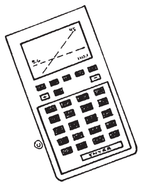

$\mathcal{ME}$ 201 ADVANCED CALCULUS

Assignment 7: *Double Integration and Surface Area*

March 2, 2018

1. Evaluate the following double integrals:

   (a) $\displaystyle\int_{-3}^{3}\int_{-\sqrt{18-2y^2}}^{\sqrt{18-2y^2}} x\, dxdy$ ANSWER: 0

   (b) $\displaystyle\int_{-1}^{0}\int_{y}^{2} (1+y)^2\, dxdy$ ANSWER: 3/4

   (c) $\displaystyle\int_{1}^{2}\int_{1}^{y} e^{x+y}\, dxdy$ ANSWER: $\dfrac{e^2(1-e)^2}{2}$

   (d) $\displaystyle\int_{-1}^{1}\int_{x}^{2x} (xy + x^3y^3)\, dxdy$ ANSWER: 0

2. Evaluate the double integrals over the region given:

   (a) $\displaystyle\int\int_R xy^2\, dA$ where R is bounded by $x + y + 1 = 0,\ x + y^2 = 1$
   ANSWER: -621/140

   (b) $\displaystyle\int\int_R (xy + y^2 - 3x^2)\, dA$ where R is bounded by $y = |x|,\ y = 1, y = 2$
   ANSWER: 0

3. Evaluate the following double integrals by reversing the order of integration:

   (a) $\displaystyle\int_{0}^{1}\int_{y}^{1} \sin(x^2)\, dxdy$ ANSWER: $\dfrac{1-\cos 1}{2}$

   (b) $\displaystyle\int_{-2}^{0}\int_{-2}^{x} \frac{x}{\sqrt{x^2+y^2}}\, dydx$ ANSWER: $2(1-\sqrt{2})$

4. The Cobb-Douglas production function for a widget is $P(x, y) = 10,000x^{0.3}y^{0.7}$, where P is the number of widgets produced each month, x is the number of employees and y is the monthly operating budget in thousands of dollars. If the company uses anywhere between 45 and 55 workers each month and its operating budget varies from $\$8,000$ to $\$12,000$ per month, what is the average number of widgets produced each month.
   ANSWER: $161,781$

5. Evaluate the double integrals in polar coordinates over the region given:

   (a) $\displaystyle\int\int_R x\,dA$ where R is bounded by $x = \sqrt{2y - y^2},\ x = 0$ ANSWER: $2/3$

   (b) $\displaystyle\int\int_R \frac{1}{\sqrt{x^2+y^2}}\,dA$ where R is the region outside $x^2+y^2 = 4$ and $x^2+y^2 = 4x$

   ANSWER: $\dfrac{4}{3(3\sqrt{3}-\pi)}$

   (c) $\displaystyle\int\int_R \sqrt{1+2x^2+2y^2}\,dA$ where R is bounded by $x^2+y^2 = 1,\ x^2+y^2 = 4$

   ANSWER: $(9-\sqrt{3})\pi$

6. Poiseuille's law states that for laminar flow through a circular pipe, the speed of the fluid, v, at a distance r from the center of the pipe is given by:

$$v = \frac{P}{4nL}\left(R^2 - r^2\right)$$

   where P is the pressure difference between the ends of the pipe, L is the length of the pipe, n is the fluid viscosity and R is the pipe radius. Find the volume flow rate through the pipe.

   ANSWER: $\dfrac{\pi P R^4}{8nL}$

7. Use double integrals to find the area of the region bounded by the following curves:

   (a) $y = x^3 + 8,\ y = 4x + 8$ ANSWER: 8

   (b) $y = xe^{-x},\ y = x,\ x = 2$ ANSWER: $1 + 3e^{-2}$

   (c) $y = x^3 - x,\ x + y + 1 = 0,\ x = \sqrt{y+1}$ ANSWER: $7/6$

   (d) Common to $r = 2,\ r^2 = 9\cos 2\theta$ ANSWER: $4\cos^{-1}(4/9) + 9 - \sqrt{65}$

8. Use double integrals to find the volume of a solid of revolution obtained by rotating the region bounded by the curves around the line:

   (a) $y = x^2 + 4,\ y = 2x^2$ about $y = 0$ ANSWER: $1024\pi/15$

   (b) $r = 1 + \sin\theta$ about the y axis ANSWER: $8\pi/3$

9. Find the surface area of the following functions for the region given:

   (a) $z = \sqrt{2xy}$ cut out by the planes $x = 1,\ x = 2, y = 1,\ y = 3$

   ANSWER: $\dfrac{4}{3}(5\sqrt{3} - 2\sqrt{6} - 3 + \sqrt{2})$

   (b) $z = \ln(1 + x + y)$ in the first octant cut off by $y = 1 - x^2$ (set up integral only)

10. Prove the following surface area relationships:

    (a) Surface area of the curved portion of a right circular cone, $\pi r\sqrt{r^2 + h^2}$

    (b) Surface area of a sphere, $4\pi r^2$

11. Determine the surface area of intersection for the part of the cylinder $x^2 + z^2 = a^2$ that lies inside the cylinder $x^2 + y^2 = a^2$. Make a suitable sketch of the two intersecting cylinders and clearly show the surface of intersection.

    ANSWER: $8a^2$

12. Find the surface area of the portion of the sphere $x^2 + y^2 + z^2 = 25$ that lies between the planes $z = 3$ and $z = 4$.

    ANSWER: 10π