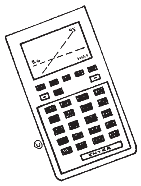

$\mathcal{ME}$ *201* ADVANCED CALCULUS

Assignment 5: *Gradient & Directional Derivative*
February 2, 2018

1. If two sides of a triangle have lengths x and y and the angle between them is θ, then the area of the triangle is $A = (1/2)xy\sin\theta$. How fast is the area changing when x is $1\ m$, y is $2\ m$ and θ is $1/3\ radian$, if x and y are each increasing at $0.5\ m/s$ and θ is decreasing at $0.1\ radian/s$?

2. Two straight roads intersect at right angles. Car A, moving on one of the roads, approaches the intersection at $40\ km/h$ and Car B, moving on the other road, approaches the intersection at $30\ km/h$. Consider the case where Car A is $650\ m$ from the intersection and Car B is $500\ m$ from the intersection.

   (a) Sketch the problem, clearly indicating your choice of coordinate system.

   (b) Use chain rule to calculate the rate of change of distance between the cars evaluated for the distances given (km/h).

3. Find the gradient vector for each of the following:

   (a) $f(x, y) = x^2y + xy^2$

   (b) $f(x, y, z) = e^{x+y+z}$

   (c) $f(x, y) = xy\ln(x + y)$ at $(4, -2)$

4. Calculate the directional derivative at the point and in the direction indicated:

   (a) $f(x, y, z) = \ln(xy+yz+xz)$ at $(1, 1, 1)$ in the direction from $(1, 1, 1)$ toward the point $(-1, -2, 3)$.

   (b) $f(x, y, z) = x^2y + y^2z + z^2x$ at $(1, -1, 0)$ along the line $x + 2y + 1 = 0,\ x - y + 2z = 2$ in the direction of decreasing z.

   (c) $f(x, y) = x^2 + y$ at $(-1, 3)$ along the curve $y = -3x^3$ in the direction of decreasing x.

5. Find the direction in which the function $f(x, y, z) = \tan^{-1}(xyz)$ increases most rapidly at the point $(3, 2, -4)$. What is the rate of change in this direction?

6. In what direction is the rate of change of the function $f(x, y, z) = xyz$ smallest at the point $(2, -1, 3)$? What is the rate of change in this direction?

7. In what direction (if any) is the rate of change of the function $f(x, y) = x^2y + y^3$ at the point $(1, -1)$ equal to:

   (a) 0

   (b) 1

   (c) 20

8. How fast is the distance to the origin changing with respect to distance traveled along the curve $x = 2\cos t,\ y = 2\sin t$ and $z = 3t$ at any point on the curve? What is the rate of change when $t = 0$? Would you expect this?