Tutorial #1

Section 1.2

#20) Find the values of m which make $\phi(x) = e^{mx}$ a solution to the given equation.

$$\phi(x) = e^{mx}$$
$$\phi'(x) = me^{mx}$$
$$\phi''(x) = m^2e^{mx}$$
$$\phi'''(x) = m^3e^{mx}$$

a) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 5y = 0$

sub in the values of $\phi''(x)$, $\phi'(x)$, and $\phi(x)$

$$m^2e^{mx} + 6me^{mx} + 5em^{x} = 0$$
$$e^{mx}(m^2 + 6m + 5) = 0$$
$$e^{mx}(m+1)(m+5) = 0$$

$\therefore$ m must be equal to -1 or -5.

b) $\frac{d^3y}{dx^2} + 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$

$$m^3e^{mx} + 3m^2e^{mx} + 2me^{mx} = 0$$
$$e^{mx}(m^3 + 3m^2 + 2m) = 0$$
$$e^{mx}(m)(m+2)(m+1) = 0$$

$\therefore$ m must equal 0, -1, or -2

#22) Verify that $\emptyset(x) = C_1e^{x} + C_2e^{-2x}$. is a solution to the ODE $y'' + y' - 2y = 0$.

$$\emptyset(x) = C_1e^{x} + C_2e^{-2x}$$
$$\emptyset'(x) = C_1e^{x} - 2C_2e^{-2x}$$
$$\emptyset''(x) = C_1e^{x} + 4C_2e^{-2x}$$

Sub these expressions into the ODE to check if $\emptyset(x)$ is a solution

LS	RS
$\cancel{C_1e^{x}} + \cancel{4C_2e^{-2x}} + \cancel{C_1e^{x}} - \cancel{2C_2e^{-2x}} - \cancel{2C_1e^{x}} - \cancel{2C_2e^{-2x}}$	0
0	0

$\therefore$ For any values of C_1 and C_2, $\emptyset(x)$ satisfies the ODE.

a) if $y(0) = 2$ then
$y'(0) = 1$

$$2 = C_1e^{0} + C_2e^{-2(0)} \qquad 1 = C_1e^{0} - 2C_2e^{-2(0)}$$
$$2 = C_1 + C_2 \qquad 1 = C_1 - 2C_2$$

Solve for C_1 and C_2

$$2 = C_1 + C_2$$
$$\underline{1 = C_1 - 2C_2}$$
$$1 = 3C_2$$
$$C_2 = \tfrac{1}{3}$$
$$\therefore C_1 = \tfrac{5}{3}$$

$\therefore$ $\emptyset(x) = \frac{1}{3}e^{x} + \frac{5}{3}e^{-2x}$ satisfies the ODE with the given boundary conditions.

b) if $y(1)=1$ Then
$y'(1)=0$

$1 = C_1 e^1 + C_2 e^{-2}$ $\qquad$ $0 = C_1 e^1 - 2C_2 e^{-2}$

solve for C_1 and C_2

$$\begin{aligned} 1 &= C_1 e^1 + C_2 e^{-2} \\ 0 &= C_1 e^1 - 2C_2 e^{-2} \\ \hline 1 &= 3C_2 e^{-2} \end{aligned}$$

$$\frac{e^2}{3} = C_2$$

$$\therefore 1 = C_1 e^1 + \frac{e^2}{3} e^{-2}$$

$$1 = C_1 e^1 + \frac{1}{3}$$

$$\frac{2}{3} = C_1 e^1$$

$$C_1 = \frac{2}{3e^1}$$

$\therefore$ $\underline{\phi(x) = \frac{2}{3e^1} e^{x} + \frac{e^2}{3} e^{-2x}}$ satisfies the ODE with the given boundary conditions

Section 1.3

Q3) *Solve for :* $\frac{dv}{dt} = 1 - \frac{v}{8}$

$$\frac{dv}{dt} = \frac{8-v}{8} = \frac{v-8}{-8}$$

$$\frac{dv}{v-8} = \frac{dt}{-8}$$

$$\ln|v-8| = \frac{-t}{8} + C$$

$$|v-8| = e^{\frac{-t}{8}+c}$$

$$if \quad |v-8| = v-8, \quad then \quad v = e^{\frac{-t}{8}+C}$$

$$if \quad |v-8| = -(v-8), \quad then \quad v = -e^{\frac{-t}{8}+C}$$