ME 353 M.M. Yovanovich

Week 11

Lecture 1

ME 353 Heat Transfer Lab begins on Monday. See signup sheet.
Forced internal laminar and turbulent convection in circular and noncircular tubes, pipes and ducts.
See handout for definitions of local and average Nusselt numbers and other pertinent dimensionless parameters.
Discuss hydrodynamic developing length, fully-developed velocity ditribution in circular tube. Hydraulic diameter D_h for noncircular tubes.
Developing thermal length with fully-developed flow.

Lecture 2

Developing lengths:

$$\frac{x_{fd,h}}{D} \simeq 0.05 Re_D, \qquad \text{Hydraulic Length}$$

and

$$\frac{x_{fd,t}}{D} \simeq 0.05 Re_D Pr, \qquad \text{Thermal Length}$$

When $Pr = 1$, then $x_{fd,h} = x_{fd,t}$, approximately.
For turbulent flow the hydrodynamic length is approximately $L/D \simeq 10$.
Simple, single tube heat exchanger. Laminar, fully-developed flow in a UWT circular tube. Wall temperature is T_w(or T_s). The bulk temperature is T_b(or T_m). The inlet and outlet temperature differences are $\Delta T_i = T_w - T_{b,i}$ and $\Delta T_o = T_w - T_{b,o}$. Temperature difference at arbitrary location x is $\Delta T(x) = T_w - T_b(x)$. Effective temperature difference over the length L is called the Log Mean Temperature Difference (LMTD) defined as

$$\Delta T_{LMTD} = \frac{\Delta T_i - \Delta T_0}{\ln\left[\dfrac{\Delta T_i}{\Delta T_o}\right]} = \frac{\Delta T_o - \Delta T_i}{\ln\left[\dfrac{\Delta T_o}{\Delta T_i}\right]}$$

Temperature rise in heat exchanger

$$\frac{T_w - T_b(x)}{T_w - T_{b,i}} = \exp\left(-\frac{Px\bar{h}}{\dot{m}c_p}\right), \qquad 0 \leq x \leq L$$

where $P = \pi D$ is the perimeter of the circular tube, $\dot{m} = \rho \bar{U} A$ is the mass flow rate through the tube of cross-section $A = \pi D^2/4$, and $\bar{h}$ is the average value of the heat transfer coefficient over the length L. The heat transfer coefficient depends on position because both the wall heat flux $q_w(x)$ and the local temperature difference $\Delta T(x)$ are variables. When $x = L$, $T_b(x = L) = T_{b,o}$. This relation can be used to solve some interesting problems.
(i) Given the temperatures: $T_w, T_{b,i}, T_{b,o}$ and $D, \dot{m}, c_p, \bar{h}$, find L.
(ii) Given the temperatures: $T_w, T_{b,i}, T_{b,o}$ and $D, L, \dot{m}, c_p$, find $\bar{h}$.
(ii) Given the temperatures: $T_w, T_{b,i}$ and $D, L, \dot{m}, c_p, \bar{h}$, find $T_{b,o}$.

Heat transfer rate to the fluid is obtained from

$$Q = \dot{m} c_p \left(T_{b,o} - T_{b,i}\right) = \rho c_p \bar{U} A \left(T_{b,o} - T_{b,i}\right)$$

Complete review of the area-average Nusselt number relations for fully-developed hydraulic, thermally developing flow conditions developed by Shah and London (1975): $Nu_{m,UWT}$ and $Nu_{m,UWF}$ as a function of the dimensionless axial position $x^{star} = x/(D Re_D Pr)$. For thermally fully-developed flow the asymptotic values for the circular tube are: $Nu_{m,UWT} = 3.656$ and $Nu_{m,UWF} = 4.354$.
See Table 8.1 on page 450 for values of $Nu_{D_h} = h D_h / k$ for UWT and UWF conditions for other noncircular tubes (pipes, ducts) such as rectangular ducts.

