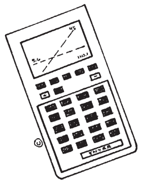


$\mathcal{ME}$ *201* ADVANCED CALCULUS

Assignment 6: *Minima & Maxima of Multivariable Functions &Least Squares*

February 9, 2018

1. Find all critical points for the following functions and classify each as yielding a relative maximum, minimum, saddle (inflection) point or none of these:

   (a) $f(x,y) = 3xy - x^3 - y^3$

   (b) $f(x,y) = x \sin y$

   (c) $f(x,y) = y^2 - 4x^2y + 3x^4$

   (d) $f(x,y,z) = xyze^{x^2+y^2+z^2}$ (find all critical points)

2. Find the maximum and minimum values of the following functions on the region given:

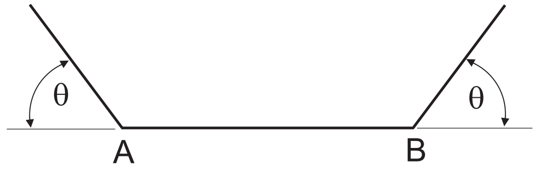
   (a) $f(x,y) = x^2 + x + 3y^2 + y$ on the region bounded by $y = x+1,\ y = 1-x,$ $y = x-1,\ y = -x-1$

   (b) $f(x,y) = x^3 - 3x + y^2 + 2y$ on the triangular region bounded by $x = 0,$ $y = 0,\ x + y = 1$

   (c) $f(x,y) = x^3 + y^3 - 3x - 3y + 2$ on the circle $x^2 + y^2 \leq 1$

3. An open tank in the form of a rectangular parallelepiped is to be built to hold $1000\ litres$ of acid. If the cost per unit area of lining the base of the tank is three times the cost of the sides, what dimensions minimize the cost of the tank lining?

4. A long piece of metal $1\ m$ wide is bent at A and B, as shown below, to form a channel with three straight sides. If the bends are equidistant from the ends, where should they be made in order to obtain maximum possible flow of fluid along the channel?



5. A right circular cone-shaped container when filled with a refreshing beverage holds a constant volume, given as

$$V = \frac{1}{3}\pi r^2 h = 100\ cm^3$$

In order to reduce the cost of producing these containers, it is necessary to find the ratio of the height, h, to the radius, r of the container that produces the minimum surface area when the volume is constrained to $100\ cm^3$. Find this optimized ratio of h/r.

6. Given the following data for average systolic blood pressure as a function of age:

Age A (years)	4	5	6	7	8	9	10	11	12	13	14	15	16
Pressure, P	85	87	90	92	95	98	100	105	108	110	112	115	118

Use a least squares method to find the equation of a straight line fitting these data.

ANSWER $P = 2.8187 \times Age + 72.967$

7. Fit a cubic polynomial $y = ax^3 + bx^2 + cx + d$ to the following data using the least squares method. Find the RMS difference between the correlation and the data.
(RMS = square $Root$ of the $Mean$ (arithmetic average) of the $Squares$ of the differences)

x	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2
y	7.06	11.34	15.62	19.50	25.62	31.94	37.02	44.32	51.56	58.72	67.08	75.91

ANSWER $y = 0.1193x^3 + 4.8113x^2 - 2.2145x - 8.6081$

8. Show that the following data for t can be represented by a correlation of the form $t = aF^b$ using the least squares method to find the coefficients a and b.

F	70	100	200	300	400
t	470	288	84	52	32

9. Use the least squares method to obtain a curve of the form $y = 1/(ax + b)$ to represent the following data:

x	5	6	7	8	9
y	1.335	1.431	1.247	1.197	1.118

ANSWER $y = 1/(0.04274x + 0.4968)$