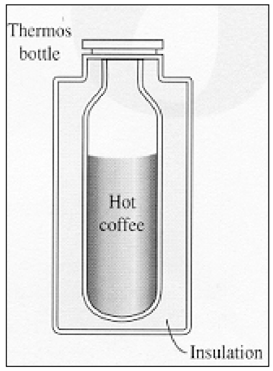
Thermodynamics *vs*. Heat Transfer

- ***Thermodynamics*** is concerned with the amount of heat transfer as a system undergoes a process from one equilibrium state to another,
- ***Thermodynamics*** gives no indication about how long the process takes,
- ***Heat Transfer*** determines how fast heat can be transferred to or from a system and thus the times of cooling or heating.



Heat Transfer Mechanisms

- **Conduction:**

$$\boxed{\dot{Q}_{cond} = kA\frac{\Delta T}{\Delta x}} \quad \text{or} \quad \boxed{\dot{Q}_{cond} = -kA\frac{dT}{dx} \quad \text{(W)}}$$

- **Convection:**

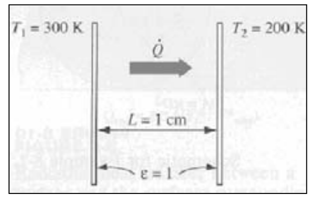
$$\boxed{\dot{Q}_{convvection} = hA(T_s - T_\infty) \quad \text{(W)}}$$

- **Radiation:**

$$\boxed{\dot{Q}_{rad} = \varepsilon\sigma A(T_s^4 - T_{surr}^4) \quad \text{(W)}}$$

Example

- Determine the rate of heat transfer between the plates per unit surface area assuming the gap between the plates is



1. filled with atmospheric air,
2. evacuated,
3. filled with urethane insulation, and
4. filled with superinsulation with k = 0.00002 W/m.°C.

Steady Heat Conduction in Plane Walls

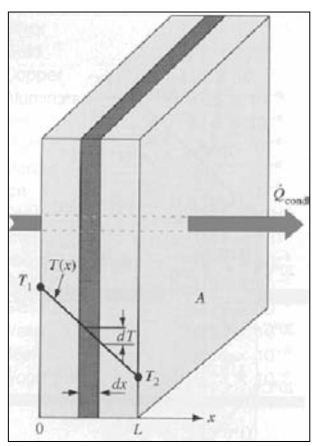
Energy Balance:

$$\begin{pmatrix}\text{rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{rate of}\\ \text{change of the}\\ \text{energy content}\\ \text{of the wall}\end{pmatrix}$$

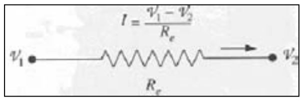
$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt}$$

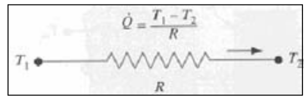
$$\dot{Q}_{cond,wall} = -kA\frac{dT}{dx} \quad \text{(W)}$$

$$\boxed{\dot{Q}_{cond,wall} = kA\frac{T_1 - T_2}{L} \quad \text{(W)}}$$



Thermal Resistance





$$\dot{Q}_{cond,wall} = \frac{T_1 - T_2}{R_{wall}} \quad \text{(W)}$$

$$\dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}} \quad \text{(W)}$$

$$\boxed{R_{wall} = \frac{L}{kA} \quad (°\text{C/W})}$$

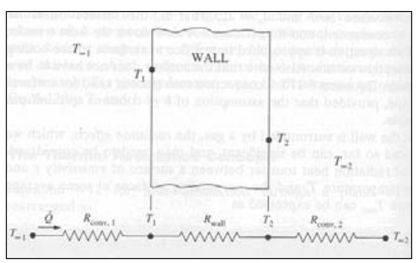
$$\boxed{R_{conv} = \frac{1}{hA} \quad (°\text{C/W})}$$

$$\dot{Q}_{rad} = \varepsilon\sigma A(T_s^4 - T_{surr}^4) = h_{rad}A(T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}} \quad \text{(W)}$$

$$\boxed{R_{rad} = \frac{1}{h_{rad}A} \quad \text{(K/W)}}$$

$$h_{rad} = \varepsilon\sigma A(T_s^2 - T_{surr}^2)(T_s - T_{surr}) \quad (\text{W/(m}^2\text{.K))}$$

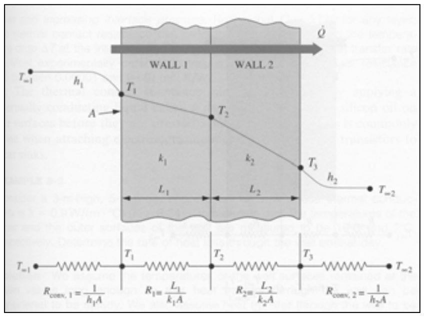
Thermal Resistance Network



$$\boxed{\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} \quad \text{(W)}}$$

$$\boxed{R_{total} = R_{conv,1} + R_{wall} + R_{conv,2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \quad (°\text{C/W})}$$

