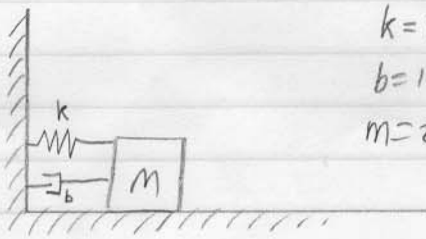
Tutorial 7

Section 4.11

#8)



$k = 200$ N/m
$b = 140$ N·sec/m
$m = 20$ kg

$x(0) = 0.25$ m
$x'(0) = -1$ m/s

a) Find equation of motion (Use already derived formulas)

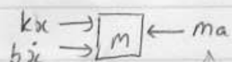
check $b^2 - 4mk$: $\quad b^2 - 4mk = 140^2 - 4(20)(200) = 3600 \quad \therefore$ over-damped.

$$\therefore r_1 = \frac{-b}{2m} + \frac{1}{2m}\sqrt{b^2-4mk} = \frac{-140}{40} + \frac{1}{40}\sqrt{3600} = -2$$

$$\therefore r_2 = \frac{-b}{2m} - \frac{1}{2m}\sqrt{b^2-4mk} = \frac{-40}{40} - \frac{1}{40}\sqrt{3600} = -5$$

$$\therefore x = Ae^{-2t} + Be^{-5t}.$$

Alternatively, Find equation of motion from basic principals.



Velocity is to left initially, so a is to the left, and kx and bẋ oppose it.

$$kx + b\dot{x} = -m\ddot{x}$$
$$20\ddot{x} + 140\dot{x} + 200x = 0$$
$$x'' + 7x' + 10x = 0$$
$$r^2 + 7r + 10 = 0$$
$$(r+5)(r+2) = 0$$
$$\therefore r = -2 \text{ and } -5.$$

$$\therefore x = Ae^{-2t} + Be^{-5t}$$

Solve Boundary Conditions:

$$\left.\begin{array}{ll} x(0) = 0.25 & 0.25 = A + B \\ x'(0) = -1 & -1 = -2A - 5B \end{array}\right\} \quad \begin{array}{l} A = \frac{1}{12} \\ B = \frac{1}{6} \end{array}$$

$$\boxed{\therefore x = \frac{1}{12}e^{-2t} + \frac{1}{6}e^{-5t}}$$

b) When will the block first return to its equilibrium position ($x=0$)?

$$0 = \tfrac{1}{12}e^{-2t} + \tfrac{1}{6}e^{-5t}$$
$$e^{-2t} = -2e^{-5t}$$
$$e^{3t} = -2$$
$$3t = \ln(-2) \leftarrow \text{this does not exist!!!}$$

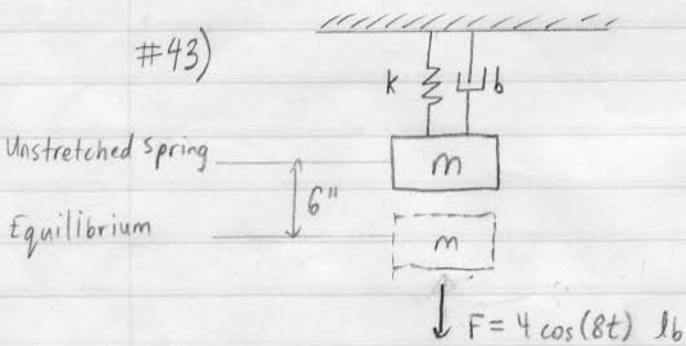
Why would that be? Rewrite the equation in the form shown below:

$$0 = \frac{1}{e^{2t}} + \frac{2}{e^{5t}}$$
$$0 = \frac{1}{e^{2t}}\left(1 + \frac{2}{e^{3t}}\right)$$

$1 + \frac{2}{e^{3t}}$ can never be 0, and $\frac{1}{e^{2t}} = 0$ when $t \to \infty$. ∴ The block never returns to its equilibrium position!! Refer to the graph for a visual representation.

Chapter 4 Review Problems

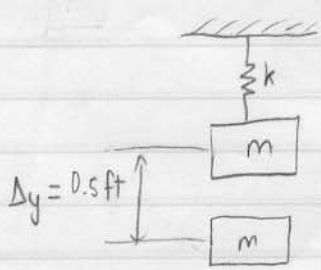
#43)



$k = ?$
$b = 2 \ \frac{lb \cdot sec}{ft}$
$m = 32 \ lb$

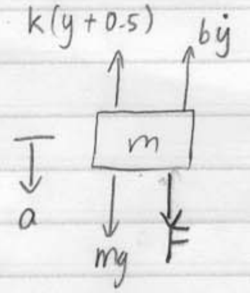
Find Equation of Motion

First look at the extended spring:



$$mg = ky$$
$$32 = k(0.5)$$
$$k = 64 \ \frac{lb}{ft}$$

Next look at Full System



$$k(y+0.5) + b\dot{y} - mg = -ma + F$$
$$m\ddot{y} + b\dot{y} + ky + \cancel{0.5(64)} \cancel{-32} = F$$

$$32\,lb\left(\frac{slug}{32.2\,lb}\right)\ddot{y} + 2\,\frac{lb\cdot sec}{ft}\,\dot{y} + 64y = 4\cos 8t$$

$$\ddot{y} + 2\dot{y} + 64 = 4\cos 8t$$

→ need to divide by gravity to go from Force to Mass

→ $32/32.2 \approx 1$

Solve homogenous:

$$r^2 + 2r + 64 = 0$$
$$r = -2 \pm \sqrt{2^2 - 4(64)}\;/2$$
$$r = -1 \pm 7.9373i$$

$$\therefore y_c = e^{-t}\left(A\cos 7.9373t + B\sin 7.9373t\right)$$

⇒ Steady state solutions involve y_p only!! ⇐

Solve Particular

$RHS = 4\cos 8t$

$\therefore y_p = C\sin 8t + D\cos 8t$

$y_p' = 8C\cos 8t - 8D\sin 8t$

$y_p'' = -64\sin 8t - 64D\cos 8t$

Sub into ODE: $\cancel{-64C\sin 8t} - \cancel{64D\cos 8t} + 16C\cos 8t - 16D\sin 8t + \cancel{64C\sin 8t} + \cancel{64D\cos 8t} = 4\cos 8t$

$$16C\cos 8t - 16D\sin 8t = 4\cos 8t$$

$\therefore 16C = 4 \Rightarrow C = \frac{1}{4}$

$\therefore D = 0$

$\therefore y_p = \frac{1}{4} \sin 8t$

$\therefore y = \frac{1}{4} \sin 8t$

Resonant Frequency $= \frac{\gamma_r}{2\pi} = \frac{\sqrt{\frac{k}{m} - \frac{b^2}{2m^2}}}{2\pi} = \frac{\sqrt{\frac{64}{1} - \frac{2^2}{2(1)^2}}}{2\pi} = 1.253$

Section 4.11

Question 8

$$x(t) := \frac{1}{12} \cdot e^{-2t} + \frac{1}{6} \cdot e^{-5t}$$