ME203 – Ordinary Differential Equations
Fall 2002 Midterm Examination

Instructor: Pete Teertstra **21 October, 2002**

Instructions:

1. Permitted aids: calculator, hand-written equation list consisting of one side of 8½”x11” paper and Schaum’s Mathematical Handbook of Formulas and Tables
2. Clear, systematic solutions are required. Part marks will be rewarded for part answers, provided that your methodology is clear
3. The time limit is 2 hours
4. Each question is worth marks indicated

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1. [10 points] Classify the following differential equations by selecting the classification given that best describes the equation. Classifications can be used more than once and some equations may have more than one classification. Some manipulation of the differential equations may be necessary. DO NOT ATTEMPT TO SOLVE THE EQUATIONS!

a) $\frac{dy}{dx} - y = 2y^3 + 4$

b) $x^2 \frac{dy}{dx} + \cos x = y$

c) $3\frac{dx^*}{dt^*} - 2x^* = 1$

d) $\left(2xy^2 + 3x^2\right)dx + \left(2x^2 y + 4y^3\right)dy = 0$

e) $\left(2y^2 + 2y + 4x^2\right)dx + \left(2xy + x\right)dy = 0$

f) $\left(y^2 - xy\right)dx + x^2\, dy = 0$

g) $\frac{dy}{dx} + \frac{y}{x-2} = 5\left(x-2\right)y^{1/2}$

h) $\frac{d^2 y}{dx^2} - 2xy = x$

i) $3\frac{d^2 y}{dx^2} + 11\frac{dy}{dx} - 7y = 0$

j) $x\frac{dy}{dx} = y \sin x$

1. Exact with integrating factor $\mu(x)$
2. First order, linear, constant coefficients
3. Bernoulli equation
4. Separable
5. Second order, linear, homogeneous with constant coefficients
6. First order, homogeneous
7. Exact
8. First order, linear
9. Second order, non-homogeneous

2. [30 points] Solve the following differential equations, including solution of the integration coefficient where initial conditions are given:

   a) $\frac{dy}{dx} + y\tan x + \sin x = 0 \quad y(0) = \pi$

   b) $\left(y^3 + 4e^x y\right)dx + \left(2e^x + 3y^2\right)dy = 0$

   c) $\frac{dy}{dx} = \frac{10-2x}{y^3 - y - 6} \quad y(2) = -1$

   d) $2xy\frac{dy}{dx} = 4x^2 + 3y^2$

   e) $\frac{dy}{dx} = (2x + y - 1)^2$ (hint: use a transformation)

   f) $2\frac{d^2x}{dt^2} - 8\frac{dx}{dt} + 8x = 0 \quad x(0) = 1,\ x'(0) = -2$

3. [10 points] Suppose that a falling hailstone with density $\rho = 1\, kg/m^3$ starts from rest with a negligible radius, $r = 0\ m$. As the hailstone falls, the radius increases as a function of time, $r = k \cdot t$, where k is a constant with units m/s. Assume that the drag and buoyancy forces can be neglected. Based on conservation of momentum, $dp/dt = \sum \text{Forces}$, where the momentum can be calculated by $p = m \cdot v$, find an equation for the velocity v of the hailstone as a function of time.

4. [10 points] A cake is removed from an oven at 100 $°C$ and left to cool at room temperature, which is 20 $°C$. After 30 minutes the temperature of the cake is 60 $°C$. The heat transfer rate Q_{out} from the cake is related to its temperature T and room temperature T_{room} by: $Q_{out} = k \cdot (T - T_{room})$, where k is a constant with units $W/°C$.

   a) Find an equation for the temperature of the cake as a function of time.

   b) When will the temperature of the cake be 38 $°C$?

5. [10 points]

   a) Find the equation of motion for the spring-mass-dashpot system:
   mass $m = 10\ kg$,
   spring constant $k = 250\ N/m$,
   damping coefficient $c = 60\ N \cdot s/m$.
   Initial conditions:
   $x(0) = 0.3\,m$ and $x'(0) = -0.1\,m/s$.

   b) What is the frequency of oscillation of the spring-mass-damper system?