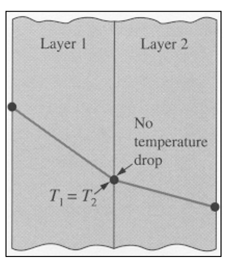
Mutilayer Plane Wall



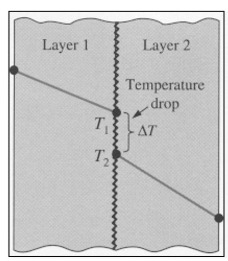
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

$$\begin{aligned} R_{total} &= R_{conv,1} + R_{wall,1} + R_{wall,2} + R_{conv,2} \\ &= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_1 A} + \frac{1}{h_2 A} \end{aligned}$$

Thermal Contact Resistance



Perfect Thermal Contact



Actual Thermal Contact

- The thermal resistance due to the roughness at the contact areas is called ***thermal contact resistance.***

Example

Given:

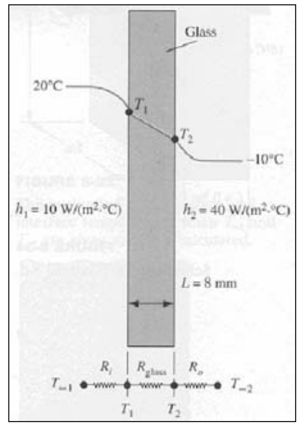
W = 1.5 m,

H = 0.8 m,

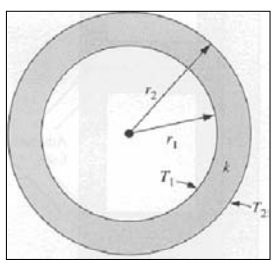
k = 0.78 W/m.°C,

Calculate:

1. Steady rate of heat transfer through this glass window and,
2. Temperature of the inner surface.



Heat Conduction in Cylinders & Spheres



$$\dot{Q}_{Cond} = \frac{T_1 - T_2}{R_{th}}$$

$$R_{th,cyl} = \frac{\ln(r_2 / r_1)}{2\pi k L} \qquad R_{th,sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 k}$$

Example

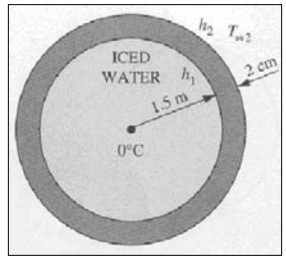
Given:

k = 15 W/(m.°C),

$T_{\infty 1}$ = 0 °C, $T_{\infty 2}$ = 22 °C,

h_1 = 80, h_2 = 10W/(m2.°C),

Latent heat of fusion = h_{if} = 333.7 kJ/kg.



Determine:

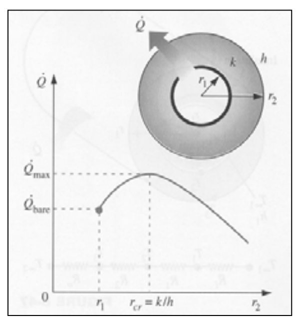
(a) Rate of heat transfer,

(b) Amount of ice at 0 °C that melts during a 24-h period.

Critical Radius of Insulation, r_{cr}

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} \Rightarrow \frac{d\dot{Q}}{dr} = 0$$

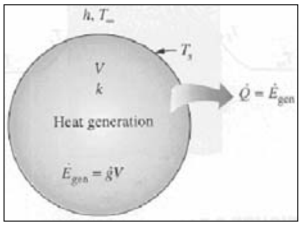
$$\boxed{r_{cr,cylinder} = \frac{k}{h} \quad (\mathrm{m})} \quad \boxed{r_{cr,sphere} = \frac{2k}{h}}$$



k: Thermal conductivity of insulation

h: Ambient heat transfer coefficient

Heat Generation in a Solid



$$\dot{Q} = \dot{g}V = hA(T_s - T_\infty)$$

$$\boxed{T_s = T_\infty + \frac{\dot{g}V}{hA}}$$

$\dot{g}$: Heat generation / unit volume, (W/m^3)

