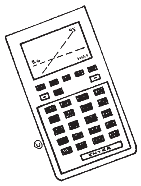

$\mathcal{ME}$ *201* ADVANCED CALCULUS

Assignment 4: *Chain Rule, Tangent Lines and Tangent Planes*

January 26, 2018

1. Solve the following partial derivatives:

   (a) $\dfrac{\partial^3 f}{\partial y^3}$ if $f(x,y) = \dfrac{2x}{y} + 3x^3y^4$

   (b) $\dfrac{\partial^2 f}{\partial y \partial z}$ if $f(x,y,z) = xyze^{x+y+z}$

   (c) $\dfrac{\partial^2 f}{\partial z^2}$ if $f(x,y,z) = \ln\sqrt{x^2+y^2+z^2}$

   (d) $\dfrac{\partial^6 f}{\partial x^2 \partial y^2 \partial z^2}$ if $f(x,y,z) = \dfrac{1}{x^2} + \dfrac{1}{y^2} + \dfrac{1}{z^2}$

2. If $z = x^2 + xy + y^2\sin(x/y)$ show that:

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z = x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial z^2}{\partial y^2}$$

3. Use chain rule to solve the following partial derivatives:

   (a) $\dfrac{\partial u}{\partial s}$ if $u = \sqrt{x^2+y^2+z^2},\ x = 2st,\ y = s^2+t^2,\ z = st$

   (b) $\dfrac{\partial z}{\partial t}$ if $z = x^2+y^2+u^2,\ x = v^3 - 3v^2,\ u = \dfrac{1}{x^2-y^2},\ v = e^t,\ y = e^{4t}$

   (c) $\dfrac{\partial^2 z}{\partial v^2}$ if $z = \sin(xy),\ x = 3\cos v,\ y = 4\sin v$

4. Find the equation for the tangent line to the curve at the point given for the following:

   (a) $x = e^{-t}\cos t,\ y = e^{-t}\sin t,\ z = t$ at $(1,0,0)$

   (b) $x^2+y^2+z^2 = 4,\ z^2 = x^2+y^2$ at $(1,1,-\sqrt{2})$

5. Find an equation for the tangent plane to the surface at the point given for the following:

   (a) $x = x^2 - y^3 z$ at $(2, -1, -2)$

   (b) $x^2 + y^2 + 2y = 1$ at $(1, 0, 3)$

6. Verify that the curve $x^2 - y^2 + z^2 = 1,\ xy + xz = 2$ is tangent to the surface $xyz - x^2 - 6y + 6 = 0$ at the point $(1, 1, 1)$.

7. Determine the following quantities:

   (a) The unit tangent vector, $\hat{T}$, for the curve of intersection of surfaces $x^2 + y^2 + z^2 = 2$ and $y = z$ at point $(0, 1, 1)$.

   (b) The directional derivative of $f(x, y, z) = 2xyz - x^2 - z^2$ along the curve from Part a) at point $(0, 1, 1)$ in the direction of increasing x.

   (c) Find the angle between the gradient vector, ∇f, and the vector, $\vec{v}$, along which the rate of change (directional derivative) of $f(x, y, z)$ at point $(0, 1, 1)$ is equal to 0, equal to 1 and is a maximum.