ME203 SUMMARY SHEET FOR METHOD OF UNDETERMINED COEFFICIENTS

This method allows us to find a **particular solution** for linear, second-order, non-homogeneous differential equation with **constant coefficients:**

$$ay'' + by' + cy = f(t)$$

1. First solve the homogenous equation $ay'' + by' + cy = 0$. This yields the complementary solution $y_c(t)$.

2. The method of undetermined coefficients works for <u>simple</u> forcing functions $f(t)$, e.g.

$$f(t) = A\sin t,\ B\cos t,\ Ce^{at},\ Dte^{at},\ Et^n,\ Fe^{at}\cos t,\ Ge^{at}\sin t, \quad etc.$$

3. If $f(t) = f_1(t) + f_2(t) + \ldots + f_n(t)$, where there are n different functions $f_i(t)$ of the above form, then we solve n non-homogeneous equations:

$$ay_p'' + by_p' + cy_p = f_i(t)$$

4. For each $f_i(t)$, assume a solution y_p consisting of a similar sin( ), cos( ), exp( ) or polynomial. If $f_i(t)$ is a solution to the homogeneous equation $ay'' + by' + cy = 0$, then the assumed form of $y_p(t)$ below is multiplied by t.

$f_i(t)$	$y_p(t)$ **(assumed form)**
t^n	$a_n t^n + a_{n-1}t^{n-1} + \ldots + a_1 t + a_0$
e^{at}	Ae^{at}
$\sin\beta t$ or $\cos\beta t$	$A\sin\beta t + B\cos\beta t$
$t^n e^{at}$	$\left(a_n t^n + a_{n-1}t^{n-1} + \ldots + a_1 t + a_0\right)e^{at}$
$t^n\sin(\beta t)$ or $t^n\cos(\beta t)$	$\left(a_n t^n + a_{n-1}t^{n-1} + \ldots + a_1 t + a_0\right)\sin(\beta t) +$ $\left(b_n t^n + b_{n-1}t^{n-1} + \ldots + b_1 t + b_0\right)\cos(\beta t)$
$e^{at}\sin\beta t$ or $e^{at}\cos\beta t$	$Ae^{at}\sin\beta t + Be^{at}\cos\beta t$
$t^n e^{at}\sin(\beta t)$ or $t^n e^{at}\cos(\beta t)$	$\left(a_n t^n + a_{n-1}t^{n-1} + \ldots + a_1 t + a_0\right)e^{at}\sin(\beta t) +$ $\left(b_n t^n + b_{n-1}t^{n-1} + \ldots + b_1 t + b_0\right)e^{at}\cos(\beta t)$

Table I. List of trial forms for the particular function $y_p(t)$ for various forms of $f(t)$.

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N.B. If $f(t)$ is a solution to the homogeneous equation, multiply the assumed form of the $y_p(t)$ particular function by t.

5. Find the coefficients for the particular solution $y_p(t)$ for each $f_i(t)$ by substituting into the differential equation and equating coefficient of like terms.

6. The particular solution is then the sum of the y_p's.

7. Add the particular solution $y_p(t)$ to the complementary solution $y_c(t)$ to get the general solution.

The basic method is fairly straightforward, but the algebra can be cumbersome, especially if f(t) is the product of exponentials and trigonometric functions, or is included in the complementary solution.