13.17

A mixture of 60% N_2, 30% Ar and 10% O_2 on a mole basis is in a cylinder at 250 kPa, 310 K and volume 0.5 m^3. Find the mass fractions and the mass of argon.

Solution:

From Eq. 13.3: $c_i = y_i M_i / \sum y_j M_j$

Eq.13.5:

$M_{mix} = \sum y_j M_j = 0.6 \times 28.013 + 0.3 \times 39.948 + 0.1 \times 31.999 = 31.992$

$c_{N2} = (0.6 \times 28.013) / 31.992 = 0.5254$

$c_{Ar} = (0.3 \times 39.948) / 31.992 = 0.3746$

$c_{O2} = (0.1 \times 31.999) / 31.992 = 0.1$, sums to 1 OK

From Eq.13.14:

$R_{mix} = \bar{R}/M_{MIX} = 8.3145 / 31.992 = 0.260$ kJ/kg K

$m_{mix} = PV/(R_{mix} T) = 250 \times 0.5 / 0.26 \times 310 = 1.551$ kg

$m_{Ar} = c_{Ar} \times m_{mix} = 0.3746 \times 1.551 =$ **0.581 kg**

13.19

A new refrigerant R-407 is a mixture of 23% R-32, 25% R-125 and 52% R-134a on a mass basis. Find the mole fractions, the mixture gas constant and the mixture heat capacities for this new refrigerant.

Solution:

From the conversion in Eq.13.4 we get:

	c_i	M_i	c_i/M_i	y_i
R-32	0.23	52.024	0.004421	0.381
R-125	0.25	120.022	0.002083	0.180
R-134a	0.52	102.03	0.0050965	0.439
			0.0116005	

Eq.13.15:

$$R_{mix} = \sum c_i R_i = 0.23 \times 0.1598 + 0.25 \times 0.06927 + 0.52 \times 0.08149$$
$$= \mathbf{0.09645\ kJ/kg\ K}$$

Eq.13.23:

$$C_{P\ mix} = \sum c_i\, C_{P\ i} = 0.23 \times 0.822 + 0.25 \times 0.791 + 0.52 \times 0.852$$
$$= \mathbf{0.8298\ kJ/kg\ K}$$

Eq.13.21:

$$C_{v\ mix} = \sum c_i C_{v\ i} = 0.23 \times 0.662 + 0.25 \times 0.721 + 0.52 \times 0.771$$
$$= \mathbf{0.7334\ kJ/kg\ K} \quad (= C_{P\ MIX} - R_{MIX})$$

13.22

A new refrigerant R-410a is a mixture of R-32 and R-125 in a 1:1 mass ratio. What are the overall molecular weight, the gas constant and the ratio of specific heats for such a mixture?

Eq.13.5:

$$M = \sum y_j M_j = = 1 / \sum (c_j / M_j) = \frac{1}{\frac{0.5}{52.024} + \frac{0.5}{120.022}} = \mathbf{72.586}$$

Eq.13.15:

$$R_{mix} = \sum c_i R_i = 0.5 \times 0.1598 + 0.5 \times 0.06927 = \mathbf{0.1145\ kJ/kg\ K}$$

$$= \bar{R}/M_{MIX} = 8.3145 / 72.586 = \text{same} \quad \text{(this is from Eq.13.14)}$$

Eq.13.23:

$$C_{P\ mix} = \sum c_i\, C_{P\ i} = 0.5 \times 0.822 + 0.5 \times 0.791 = 0.8065 \text{ kJ/kg K}$$

Eq.13.21:

$$C_{V\ mix} = \sum c_i C_{V\ i} = 0.5 \times 0.662 + 0.5 \times 0.722 = 0.692 \text{ kJ/kg K}$$

$$(= C_{P\ mix} - R_{mix})$$

$$k_{mix} = C_{P\ mix} / C_{V\ mix} = 0.8065 / 0.692 = \mathbf{1.1655}$$

13.30

A rigid insulated vessel contains 12 kg of oxygen at 200 kPa, 280 K separated by a membrane from 26 kg carbon dioxide at 400 kPa, 360 K. The membrane is removed and the mixture comes to a uniform state. Find the final temperature and pressure of the mixture.

