ME203 PROBLEM SET #4

Issued: October 2, 2002
Solutions Available: October 8, 2002
http://www.mhtl.uwaterloo.ca/courses/me203/

At least one of these problems should be attempted in Mathcad. If successful, please email Mathcad worksheet to pmt@mhtlab.uwaterloo.ca to be included on course website.

1. A ball of mass m is thrown upward with initial velocity V_0. Assuming that the air resistance is proportional to the instantaneous velocity, with constant of proportionality k, show that the maximum height obtained is:

$$z_{\max} = \frac{mV_0}{k} - \frac{m^2 g}{k^2}\ln\left(1 + \frac{kV_0}{mg}\right)$$

2. Consider a tank in the form of a paraboloid of revolution, with radius $r \propto y^{1/2}$. If y_0 is the initial depth of water in the tank, then the radius of the tank at each y is:

$$r(y) = R_0\sqrt{\frac{y}{y_0}}$$

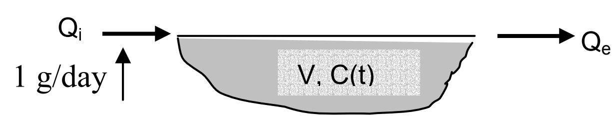
Find the length of time it takes the tank to drain if the initial height of water in the tank is y_0 and there is a hole of radius r_h at the bottom of the tank.

3. A chemical company has been releasing excessive amounts (50 g/day) of a toxic substance X into a stream for several years. The stream enters into a lake, where the concentration of X, C(t), has reached a steady-state value of 10^{-3} g/m^3, which is considered harmful to human health. As a result, another stream, which exits the lake, cannot be used for drinking water because of the high concentration of the contaminant X.

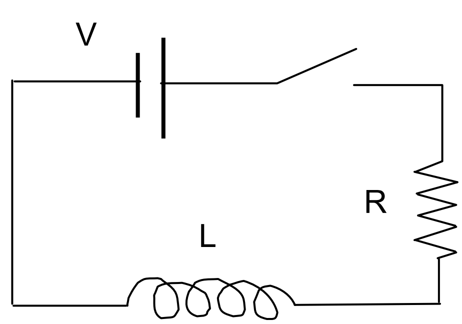
Eventually the local environmental enforcement officer locates the source of the contaminant. Along with a big fine, the company is forced to reduce its discharge rate to 1 g/day. Assuming well mixed conditions in the lake, how long will it take for the exit stream concentration to drop to a level of 10^{-4} g/m^3 of X, which is considered safe for human consumption?

Use the following data:

Original rate of discharge of X:	50 g/day
New rate of discharge of X:	1 g/day
Inlet stream flow rate Q_i:	50 000 m^3/day
Exit stream flow rate Q_e:	50 000 m^3/day
Volume of Lake V:	10^7 m^3



4. A simple resistor-inductor circuit is shown below. At $t = 0$ the knife switch is closed and current $I(t)$ flows through the circuit due to the applied voltage V. Find a differential equation for $I(t)$ if the voltage drop across the inductor is $L dI/dt$. How long does it take for the current to achieve half its steady-state value if L = 0.1 Henry and R = 100 Ω ?



5. Non-dimensionalize the ODE from Problem 4, solve the non-dimensionalized differential equations and express the solution in terms of the dimensionless variables $I^* = I / I_{ref}$ and $t^* = t / t_{ref}$