Tutorial #2

Section 2.3

#11) Find function that satisfies $(t+y+1)\,dt - dy = 0$

$$t+y+1 = \frac{dy}{dt}$$
$$\frac{dy}{dt} - y = t+1 \qquad \therefore P(t) = -1 \text{ and } Q(t) = t+1$$

$$u = \exp\left(\int P(t)\,dt\right) = \exp\int -1\,dt = e^{-t}$$

$$\therefore \frac{d}{dt}\left(y(u)\right) = (u)Q(t)$$
$$\frac{d}{dt}\left(ye^{-t}\right) = te^{-t} + e^{-t}$$
$$ye^{-t} = \int te^{-t}\,dt + \int e^{-t}\,dt$$
$$ye^{-t} = -te^{-t} - e^{-t} - e^{-t} + C$$
$$ye^{-t} = -te^{-t} - 2e^{-t} + C$$
$$\underline{\underline{y = -t - 2 + Ce^{t}}}$$

Check: $\frac{dy}{dt} = -1 + Ce^{t}$ sub this into the original ODE

LS	RS
$\frac{dy}{dt} - y$	$t+1$
$-1 + \cancel{Ce^{t}} + t + 2 - \cancel{Ce^{t}}$	$t+1$
$t+1$	$t+1$

$\therefore$ The function is a solution.

$$\boxed{\therefore \text{The general solution is: } y = -t - 2 + Ce^{t}}$$

#19) find a function that satisfies $t^3 \frac{dx}{dt} + 3t^2 x = t$ at $x(2)=0$

$\Rightarrow$ put in standard form $\left(\frac{dy}{dx} + P(x)y = Q(x) \right)$, so must divide both sides by t^3

$$\frac{dx}{dt} + \frac{3}{t}x = \frac{1}{t^2} \qquad \therefore P(x) = \frac{3}{t} \qquad Q(x) = \frac{1}{t^2}$$

$$u = \exp\int \frac{3}{t}\,dt = e^{3\ln t} = t^3$$

So...
$$\frac{d}{dt}\left(xt^3\right) = t^3\left(\frac{1}{t^2}\right)$$
$$xt^3 = \int t\,dt$$
$$xt^3 = \frac{t^2}{2} + C$$
$$x = \frac{1}{2t} + \frac{C}{t^3} \qquad \text{and} \qquad \frac{dx}{dt} = \frac{-1}{2t^2} - \frac{3C}{t^4}$$

check:

LS	RS
$t^3\left(\frac{-1}{2t^2} - \frac{3C}{t^4}\right) + 3t^2\left(\frac{1}{2t} + \frac{C}{t^3}\right)$	t
$-\frac{t}{2} - \cancel{\frac{3C}{t}} + \frac{3t}{2} + \cancel{\frac{3C}{t}}$	t
t	t

$\therefore$ The function is a solution

Sub in for $x(2) = 0$

$$0 = \frac{1}{4} + \frac{C}{8} \Rightarrow C = -2$$

$$\boxed{\therefore \text{The specific solution is: } x = \frac{1}{2t} - \frac{2}{t^3}}$$

Section 2.4

#23) Find the function that satisfies $(e^t y + te^t y)\,dt + (te^t + 2)\,dy = 0$ at $y(0) = -1$.

$M(t,y) = e^t y + te^t y$
$N(t,y) = te^t + 2$

Check to see if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$

$\frac{\partial M}{\partial y} = e^t + te^t$ $\qquad\qquad$ $\frac{\partial N}{\partial t} = e^t + te^t$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$, The equation is exact.

$F(t,y) = \int M\,dt = \cancel{e^t y} + te^t y - \cancel{e^t y} + g(y)$

$F(t,y) = \int N\,dy = te^t y + 2y + h(t)$

$\therefore F(t,y) = te^t y + 2y = C$

$y = \frac{C}{2 + te^t}$ $\qquad$ $\therefore \frac{dy}{dt} = \frac{-C(e^t + te^t)}{(2 + te^t)^2}$

Check: rewrite original equation as: $\frac{dy}{dt} = \frac{y(e^t + te^t)}{-(2 + te^t)}$

LS	RS
$\Rightarrow \frac{-C(e^t + te^t)}{(2 + te^t)^2}$	$\Rightarrow \frac{y(e^t + te^t)}{-(2 + te^t)}$
$\Rightarrow \frac{-C(e^t + te^t)}{(2 + te^t)^2}$	$\Rightarrow \frac{C(e^t + te^t)}{-(2 + te^t)^2}$

$\therefore$ The function is a solution.

Sub in $y(0) = -1$

$-1 = \frac{C}{2} \Rightarrow \therefore C = -2$

$\boxed{\therefore \text{The specific solution is: } y = \frac{-2}{2 + te^t}}$