{h_r = \epsilon_1 \sigma (T_1^2 + T_2^2)(T_1 + T_2)}$;
compute h_r for black-body radiation: $\epsilon_1 = 1, T_2 = 300\,K$, and $\Delta T = (T_1 - T_2) = 1, 10, 100\,K$; see Web Site;
see Example 1.5 for system with conduction, convection and radiation; see corresponding Maple worksheet on Web Site.

Lecture 2

Read Chapter 2: Sections: 2.1-2.4.
Fourier's Rate Equation: $\vec{q} = -k \nabla T$
Heat flux vector: $\vec{q} = \vec{i}\, q_x'' + \vec{j}\, q_y'' + \vec{k}\, q_z''$
Temperature gradient: $\vec{i}\, \partial T/\partial x + \vec{j}\, \partial T/\partial y + \vec{k}\, \partial T/\partial z$
Heat flux components: $q_x'' = -k\, \partial T/\partial x,\ q_y'' = -k\, \partial T/\partial y,\ q_z'' = -k\, \partial T/\partial z$

Thermal conductivity: $k = k(T), [W/(m \cdot K)]$; See Figures 2.4-2.7 for values for various substances such as gases, liquids, non-metals, alloys and pure metals.
Thermal diffusivity: $\alpha = k/(\rho c_p)$ $[m^2/s]$ is important in transient conduction.

Heat Diffusion Equations: Based on

Conduction
Energy generation: $\dot{E}_g = \dot{q} dV$
Energy storage: $\dot{E}_{st} = \rho c_p \frac{\partial T}{\partial t} dV$

Select differential control volume: dV in
– cartesian coordinates: (x, y, z)
– cylindrical coordinates: (r, ϕ, z)
– spherical coordinates: (r, ϕ, θ)

Apply conservation of energy principle to dV to get Heat Conduction Equation.

$$\left\{\frac{\text{Conduction}}{\text{Rate Into dV}}\right\} - \left\{\frac{\text{Conduction}}{\text{Rate Out of dV}}\right\} + \left\{\frac{\text{Generation}}{\text{Rate Within dV}}\right\} = \left\{\frac{\text{Energy Storage}}{\text{Rate Within dV}}\right\}$$

This gives the general non-linear Conduction Equation in vector notation:

$$\nabla \cdot (k \nabla T) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

For constant thermal conductivity equation becomes:

$$\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Special cases:
1) Steady-state: $(\partial T / \partial t = 0)$, source free $(\dot{q} = 0)$

$$\nabla^2 T = 0, \qquad \text{Laplace equation}$$

2) Steady-state: $(\partial T / \partial t = 0)$, with sources $(\dot{q} > 0)$

$$\nabla^2 T = -\frac{\dot{q}}{k}, \qquad \text{Poisson equation}$$

3) Transient: $(\partial T / \partial t \neq 0)$, source free $(\dot{q} = 0)$

$$\nabla^2 T = \frac{1}{\alpha} \frac{\partial T}{\partial t}, \qquad \text{Diffusion equation}$$

IC (Initial Condition) and BCs (Boundary Conditions):
1 IC
2 BCs for 1D Conduction; eg, $T = T(x, t)$
4 BCs for 2D Conduction; eg, $T = T(x, y, t)$

6 BCs for 3D Conduction; eg, $T = T(x, y, z, t)$

Three types of BCs for $t > 0$:
1) Dirichlet (BC of First Kind): Temperature is prescribed on the boundary.
2) Neumann (BC of Second Kind): Temperature gradient is prescribed on the boundary.
3) Robin (BC of Third Kind): Linear superposition of temperature and its gradient with appropriate coefficients are prescribed on the boundary.

See text for derivation of general differential equation, and differential equations in the three coordinate systems: Cartesian, cylindrical and spherical.

Lecture 3

Read Chapter 3: Sections: 3.1-3.7.
Solutions of one-dimensional Laplace and Poisson equations in Cartesian, cylindrical and spherical coordinates; applications of the three types of boundary conditions; three methods to find thermal resistances of solids R: (i) full solution of governing equation with Dirichlet boundary conditions; (ii) integration of Fourier equation of conduction after separation of variables; (iii) physical approach; text discusses the first two methods.

Lecture 4 (Makeup Lecture 1)
Solutions of 1D Laplace equation $\nabla^2 T = 0$.
The Laplacian operator in Cartesian coordinates:

$$\nabla^2 = \nabla \cdot \nabla = (\vec{i}\,\partial T/\partial x + \vec{j}\,\partial T/\partial y + \vec{k}\,\partial T/\partial z) \cdot (\vec{i}\,\partial T/\partial x + \vec{j}\,\partial T/\partial y + \vec{k}\,\partial T/\partial z)$$

Therefore,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad \left[\frac{1}{m^2}\right]$$

Thermal resistance: R, and shape factor: $S = 1/(kR)$
- plane wall: $(0 \le x \le L)$ where $A =$ constant,
- long cylindrical shell of length $L >> r_2 > r_1$ with $(r_1 \le r \le r_2)$ where $A(r) = 2\pi r L$,
- spherical shell: $(r_1 \le r \le r_2)$ where $A(r) = 4\pi r^2$.

Physical approach based on $dR = dr/(kA(r))$ and $R = \int dR$;
- plane wall: $\boxed{R = L/(kA)}$; $\boxed{S = A/L}$;

• cylindrical shell: $\boxed{R = \ln(r_2/r_1)/(2\pi Lk)}$; $\boxed{S = 2\pi L/\ln(r_2/r_1)}$;

• spherical shell: $\boxed{R = 1/(4\pi k)(1/r_1 - 1/r_2)}$; $\boxed{S = 4\pi/(1/r_1 - 1/r_2)}$;

special case: isolated sphere of radius r_1 in an infinite domain $r_2 \to \infty$ of conductivity k: $\boxed{R = 1/(4\pi k r_1)}$; $\boxed{S = 4\pi r_1}$;

see Table 3.3 for summary of one-dimensional solutions, in plane wall, cylindrical shell and spherical shell.

Example of compound cylinder: $a \le r \le b$ of thermal conductivity k_1 and $b \le r \le c$ of thermal conductivity k_2 with heat transfer coefficient h_1 on inner surface $A_1 = 2\pi aL$ and heat transfer coefficient h_2 on outer surface $A_2 = 2\pi cL$. There is contact resistance R_c at interface at $r = b$. There are 5 resistances in series: 2 solid resistances: R_{s1}, R_{s2}, 2 film resistances: R_{f1}, R_{f2} and the contact resistance.

Total resistance of the system: $R_{total} = R_{f1} + R_{s1} + R_c + R_{s2} + R_{f2}$ with

$R_{s1} = \frac{1}{2\pi k_1 L}\ln(b/a)$

$R_{s2} = \frac{1}{2\pi k_2 L}\ln(c/b)$

$R_{f1} = \frac{1}{h_1 2\pi aL}$

$R_{f2} = \frac{1}{h_2 2\pi cL}$

$R_c = \frac{1}{h_c 2\pi bL}$

Heat transfer rate through system:

$$Q_{sys} = \frac{(T_{f1} - T_{f2})}{R_{total}}$$
