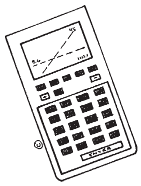


# $\mathcal{ME}$ *201* ADVANCED CALCULUS

**Assignment 3:** *Curvature, Acceleration & Partial Derivatives*
*January 19, 2018*

1. Find parametric equations, the position vector and plot the following curves:

   (a) $x^2 + y^2 = 2,\ z = 4$ directed so that $y$ increases in the first octant.

   (b) $z = \sqrt{x^2 + y^2},\ y = x$ directed so that $y$ increases when $x$ is positive.

   (c) $z = \sqrt{4 - x^2 - y^2},\ x^2 + y^2 - 2y = 0$ directed so that $z$ decreases when $x$ is positive.

   (d) $z = \sqrt{x^2 + y^2},\ y = x^2$ directed so that $y$ decreases in the first octant.

2. Find the normal vectors $\hat{N}$ and $\hat{B}$ for the following:

   (a) At point $(2\sqrt{2}, 3\sqrt{2}, \sqrt{2})$ on the line $x = 4\cos t,\ y = 6\sin t,\ z = 2\sin t$ from $-\infty < t < \infty$.

   (b) At point $(1, 1, \sqrt{2})$ on the line $x^2 + y^2 + z^2 = 4,\ z = \sqrt{x^2 + y^2}$ directed so that $x$ increases when $y$ is positive.

3. Find the curvature and radius of curvature and plot the following curves:

   (a) $x = e^t \cos t,\ y = e^t \sin t,\ z = t$ for $t \geq 0$.

   (b) $x = t + 1,\ y = t^2 - 1,\ z = t + 1,$ for $-\infty < t < \infty$.

4. At which points on the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ where $a > b$ is the curvature a maximum and at what points is the curvature a minimum?

5. Find the velocity, speed and acceleration of a particle moving along the curve described by the parametric equations:

$$x = t^2 + 1,\ y = 2te^t,\ z = \frac{1}{t^2} \quad \text{for} \quad 1 \leq t \leq 5.$$

6. If a particle starts at rest (zero velocity) from position $(1, 2, -1)$ at time $t = 0$ and experiences acceleration $\vec{a} = 3t^2\hat{\imath} + (t+1)\hat{\jmath} - 4t^3\hat{k}$ for $t \geq 0$, find an expression for the position vector.

7. Find the normal and tangential components of acceleration, $a_N$ and $a_T$, for a particle moving with position defined by the parametric equations:

$$x = \cos t, \; y = \sin t, \; z = t, \quad \text{for} \quad t \geq 0$$

8. A particle travels counterclockwise around a circle $(x-h)^2 + (y-k)^2 = R^2$, where $R$ is the radius and $h$ and $k$ are the $x-$ and $y-$coordinates at the center of the circle. Show that the speed of the particle at any time $t$ is $|\vec{v}| = \omega R$, where $\omega$ is the angular speed of the particle in $rad/s$.

9. A particle follows a trajectory in space given by:

$$x(t) = 2\cos t, \;\; y(t) = 2\sin t, \;\; z(t) = 2\pi - t$$

where $x, y$ and $z$ are in meters and $0 \leq t \leq 2\pi$.

   (a) Calculate the local vectors $\hat{T}, \hat{N}$ and $\hat{B}$ to the curve at any time $t$.

   (b) Calculate the length of the curve.

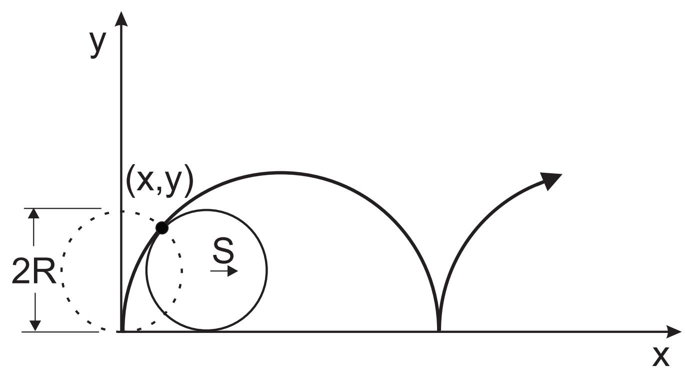
   (c) Find the curvature of the curve at any time $t$.

   (d) Express the particle velocity and acceleration (normal and tangent components).

10. A stone embedded in the tread of a rolling tire follows a path called a cycloid, shown in the figure. $S$ is the speed of the center of the wheel in the $x$ direction and radius $R$ is the radius of the tire. If a coordinate system is defined such that at $t = 0$ the stone is located at the origin, parametric equations can be formed to describe the position of the stone:

$$x = R(\theta - \sin\theta) \qquad y = R(1 - \cos\theta)$$

where $\theta$ is the angle of rotation of the wheel, which is related to the speed and radius by $\theta = St/R$




(a) Find the velocity, speed and acceleration of the stone in terms of the variables $\theta, S, R$.

(b) Find the tangential and normal components of the acceleration, $a_T$ and $a_N$.

11. Plot the following functions:

(a) $f(x, y) = x^2 + y^2$

(b) $f(x, y) = x^2 - y^2$

(c) $f(x, y) = \ln(x^2 + y^2)$

12. Evaluate partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ for the following:

(a) $f(x, y) = 3xy - 4x^4y^4$

(b) $f(x, y) = \sin(xy)$

(c) $f(x, y) = \ln(x^2 + y^2)$

(d) $f(x, y) = \ln(\sec\sqrt{x + y})$