Turbulent, fully-developed flow $Re_D > 2300$. Correlation equation, Eq. (8.63) can be used

$$Nu_D = \frac{(f/8)\left(Re_D - 1000\right) Pr}{1 + 12.7 (f/8)^{1/2} \left(Pr^{2/3} - 1\right)}, \qquad \frac{L}{D} \geq 10$$

for $0.5 < Pr < 2000$ and $3000 \leq Re_D \leq 5 \times 10^6$ with Eq. (8.21)

$$f = \left(0.790 \ln Re_D - 1.64\right)^{-2}$$

See Table 8.4 on page 460 for other relationships.
For noncircular tubes, pipes and ducts use the relations for the circular tube, but replace the tube diameter with the hydraulic diameter of the given cross-section:

$$D_h = \frac{4A}{P}$$

Lecture 3

External natural convection heat transfer correlations. Definitions of dimensionless groups such as local and average Nusselt Nu, Grashof Gr and Rayleigh Ra numbers.

Temperature and velocity distributions across thermal and hydrodynamic boundary layers adjacent to a vertical isothermal plate in contact with a stagnant fluid of large extent.
Show $Nu - Ra$ plot for arbitrary geometry over the Rayleigh number range: $0 \le Ra_{\mathcal{L}} < 10^{12}$. Discuss the different regions: pure conduction, laminar boundary layer flow, turbulent boundary layer flow and the transitions.
Present general correlation equation for isothermal convex geometries:

$$Nu_{\mathcal{L}} = S_{\mathcal{L}}^{\star} + F(Pr) G_{\mathcal{L}} Ra_{\mathcal{L}}^{1/4}$$

where $S_{\mathcal{L}}^{\star}$ is the dimensionless shape factor for pure conduction. The Prandtl number function is defined as

$$F(Pr) = \frac{0.670}{\left[1 + (0.5/Pr)^{9/16}\right]^{4/9}}, \qquad 0 \le Pr < \infty$$

Note the text uses 0.492 in place of 0.5. For air $Pr = 0.71$, the Prandtl number function gives $F(Pr = 0.71) = 0.513$.
Different length scales have been proposed for different geometries:
• Vertical plate: $\mathcal{L} = L$, the plate height parallel to the gravity vector.
• Sphere: $\mathcal{L} = a$, the sphere radius or $\mathcal{L} = D$, the sphere diameter which is used most frequently.
• Finite circular cylinder of length L and diameter D:
−vertical orientation: $\mathcal{L} = L$
−horizontal orientation: $\mathcal{L} = D$

Yovanovich (1987) proposed the use of the square root of the total active surface area: $\mathcal{L} = \sqrt{A}$. For the finite circular cylinder this gives:

$$\mathcal{L} = \sqrt{A} = \sqrt{\pi D L + 2\pi D^2/4}$$

where $\pi D L$ is the side surface area and $2\pi D^2/4$ represents the two end surface areas. This length scale contains the two length scales used by other researchers. It does not change with the orientation.
When $\mathcal{L} = \sqrt{A}$ is used, then $3.19 \le S_{\sqrt{A}}^{\star} \le 4.4$ lies in a narrow range. Also the body gravity-function now lies in a narrow range as shown in Table 1.

The body-gravity function $G_{\sqrt{A}}$ is a relatively weak function of body shape and orientation.

See the handout notes for details of the body-gravity function as applied to:
• Cuboids
• Horizontal rectangular plates (both sides active)

Table 1: Short Table of Body-Gravity Functions

Geometry	Orientation	$G_{\sqrt{A}}$
Sphere		1.014
Cylinder ($L/D = 1$)	Vertical Axis	0.967
Cylinder ($L/D = 1$)	Horizontal Axis	1.051
Cube	Horizontal	0.984
Cube	On Edge	1.080
Cube	On Corner	1.091

- Vertical rectangular plates (both sides active)
- Long horizontal square prisms with active ends