Solution:

C.V. Total vessel. Control mass with two different initial states.

Mass: $m = m_{O2} + m_{CO2} = 12 + 26 = 38$ kg

Process: V = constant (rigid) => W = 0, insulated => Q = 0

Energy: $U_2 - U_1 = 0 - 0 = m_{O2}\, C_{V\,O2}(T_2 - T_{1\,O2}) + m_{CO2}C_{V\,CO2}(T_2 - T_{1\,CO2})$

Initial state from ideal gas Table A.5

$C_{V\,O2} = 0.662$ kJ/kg, $C_{V\,CO2} = 0.653$ kJ/kg K

O_2 : $V_{O2} = mRT_1/P = 12 \times 0.2598 \times 280/200 = 4.3646\ m^3$,

CO_2 : $V_{CO2} = mRT_1/P = 26 \times 0.1889 \times 360/400 = 4.4203\ m^3$

Final state mixture

$R_{MIX} = \sum c_iR_i = [12 \times 0.2598 + 26 \times 0.1889\ ]/38 = 0.2113$ kJ/kg K

The energy equation becomes

$$m_{O2}\, C_{V\,O2}\, T_2 + m_{CO2}C_{V\,CO2}\, T_2$$
$$= m_{O2}\, C_{V\,O2}\, T_{1\,O2} + m_{CO2}C_{V\,CO2}\, T_{1\,CO2}$$

$$(7.944 + 16.978\ )\, T_2 = 2224.32 + 6112.08 = 8336.4 \text{ kJ}$$

$$\Rightarrow\ T_2 = \mathbf{334.5\ K}$$

From mixture gas constant and total volume

$P_2 = mR_{mix}T_2/V = 38 \times 0.2113 \times 334.5 / 8.7849 =$ **305.7 kPa**

13.39

Natural gas as a mixture of 75% methane and 25% ethane by mass is flowing to a compressor at 17°C, 100 kPa. The reversible adiabatic compressor brings the flow to 250 kPa. Find the exit temperature and the needed work per kg flow.

Solution:

C.V. Compressor. Steady, adiabatic q = 0, reversible $s_{gen} = 0$

Energy Eq.6.13: $-w = h_{ex} - h_{in}$; Entropy Eq.9.8: $s_i + s_{gen} = s_e$

Process: reversible => $s_{gen} = 0$ => $s_e = s_i$

Assume ideal gas mixture and constant heat capacity, so we need k and C_P

From Eq.13.15 and 13.23:

$$R_{mix} = \sum c_i R_i = 0.75 \times 0.5183 + 0.25 \times 0.2765 = 0.45785 \text{ kJ/kg K}$$

$$C_{P\ mix} = \sum c_i C_{Pi} = 0.75 \times 2.254 + 0.25 \times 1.766 = 2.132 \text{ kJ/kg K}$$

$$C_V = C_{P\ mix} - R_{mix} = 2.132 - 0.45785 = 1.6742 \text{ kJ/kg K}$$

Ratio of specific heats: $k = C_p / C_v = 1.2734$

The isentropic process gives Eq.8.23

$$T_e = T_i\, (P_e / P_i)^{(k-1)/k} = 290\, (250/100)^{\,0.2147} = \mathbf{353\ K}$$

Work from the energy equation:

$$w_{c\ in} = C_P\, (T_e - T_i) = 2.132\, (353 - 290) = \mathbf{134.3\ kJ/kg}$$

13.53

A flow of 1.8 kg/s steam at 400 kPa, 400°C is mixed with 3.2 kg/s oxygen at 400 kPa, 400 K in a steady flow mixing-chamber without any heat transfer. Find the exit temperature and the rate of entropy generation.

