Introduction to Thermodynamics and Heat Transfer

E&CE309, Spring 2003, Project #2

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- *Due date for this project is on Saturday, August 2, 03.*

A hot fluid is contained in a spherical shell of inner radius r_i and outer radius r_o. The thermal conductivity of the spherical wall is k, which is assumed to be constant. The temperature of the hot fluid is T_{f1} and the heat transfer coefficient at the inner boundary is h_1. The temperature of the fluid at the outer boundary is $T_{f2} < T_{f1}$ and the heat transfer coefficient is h_2. Since there are no distributed source or sinks within the spherical wall, and the temperature is steady-state, *i.e.* $T = T(r)$, the governing equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \qquad r_i \le r \le r_o$$

(a) Specify the two boundary conditions.

(b) Obtain the temperature distribution within the spherical wall, and put your results in the form:

$$\frac{T_{f1} - T(r)}{T_{f1} - T_{f2}} = ?$$

Give a physical interpretation of the terms which appear in the numerator and the denominator of the right-hand side of the solution.

(c) Obtain the expression for the heat transfer rate through the spherical wall using the Fourier law of conduction:

$$\dot{Q} = -k4\pi r^2\frac{dT}{dr}$$

(d) Use the definition of the total thermal resistance of the system:

$$R_{total} = \frac{(T_{f1} - T_{f2})}{\dot{Q}}$$

to demonstrate that the total resistance consists of the sum of the inner film resistance, the shell resistance and the outer film resistance.

(e) Sketch the thermal circuit showing clearly the nodes, the thermal resistors and the throughput. Label the nodes and resistors.