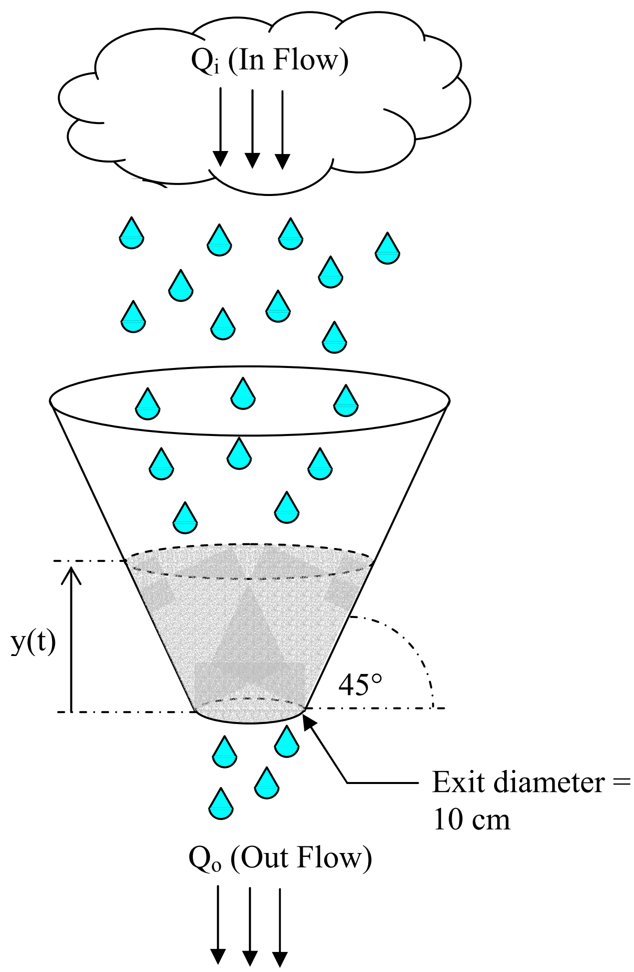
Tutorial 4

Q1) Oil is poured into the conical container shown below at a constant rate Q_1 cm^3/s. The outflow is given by the equation:

$$Q_0 = cA_0\sqrt{2gy}\ cm^3/s$$

where: c = constant = 0.6
A_0 = exit area
g = gravitational acceleration

a. Apply conservation of mass to find an ODE for the oil level *y* in the container versus time *t*. Initially the container is empty.
b. From part (a), develop an integral expression for the time required for the oil to reach a specified level *L* in the container. Do not try to evaluate the integral.
c. It is observed that a steady level is maintained at *y* = 30 cm. What is the inflow rate Q_i in litres/sec?
d. For the case (c) situation, how long will it take for the container to empty if the inflow suddenly stops?



Q2) The pressure variation in the atmosphere obeys the differential equation:

$$\frac{dp}{dz} = -\frac{pg}{R(T_0 - \gamma z)}$$

where p = pressure (N/m^2)
T_0 = surface temperature (K)
P_0 = surface pressure (N/m^2)
z = altitude (m)
R, γ = constants
g = gravity (m/s^2)

and the boundary condition is: $p = p_0$ at $z = 0$

a. What are the S.I. units of R and γ ?
b. Introduce dimensionless variables

$$p^* = \frac{p - p_r}{\delta_p} \qquad z^* = \frac{z - z_r}{\delta_z}$$

where p_r, δ_p, z_r, and δ_z are constants to be found. Derive the dimensionless equation for the ODE with the given boundary condition in terms of $p^*(z^*)$.
c. Solve the ODE.