C.V. Mixing chamber, steady flow, no work, no heat transfer. To do the entropies we need the mole fractions.

$$\dot{n}_{H2O} = \frac{\dot{m}_{H2O}}{M_{H2O}} = \frac{1.8}{18.015} = 0.1 \text{ kmol/s}; \quad \dot{n}_{O2} = \frac{\dot{m}_{O2}}{M_{O2}} = \frac{3.2}{31.999} = 0.1 \text{ kmol/s}$$

$$y_{H2O} = y_{O2} = 0.5$$

Energy Eq.: $\dot{m}_{H2O}\, h_1 + \dot{m}_{O2}\, h_2 = \dot{m}_{H2O}\, h_{3\ H2O} + \dot{m}_{O2}\, h_{3\ O2}$

Entropy Eq.: $\dot{m}_{H2O}\, s_1 + \dot{m}_{O2}\, s_2 + \dot{S}_{gen} = \dot{m}_{H2O}\, s_{3\ H2O} + \dot{m}_{O2}\, s_{3\ O2}$

Solve for T from the energy equation

$$\dot{m}_{H2O}\,(h_{3\ H2O} - h_1) + \dot{m}_{O2}\,(h_{3\ O2} - h_2) = 0$$

$$\dot{m}_{H2O}\, C_{P\ H2O}(T_3 - T_1) + \dot{m}_{O2}\, C_{P\ O2}(T_3 - T_2) = 0$$

$$1.8 \times 1.872\,(T_3 - 400 - 273.15) + 3.2 \times 0.922(T_3 - 400) = 0$$

$$T_3 = \mathbf{545.6\ K}$$

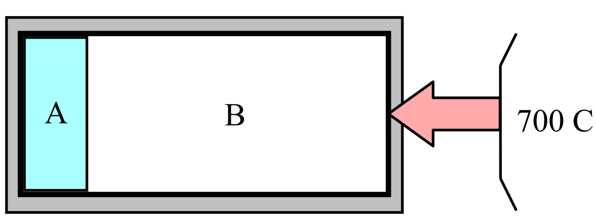
$$\dot{S}_{gen} = \dot{m}_{H2O}\,(s_{3\ H2O} - s_1) + \dot{m}_{O2}\,(s_{3\ O2} - s_2)$$

$$= \dot{m}_{H2O}\,[\, C_{P\ H2O} \ln \frac{T_3}{T_1} - R \ln y_{H2O}\,] + \dot{m}_{O2}\,[\, C_{P\ O2} \ln \frac{T_3}{T_2} - R \ln y_{O2}\,]$$

$$= 1.8\,[\, 1.872 \ln \frac{545.6}{673.15} - 0.4615 \ln 0.5\,]$$

$$+ 3.2\,[\, 0.922 \ln \frac{545.6}{400} - 0.2598 \ln 0.5\,]$$

$$= -\,0.132 + 1.492 = \mathbf{1.36\ kW/K}$$



13.69

Consider 100 m^3 of atmospheric air which is an air–water vapor mixture at 100 kPa, 15°C, and 40% relative humidity. Find the mass of water and the humidity ratio. What is the dew point of the mixture?

Solution:

Air-vapor P = 100 kPa, T = 15 ^{o}C, ϕ = 40%

Use Table B.1.1 and then Eq.13.25

$P_g = P_{sat15} = 1.705$ kPa => $P_v = \phi\ P_g = 0.4\times1.705 = 0.682$ kPa

$$m_v = \frac{P_v V}{R_v T} = \frac{0.682\times100}{0.461\times288.15} = \mathbf{0.513\ kg}$$

$P_a = P_{tot} - P_{v1} = 100 - 0.682 = 99.32$ kPa

$$m_a = \frac{P_a V}{R_a T} = \frac{99.32\times100}{0.287\times288.15} = 120.1 \text{ kg}$$

$$w_1 = \frac{m_v}{m_a} = \frac{0.513}{120.1} = \mathbf{0.0043}$$

T_{dew} is T when $P_v = P_g = 0.682$ kPa;

Table B.1.2 gives T = **1.4 ^{o}C**

13.78

A rigid container, 10 m^3 in volume, contains moist air at 45°C, 100 kPa, ϕ = 40%. The container is now cooled to 5°C. Neglect the volume of any liquid that might be present and find the final mass of water vapor, final total pressure and the heat transfer.

