Problem Set #1

Section 1.1

5) $\frac{dp}{dt} = kp(P-p)$ $\qquad$ P, k constants

- This is an ODE because the derivative of p is only with respect to t
- This is a first order equation, since the highest derivative is of the first power
- The independent variable is t, and the dependent is p
- This equation can be rewritten as:

$\frac{dp}{dt} = kPp - kp^2$ $\quad$ since there is a p^2 term, the ODE is non-linear (see page 5 for further explanation)

7) $y\left[1 + \left(\frac{dy}{dx}\right)^2\right] = C$ $\quad$ C is constant

- This is an ODE
- This is 1^{st} order (note that $\frac{\partial^2 y}{\partial x^2} \neq \left(\frac{\partial y}{\partial x}\right)^2$)
- Independent variable is x, dependent variable is y
- non-linear

9) $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$

- ODE
- 2^{nd} order
- independent: x , dependent: y
- linear

15) $\frac{dT}{dt} = k\left(M(t) - T(t)\right)$ $\qquad$ k is constant

Section 1.2

3) is $y=\sin x+x^2$ a solution to $\frac{d^2y}{dx^2}+y=x^2+2$?

$$\left.\begin{aligned} y'&=\frac{dy}{dx}=\cos x+2x \\ y''&=\frac{d^2y}{dx^2}=-\sin x+2 \end{aligned}\right\}$$ defined on the interval $(-\infty,\infty)$

Sub $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ into the differential equation

LS	RS
$-\sin x+2+\sin x+x^2$	x^2+2
$2+x^2$	x^2+2

Since LS = RS and y, y' and y'' are all defined on $(-\infty,\infty)$, the given function is a solution to the differential equation

8) $y=3\sin 2x+e^{-x}$ $\qquad y''+4y=5e^{-x}$

$$\left.\begin{aligned} y'&=6\cos 2x-e^{-x} \\ y''&=-12\sin 2x+e^{-x} \end{aligned}\right\}$$ defined on interval $(-\infty,\infty)$

LS	RS
$-12\sin 2x+e^{-x}+12\sin 2x+4e^{-x}$	$5e^{-x}$
$5e^{-x}$	$5e^{-x}$

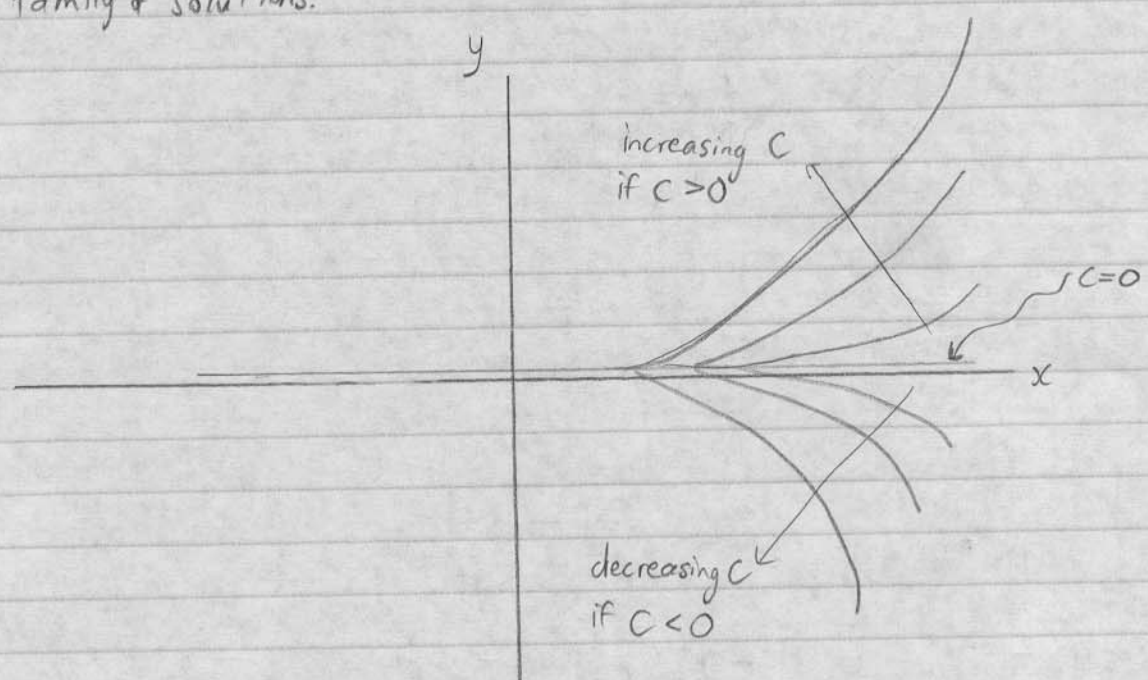
Since LS = RS, and y, y', and y'' are all defined on $(-\infty,\infty)$, the given function is a solution to the differential equation

15) show that $\emptyset(x) = Ce^{3x} + 1$ is a solution to
$\frac{dy}{dx} - 3y = -3$ for any value of C.

$$\left.\begin{aligned} y = \emptyset(x) &= Ce^{3x} + 1 \\ y' &= 3Ce^{3x} \end{aligned}\right\} \text{valid on } (-\infty, \infty)$$

LS	RS
$3Ce^{3x} - 3Ce^{3x} - 3$	-3
-3	-3

$\therefore$ The function is a solution, and $Ce^{3x}+1$ is a one-parameter family of solutions.



