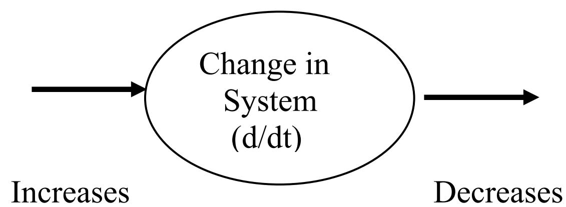
**ME203: Ordinary Differential Equations** **September 24, 2002**

## Derivation of First Order Differential Equations Based on the Conservation Laws of Engineering

A fundamental principle used in the analysis of many engineering problems is the idea of a **conservation law**, which usually takes the form:

$$Change\ in\ System\ Property = Increases - Decreases \qquad (1)$$

This expression can refer to changes in momentum, energy, or mass of a system. An equation of this form can be used to predict changes with respect to time and yield the time-dependent behaviour of the system of interest.




At equilibrium, there is no net change in the system, so we must have:

$$Increases - Decreases = 0, \qquad (2)$$

i.e., the increases and decreases are in balance. The solution in this case is known as the *steady-state* condition, for example in the case of a sphere falling into a body of fluid, the steady state occurs when the acceleration (rate of change of momentum) is zero, and:

downward force = upward force

or, $mg = cv$

This immediately gives us the terminal velocity $v = \dfrac{mg}{c}$

where $c$ is a coefficient for the drag force.

Depending on the form of the various terms on the right hand side of equation (1), the conservation laws may lead to non-linear differential equations that are difficult to solve. However, the concept of a conservation law often leads to simple mathematical models used in several engineering disciplines. These include (for example):

1. Chemical Engineering:

**Conservation of Mass:** $\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$

(m = mass)

2. Mechanical Engineering:

**Newton's Second Law:** $\frac{dp}{dt} = \sum Forces$

(p = momentum)

**Heat Transfer:** $\frac{dT}{dt} = \frac{1}{m\,c_p}\left(Q_{in} - Q_{out}\right)$

(T = temperature, Q = heat flow rate [W], cp = specific heat)

3. Electrical Engineering:

**Kirchoff's Law:** $emf = \sum voltage\ \ drops$

(emf = source voltage)