Solution:

CV container. $m_2 = m_1$; $m_2u_2 - m_1u_1 = {}_1Q_2$

State 1: 45°C, ϕ = 40% => $w_1 = 0.0236$, $T_{dew} = 27.7°C$

Final state $T_2 < T_{dew}$ so condensation, $\phi_2 = 100\%$

$P_{v1} = 0.4\ P_g = 0.4 \times 9.593 = 3.837$ kPa, $P_{a1} = P_{tot} - P_{v1} = 96.163$ kPa

$m_a = P_{a1}V/RT_1 = 10.532$ kg, $m_{v1} = w_1\ m_a = 0.248$ kg

$P_{v2} = P_{g2} = 0.8721$ kPa, $P_{a2} = P_{a1}T_2/T_1 = 84.073$ kPa

$P_2 = P_{a2} + P_{v2} =$ **84.95 kPa**

$m_{v2} = P_{v2}V/R_vT_2 =$ **0.06794 kg** (= V/v_g = 0.06797 steam table)

$m_{f2} = m_{v1} - m_{v2} = 0.180$ kg

The heat transfer from the energy equation becomes

$${}_1Q_2 = m_a(u_2-u_1)_a + m_{v2}u_{g2} + m_{f2}u_{f2} - m_{v1}u_{g1}$$

$$= m_a\ C_v(T_2 - T_1) + m_{v2}\ 2382.3 + m_{f2}\ 20.97 - m_{v1}\ 2436.8$$

$$= -302.06 + 161.853 + 3.775 - 604.33 = \mathbf{-\ 740.8\ kJ}$$

13.81

A flow moist air at 100 kPa, 40°C, 40% relative humidity is cooled to 15°C in a constant pressure device. Find the humidity ratio of the inlet and the exit flow, and the heat transfer in the device per kg dry air.

Solution:

C.V. Cooler. $\dot{m}_{v1} = \dot{m}_{liq} + \dot{m}_{v2}$

Tables: $P_{g1} = 7.384$ kPa, $P_{v1} = \phi\, P_g = 0.4 \times 7.384 = 2.954$ kPa,

$\omega_1 = 0.622 \times 2.954 /(100 - 2.954) = 0.0189$

$T_2 < T_{dew}$ [from $P_g(T_{dew}) = 2.954$] => $P_{v2} = 1.705$ kPa $= P_{g2}$ =>

$\omega_2 = 0.622 \times 1.705 /(100 - 1.705) = 0.0108$

$h_{v1} = 2574.3$ kJ/kg, $h_{v2} = 2528.9$ kJ/kg, $h_f = 62.98$ kJ/kg

$\bar{q}_{out} = C_P(T_1 - T_2) + \omega_1 h_{v1} - \omega_2 h_{v2} - (\omega_1 - \omega_2) h_f$

$= 1.004(40 - 15) + 0.0189 \times 2574.3 - 0.0108 \times 2528.9 - 0.0073 \times 62.98$

