$\mathcal{ME}$ 201   ADVANCED CALCULUS

Assignment 1: *Vectors, Vector Operations and Vector Representations*
January 5, 2018

1. If $\vec{u} = (1, 3, 6)$, $\vec{v} = (-2, 0, 4)$ and $\vec{w} = (4, 3, -2)$, solve the following vector operations, expressing your answers in triple notation.

   (a) $2\vec{w} + 3\vec{v}$

   (b) $|\vec{v}|\vec{v} - 2|\hat{v}|\vec{w}$

   (c) $\dfrac{\vec{v} - \vec{w}}{|\vec{v} + \vec{w}|}$

2. If $\vec{u} = 2\hat{i} + \hat{j}$ and $\vec{v} = -\hat{i} + 3\hat{j}$, solve the following vector operations, expressing your answers in vector notation. Plot the resulting vector in $2D$ (x, y) space.

   (a) $\vec{u} + \vec{v}$

   (b) $\hat{v} + \hat{u}$

3. If P, Q and R are the points with coordinates $(3, 2, -1)$, $(0, 1, 4)$ and $(6, 5, -2)$, respectively, find the coordinates of a point S such that $\overrightarrow{PQ} = \overrightarrow{RS}$

4. Section 11.3   Problem 37.

5. If $\vec{u} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{v} = \hat{j} - \hat{k}$ and $\vec{w} = 6\hat{i} - 2\hat{j} + 3\hat{k}$, solve the following vector products, expressing your answers in vector notation.

   (a) $(\vec{v} \cdot \vec{w})\vec{u}$

   (b) $(3\hat{u} - 4\hat{w}) \cdot (2\hat{i} + 3\hat{u} - 2\hat{v})$

6. If $\vec{u} = (3, 1, 4)$, $\vec{v} = (-1, 2, 0)$ and $\vec{w} = (-2, -3, 5)$, solve the following vector operations, expressing your answers in triple notation.

   (a) $\vec{u} \times (3\vec{v} - \vec{w})$

   (b) $\vec{u} \times (\vec{v} \times \vec{w})$

7. Section 11.4   Problems 26, 30.

8. Section 11.4   Problems 31, 32.

9. Find the equation for the following planes:

   (a) Through the point $(2, 1, 5)$ and normal to the vector joining $(2, 1, 5)$ and $(4, 2, 3)$.

   (b) Containing the points $(1, 3, 2)$, $(-2, 0, -2)$ and $(1, 4, 3)$.

   (c) Containing the lines $\dfrac{x-1}{6} = \dfrac{y}{8} = \dfrac{z+2}{2}$ and $\dfrac{x+1}{3} = \dfrac{y-2}{4} = z + 5$.

10. Find vector, parametric and symmetric (if possible) equations for the following lines:

    (a) Through the points $(2, -3, 4)$ and $(5, 2, -1)$.

    (b) Through the point $(-2, 3, 1)$ and parallel to the line $x + y = 3$, $2x - y + z = -2$.

    (c) For the intersection of surfaces $2x - y = 5$ and $3x + 4y + z = 10$.