# $\mathcal{ME}$ *201* ADVANCED CALCULUS

**Assignment 2:** *Applications of Vectors and Vector Calculus*
*January 12, 2018*

1. Find the area of the following:

   (a) A triangle with vertices $(1, 2, 3)$, $(3, 5, 10)$ and (-3,-4,-11).

   (b) A parallelogram with vertices $(1, -2, 4)$, $(3, 5, 7)$, $(4, 6, 8)$ and $(2, -1, 5)$.

2. Find the shortest distance between the following:

   (a) From point $(-2, 3, -5)$ to the plane $2x + y + 4z = 6$.

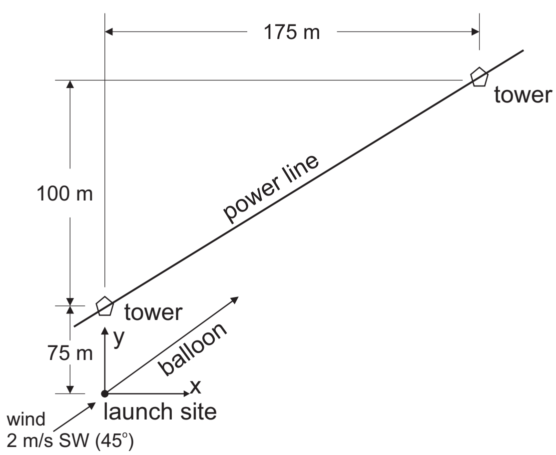
   (b) From point $(3, -2, 0)$ to the line $x = t,\ y = 3 - 2t,\ z = 4 + t$.

   (c) From point $(1, 2, -3)$ to the line $x = 2(y + 1) = (z - 4)/2$.

   (d) Between the lines $x = t,\ y = 3t - 1,\ z = 1 + 2t$ and $x = 2t + 1,\ y = 1 - t,\ z = 4 + 2t$.

   (e) Between the lines $x + y - z = 4,\ 2x - z = 4$ and $x = \dfrac{y + 1}{2} = \dfrac{z - 1}{3}$.

3.




A hot air balloon is being launched from a field adjacent to a high voltage power line. As the balloon leaves the ground, a $2\ m/s$ gust of wind from the south-west blows the balloon towards the power lines. Assume that the balloon ascends at a steady rate of $0.5\ m/s$ and the power lines are at a constant height of $30\ m$ (no line sag between the towers). A plan view of the launch site and the power line is shown in the figure.

   (a) Derive a set of parametric equations for the position of the balloon as a function of time relative to the coordinate system given in the diagram. $(m)$

   (b) How close does the balloon get to the power line? $(m)$

4. Calculate the moment of the force for the following:

   (a) $\vec{F} = 3\hat{i} - \hat{j} + 4\hat{k}$ at $(1, 1, 0)$ about the point $(2, 1, -5)$

   (b) $\vec{F} = 6\hat{i} - 5\hat{j} + \hat{k}$ at $(-2, 3, 1)$ about the line $\dfrac{x-3}{2} = y + 1 = \dfrac{z}{4}$

5. If

$$\begin{aligned} f(t) &= t^2 + 3 \\ \vec{u}(t) &= t\hat{i} - t^2\hat{j} + 2t\hat{k} \\ \vec{v}(t) &= \hat{i} - 2t\hat{j} + 3t^2\hat{k} \end{aligned}$$

   solve the following derivative and integral expressions:

   (a) $\dfrac{d}{dt}(3\vec{u} + 4\vec{v})$

   (b) $\displaystyle\int \vec{u}\, dt$

   (c) $\dfrac{d}{dt}\left[t\,(\vec{u} \times \vec{v})\right]$

   (d) $\displaystyle\int \left[f(t)\, \vec{u} \cdot \vec{v}\right]\, dt$

6. Express the curve in vector form and find the unit tangent vector $\hat{T}$ at each point on the curve for the following:

   (a) $x = t,\ y = t^2, z = t^3,\ \ t \geq 0$

   (b) $x + y = 5,\ x^2 - y = z$ from $(5, 0, 25)$ to $(0, 5, -5)$

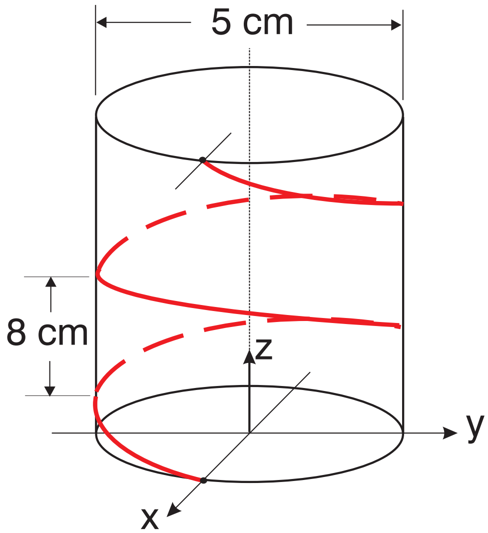
7. Find the length of the curve and plot the curve in $3D$:

   (a) $x = 2\cos t,\ y = 2\sin t,\ z = 3t,\ \ 0 \leq t \leq 2\pi$

   (b) $x = 2 - 5t,\ y = 1 + t,\ z = 6 + 4t,\ \ -1 \leq t \leq 0$

   (c) $x = t,\ y = t^{3/2},\ z = 4t^{3/2},\ \ 1 \leq t \leq 4$

8.




To protect a water pipe from freezing in winter temperatures, a heat cable is wrapped around the pipe. The wire is wrapped in a circular helix around the $\mathbf{5}\ \boldsymbol{cm}$ diameter pipe with a complete turn around the pipe every $\mathbf{8}\ \boldsymbol{cm}$. Assume that the wire diameter is negligible.

(a) Derive a set of parametric equations for the position of the heating wire relative to the coordinate system given in the diagram.

(b) What is the length of wire required to heat a $\mathbf{2}\ \boldsymbol{m}$ long section of pipe? $(\boldsymbol{m})$