= **45.98 kJ/kg dry air**

Psychrometric chart: State 2: $T < T_{dew} = 23$°C => $\phi_2 = 100\%$

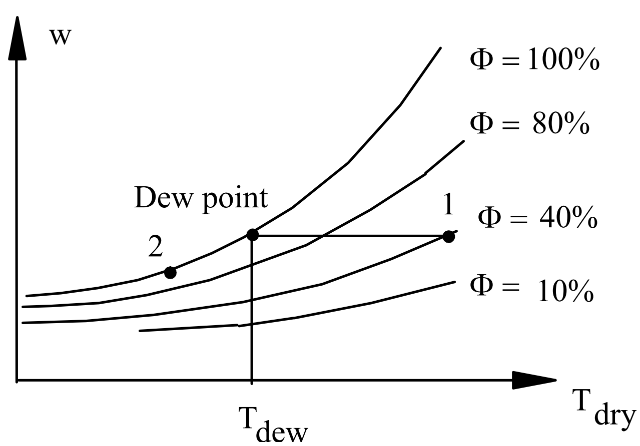
$\dot{m}_{v1}/\dot{m}_a = \omega_1 = 0.018$, $\tilde{h}_1 = 106$; $\dot{m}_{v2}/\dot{m}_a = \omega_2 = 0.0107$, $\tilde{h}_2 = 62$

$\dot{m}_{liq}/\dot{m}_a = \omega_1 - \omega_2 = 0.0073$, $h_f = 62.98$ kJ/kg

$\dot{m}_a\, \bar{q}_{out} = \dot{m}_a \tilde{h}_1 - \dot{m}_{liq}\, h_f - \dot{m}_a\, \tilde{h}_2$ =>

$\bar{q}_{out} = \tilde{h}_1 - (\omega_1 - \omega_2)\, h_f - \tilde{h}_2 = 106 - 0.0073 \times 62.98 - 62$

= **43.54 kJ/kg-dry air**



13.84

A flow, 0.2 kg/s dry air, of moist air at 40°C, 50% relative humidity flows from the outside state 1 down into a basement where it cools to 16°C, state 2. Then it flows up to the living room where it is heated to 25°C, state 3. Find the dew point for state 1, any amount of liquid that may appear, the heat transfer that takes place in the basement and the relative humidity in the living room at state 3.

Solve using psychrometric chart:

a) $T_{dew} = 27.2$ ($w = w_1$, $\phi = 100\%$) $\quad w_1 = 0.0232$, $\tilde{h}_1 = 118.2$ kJ/kg air

b) $T_2 < T_{dew}$ so we have $\phi_2 = 100\%$ liquid water appear in the basement.

$\Rightarrow w_2 = 0.0114 \qquad \tilde{h}_2 = 64.4$ and from steam tbl. $h_f = 67.17$

$\dot{m}_{liq} = \dot{m}_{air}(w_1 - w_2) = 0.2(0.0232-0.0114) = 0.00236$ kg/s

c) Energy equation: $\dot{m}_{air}\,\tilde{h}_1 = \dot{m}_{liq}\,h_f + \dot{m}_{air}\,\tilde{h}_2 + \dot{Q}_{out}$

$$\dot{Q}_{out} = 0.2[118.2 - 64.4 - 0.0118\times 67.17] = 10.6 \text{ kW}$$

d) $w_3 = w_2 = 0.0114$ & 25°C $\Rightarrow \phi_3 = 58\%$.

If you solve by the formulas and the tables the numbers are:

$P_{g40} = 7.384$ kPa; $P_{v1} = \phi\, P_{g40} = 0.5 \times 7.384 = 3.692$ kPa

$w_1 = 0.622 \times 3.692 / (100 - 3.692) = 0.02384$

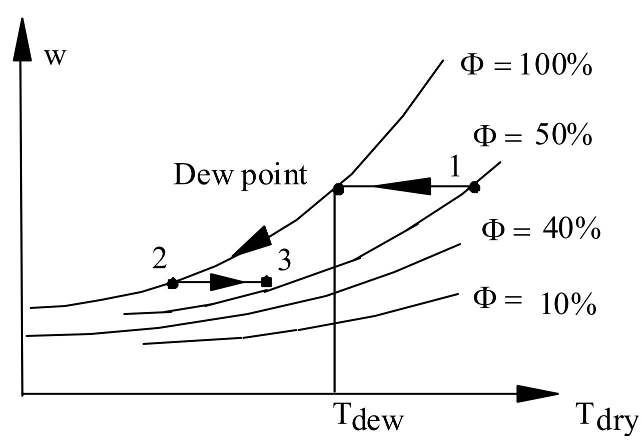
$P_{v1} = P_g\,(T_{dew}) \Rightarrow T_{dew\,1} = 27.5$ °C

