ME 203 – Summary First Order ODE Solution Techiques 23 September 2002

1. First Order Linear Differential Equations

General form: $\frac{dy}{dt} + P(t)y = Q(t)$ => integrating factor $\mu(t) = e^{\int P(t)dt}$

General Solution is: $y(t) = \frac{\int \mu(t)Q(t)dt + c}{\mu(t)}$ (Solution valid over any interval where P(t) and Q(t) are continuous.)

If constant coefficients: $\frac{dy}{dt} = -py + q$ => $y(t) = \frac{q}{p} + ce^{-pt}$ *and* $\lim_{t \to \infty} y(t) = \frac{q}{p}$

2. Nonlinear, First Order Differential Equations

Given: $\frac{dy}{dx} = f(y,x)$, where *f(y,x)* is a nonlinear function of *y*.

There are several solution methods available:

a) **Separation of variables** if possible, to obtain: $h(y)dy = g(x)dx$

b) **Homogeneous equations** if $\frac{dy}{dx} = G\left(\frac{y}{x}\right)$. Let $y = vx$ and transform to solve.

c) **Exact equations** of form $Mdx + Ndy = 0$. If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, easy to solve.

d) Equations made exact by an **integrating factor**:

$$\text{If } \frac{M_y - N_x}{N} = \varphi(x) \text{ then } \mu(x) = e^{\int \varphi(x)dx}$$

$$\text{If } \frac{N_x - M_y}{M} = \psi(y) \text{ then } \mu(y) = e^{\int \psi(y)dy}$$

e) **Transformation** of dependent variable: Let $y = f(v)$, $\frac{dy}{dx} = f'(v)\frac{dv}{dx}$, etc.