$$T_{s,\text{ plane wall}} = T_\infty + \frac{\dot{g}L}{h}$$

$$\Delta T_{\text{max, plane wall}} = \frac{\dot{g}L^2}{2k}$$

$$T_{s,\text{ cylinder}} = T_\infty + \frac{\dot{g}r_o}{2h}$$

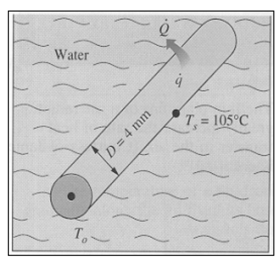
$$\Delta T_{\text{max, cylinder}} = T_o - T_s = \frac{\dot{g}r_o^2}{4k}$$

$$T_{s,\text{ sphere}} = T_\infty + \frac{\dot{g}r_o}{3h}$$

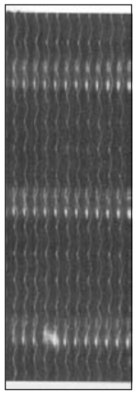
$$\Delta T_{\text{max, sphere}} = \frac{\dot{g}r_o^2}{6k}$$

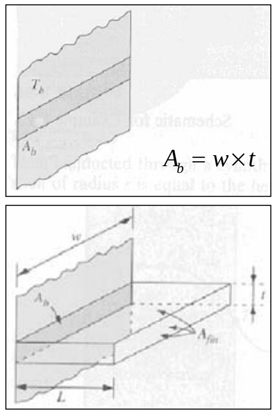
Example

Given:
2-kW resistance heater wire,
k = 15 W/(m.°C),
D = 4 mm,
L = 0.5 m,
T_s = 105 °C,
Calculate: Centerline temperature.

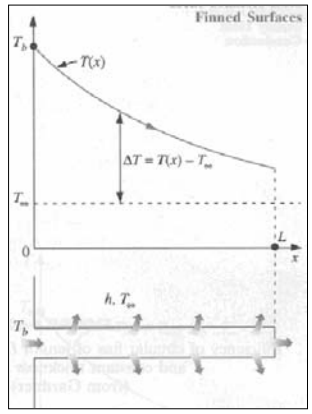


Heat Transfer from Finned Surfaces





$$A_b = w \times t$$



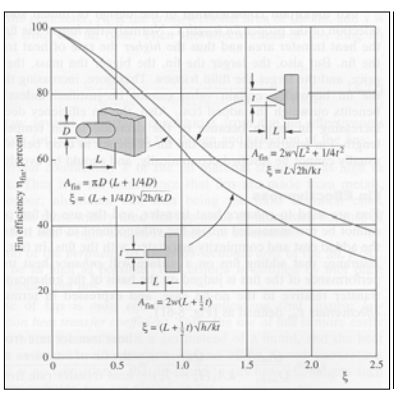
$$\dot{Q}_{conv} = \int_L h(T - T_\infty)\, dA$$

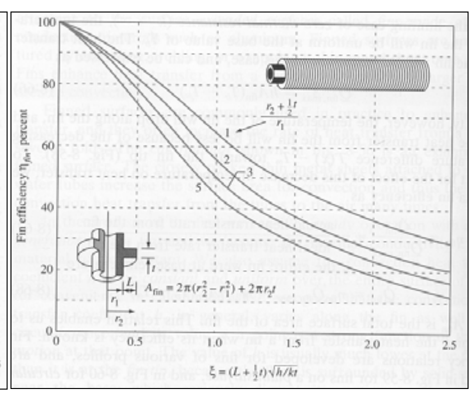
$$A_{rec.\ fin} = 2 \times w \times L + w \times t$$

$$A_{pin\ fin} = \pi DL + \pi D^2 / 4$$

Fin Efficiency, η_{fin}

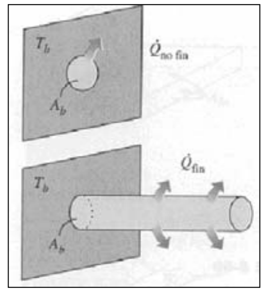
$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,\max}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal (maximum) heat transfer rate from the fin}}$$





Fin Effectiveness, ε_{fin}

- Thermal conductivity of the fin material should be as high as possible. The most widely used fins are made of aluminum.
- The ratio of the perimeter to the cross-sectional area of the fin, p/A_c, should be as high as possible.
- The use of fins is most effective in applications that involve low convection heat transfer coefficient (gases not liquids).

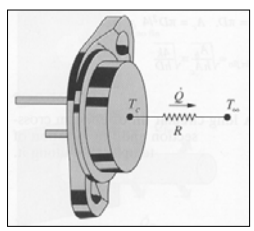


$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}}$$

Example

Given

- Case-to-ambient thermal resistance = 20 °C/W,
- Maximum power rating = 10W,
- Maximum allowable temperature = 85 °C,
- Ambient temperature = 25 °C.



Determine:

- Power at which this transistor can be operated safely.