2: $\phi = 100\%$, $P_{v2} = P_{g2} = 1.832$ kPa, $w_2 = 0.622\times 1.832/98.168 = 0.0116$

$\dot{m}_{liq} = \dot{m}_{air}\,(w_1 - w_2) = 0.2\times 0.01223 = 0.00245$ kg/s

3: $w_3 = w_2 \Rightarrow P_{v3} = P_{v2} = 1.832$ kPa & $P_{g3} = 3.169$ kPa

$\phi_3 = P_v/P_g = 1.832/3.169 = 57.8\%$

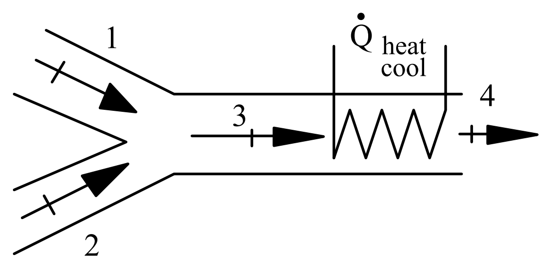


13.108

A flow of moist air at 21°C, 60% relative humidity should be produced from mixing of two different moist air flows. Flow 1 is at 10°C, relative humidity 80% and flow 2 is at 32°C and has T_{wet} = 27°C. The mixing chamber can be followed by a heater or a cooler. No liquid water is added and P = 100 kPa. Find the two controls one is the ratio of the two mass flow rates $\dot{m}_{a1}/\dot{m}_{a2}$ and the other is the heat transfer in the heater/cooler per kg dry air.

Solution:

C.V : Total Setup
state 3 is internal to CV.



Continuity Eq.: $\dot{m}_{a1}\, w_1 + \dot{m}_{a2}\, w_2 = (\dot{m}_{a1} + \dot{m}_{a2})\, w_4$

Energy Eq. $\dot{m}_{a1}\, \tilde{h}_1 + \dot{m}_{a2}\, \tilde{h}_2 + \dot{Q}_{a1} = (\dot{m}_{a1} + \dot{m}_{a2})\, \tilde{h}_4$

Define $x = \dot{m}_{a1}/\dot{m}_{a2}$ and substitute into continuity equation

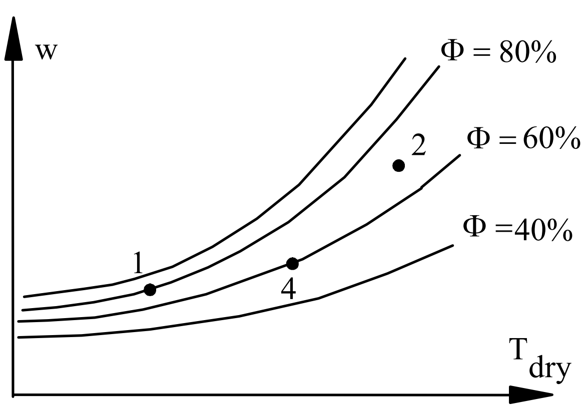
$$\Rightarrow \quad x\, w_1 + w_2 = (1+x)\, w_4 \quad \Rightarrow \quad x = \frac{w_4 - w_2}{w_1 - w_4} = \mathbf{3.773}$$