21 a) $3x^2 \frac{d^2y}{dx^2} + 11x \frac{dy}{dx} - 3y = 0$

$y = x^m$
$y' = mx^{m-1}$
$y'' = (m)(m-1)x^{m-2}$

$0 = 3x^2(m)(m-1)(x^{m-2}) + 11x(m)(x^{m-1}) - 3x^m$
$0 = 3m^2x^2x^{m-2} - 3mx^2x^{m-2} + 11mxx^{m-1} - 3x^m$
$0 = 3m^2x^m - 3mx^m + 11mx^m - 3x^m$
$0 = x^m(3m^2 + 8m - 3)$
$0 = x^m(3m^2 + 9m - m - 3)$
$0 = x^m(3m(m+3) - 1(m+3))$
$0 = x^m(3m-1)(m+3)$

$\therefore$ m must be either -3 or $\frac{1}{3}$

b) $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 5y = 0$

$y = x^m$
$y' = (m)x^{m-1}$
$y'' = (m)(m-1)(x^{m-2})$

$0 = x^2(m)(m-1)x^{m-2} - x(m)x^{m-1} - 5x^m$
$0 = x^2m^2x^{m-2} - x^2mx^{m-2} - mx^m - 5x^m$
$0 = m^2x^m - mx^m - mx^m - 5x^m$
$0 = x^m(m^2 - 2m - 5)$
$\therefore m = \frac{2 \pm \sqrt{4 + 4(5)}}{2} = \frac{2 \pm \sqrt{24}}{2} = 1 \pm \sqrt{6}$

$\therefore$ m must be $1 \pm \sqrt{6}$

Question 3

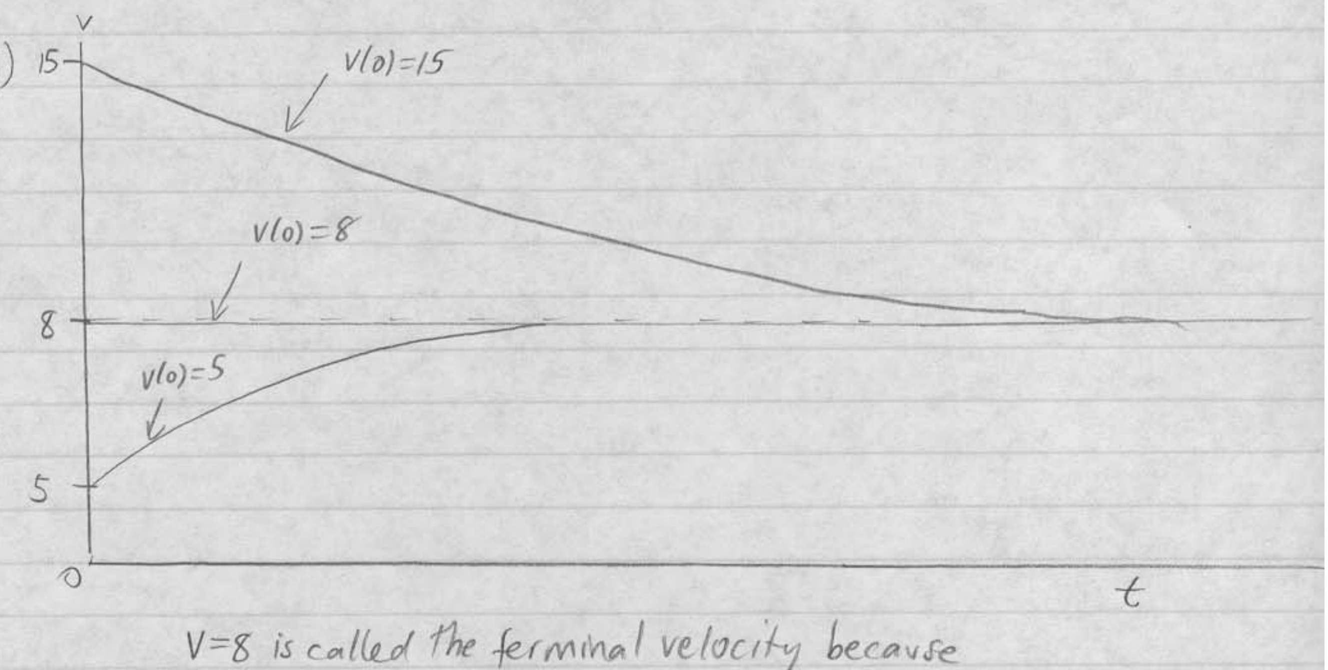
$$t^2y'' - 4ty' + 4y = 0$$

$y = t^r$
$y' = rt^{r-1}$
$y'' = (r)(r-1)t^{r-2}$

$$t^2(r)(r-1)t^{r-2} - 4t(r)t^{r-1} + 4t^r = 0$$
$$t^r(r^2-r) - t^r(4r) + t^r(4) = 0$$
$$t^r(r^2-5r+4) = 0$$
$$t^r(r-4)(r-1) = 0$$

$\therefore$ r must be 4 or 1

Section 1.3

3) 

V=8 is called the terminal velocity because all of the family of curves converge on V=8.