Energy equation scaled to total flow of dry air

$$\tilde{q} = \dot{Q}_{a1}/(\dot{m}_{a1} + \dot{m}_{a2}) = \tilde{h}_4 - [x/(1+x)]\, \tilde{h}_1 - [1/(1+x)]\, \tilde{h}_2$$

$$= 64 - 0.7905 \times 45 - 0.2095 \times 105$$

= **6.43 kJ/kg-dry air**



State 1:
$w_1 = 0.006$, $\tilde{h}_1 = 45$

State 2:
$w_2 = 0.0208$, $\tilde{h}_2 = 105$

State 4:
$w_4 = 0.0091$, $\tilde{h}_4 = 64$,
$T_{dew\,4} = 12.5°C$

13.109

In a hot and dry climate, air enters an air-conditioner unit at 100 kPa, 40°C, and 5% relative humidity, at the steady rate of 1.0 m^3/s. Liquid water at 20°C is sprayed into the air in the AC unit at the rate 20 kg/hour, and heat is rejected from the unit at at the rate 20 kW. The exit pressure is 100 kPa. What are the exit temperature and relative humidity?

State 1: $T_1 = 40°C$, $P_1 = 100$ kPa, $\phi_1 = 5\%$, $\dot{V}_{a1} = 1\ m^3/s$

$P_{g1} = 7.3837$ kPa, $P_{v1} = \phi_1 P_{g1} = 0.369$ kPa, $P_{a1} = P - P_{v1} = 99.63$ kPa

$$\omega_1 = 0.622 \frac{P_{v1}}{P_{a1}} = 0.0023,\ \dot{m}_{a1} = \frac{P_{a1}\dot{V}_{a1}}{RT_{a1}} = 1.108\ \text{kg/s},\quad h_{v1} = 2574.3\ \text{kJ/kg}$$

State 2 : Liq. Water. 20°C, $\dot{m}_{f2} = 20$ kg/hr = 0.00556 kg/s, $h_{f2} = 83.9$ kJ/kg

Conservation of Mass: $\dot{m}_{a1} = \dot{m}_{a3}$, $\dot{m}_{v1} + \dot{m}_{12} = \dot{m}_{v3}$

$\omega_3 = (\dot{m}_{f2} / \dot{m}_{a1}) + \omega_1 = (0.00556/1.108) + 0.0023 = 0.0073$

State 3 : $P_3 = 100$ kPa and $P_{v3} = P_3\omega_3/(0.622 + \omega_3) = 1.16$ kPa

Energy Eq. with $\dot{Q} = -20$ kW :

$$\dot{Q} + \dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{f2}h_{f2} = \dot{m}_{a3}h_{a3} + \dot{m}_{v3}h_{v3};$$

$$(h_{a3}-h_{a1}) + \omega_3 h_{v3} = \omega_1 h_{v1} + (\dot{m}_{f2}h_{f2} + \dot{Q})/\dot{m}_{a1}$$

$$= 0.0023 \times 2574.3 + (0.00556\times83.9 - 20)/1.108 = -11.7$$

Unknowns: h_{a3}, h_{v3} implicitly given by a single unknown: T_3

Trial and Error for T_3; T_3 = **10°C**, $P_{g3} = 1.23$ kPa, $\phi_3 = \frac{P_{v3}}{P_{g3}} = \mathbf{0.94}$

If we solved with the psychrometric chart we would get:

State 1: $\dot{m}_{v1}/\dot{m}_a = \omega_1 = 0.002$, $\tilde{h}_1 = 65$ kJ/kg dry air;

State 3: $\omega_3 = (\dot{m}_{f2} / \dot{m}_{a1}) + \omega_1 = (0.00556/1.108) + 0.002 = 0.007$

Now the energy equation becomes

$$\tilde{h}_3 = \tilde{h}_1 + (\dot{m}_{f2}h_{f2} + \dot{Q})/\dot{m}_{a1} = 65 + (0.00556\times83.9 - 20)/1.108 = 47.4$$

Given ω_3 we find the state around 10°C and $\phi_3 = 90\%$