Introduction to Thermodynamics and Heat Transfer (ECE 309)
Suggested Problems for Chapter 4

1. Water flows through a shower head steadily at a rate of 10 L/min. An electric resistance heater placed in the water pipe heats the water from 16 °C to 43 °C. Taking the density of water to be 1 kg/L, determine the electric power input to the heater, in kW.

In an effort to conserve energy, it is proposed to pass the drained warm water at a temperature of 39°C through a heat exchanger to preheat the incoming cold water. If the heat exchanger has an effectiveness of 0.50 (that is, it recovers only half of the energy which can possibly be transferred from the drained water to incoming cold water), determine the electric power input required in this case. If the price of the electric energy is 8.5 cents/kWh, determine how much money is saved during a 10 min shower as a result of installing this heat exchanger.

We treat the water as an incompressible substance with $\rho = 1$ kg/L. The required electric power input is determined from the steady-flow energy equation,

$$\dot{Q}^{\nearrow 0} - \dot{W} = \dot{m}\left(\Delta h + \Delta ke^{\nearrow 0} + \Delta pe^{\nearrow 0}\right) \longrightarrow \dot{W}_{in} = \dot{m}C(T_2 - T_1)$$

with

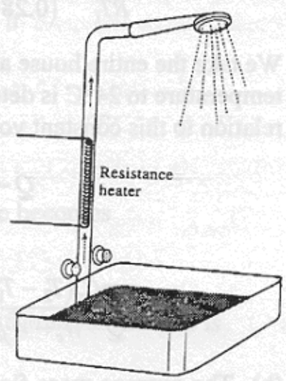
$$\dot{m} = \rho \dot{V} = (1\ \text{kg/L})(10\ \text{L/min}) = 10\ \text{kg/min}$$

Thus,

$$\dot{W}_{in} = (10/60\ \text{kg/s})(4.184\ \text{kJ/kg}\cdot{}^\circ\text{C})(43-16)^\circ\text{C} = \mathbf{18.8\ kW}$$

The energy recovered by the heat exchanger is

$$\dot{Q}_{saved} = \varepsilon \dot{Q}_{max} = \varepsilon \dot{m}C(T_{max} - T_{min})$$
$$= 0.5(10/60\ \text{kg/s})(4.184\ \text{kJ/kg}^\circ\text{C})(39-16)^\circ\text{C}$$
$$= 8.0\ \text{kJ/s} = 8.0\ \text{kW}$$



Therefore, 8.0 kW less energy is needed in this case, and the required electric power in this case reduces to

$$\dot{W}_{in,new} = \dot{W}_{in,out} - \dot{Q}_{saved} = 18.8 - 8.0 = \mathbf{10.8\ kW}$$

The money saved during a 10-min shower as a result of installing this heat exchanger is

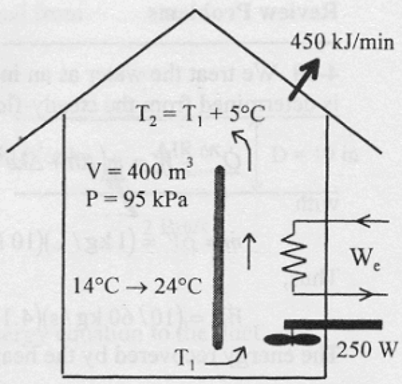
$$(8.0\ \text{kW})(10/60\ \text{h})(8.5\ \text{cents/kWh}) = \mathbf{11.3\ cents}$$

2. A building with an internal volume of 400 m^3 is to be heated by a 30-kW electric resistance heater placed in the duct inside the building. Initially, the air in the building is at 14 °C, and the local atmospheric pressure is 95 kPa. The building is losing heat to the surroundings at a steady rate of 450 kJ/min. Air is forced to flow through the duct and the heater steadily by a 250-W fan, and it experiences a temperature rise of 5 °C each time it passes through the duct, which may be assumed to be adiabatic.

(a) How long will it take for the air inside the building to reach an average temperature of 24 °C?

(b) Determine the average mass flow rate of air through the duct.

(a) We assume air to be an ideal gas with constant specific heats at room temperature, C_p = 1.005 and C_v = 0.718 kJ/kg·K. The total mass of air in the house is



$$m = \frac{P_1 V_1}{RT_1} = \frac{(95\text{ kPa})(400\text{ m}^3)}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(287\text{ K})} = 461.4\text{ kg}$$

We take the entire house as our system. The time required to raise the air temperature to 24°C is determined by applying the conservation of energy relation to this constant volume closed system:

$$Q - W_e - W_{fan} - W_b^{\nearrow 0} = \Delta U + \Delta KE^{\nearrow 0} + \Delta PE^{\nearrow 0}$$

$$\Delta t\left(\dot{Q} - \dot{W}_e - \dot{W}_{fan}\right) = mC_v(T_2 - T_1)$$

$$\Delta t = \frac{mC_v(T_2 - T_1)}{\dot{Q} - \dot{W}_e - \dot{W}_{fan}} = \frac{(461.4\text{ kg})(0.718\text{ kJ/kg}\cdot°\text{C})(24-14)°\text{C}}{-(450/60\text{ kJ/s}) - (30\text{ kJ/s}) - (-0.25\text{ kJ/s})} = \mathbf{146\text{ s}}$$

(b) The average mass flow rate of air through the duct is determined by applying the steady-flow energy equation to the duct:

$$\dot{Q}^{\nearrow 0} - \dot{W} = \dot{m}\left(\Delta h + \Delta ke^{\nearrow 0} + \Delta pe^{\nearrow 0}\right)$$

Thus,

$$\dot{m} = \frac{-\dot{W}}{C_p \Delta T} = \frac{-(-30 - 0.25)\text{ kJ/s}}{(1.005\text{ kJ/kg}\cdot°\text{C})(5°\text{C})} = \mathbf{6.02\text{ kg/s}}$$

3. The ventilating fan of the bathroom of a building has a volume flow rate of 30 L/s, and runs continuously. The building is located in San Francisco, California where the average winter temperature is 12.2 °C, and is maintained at 22 °C at all times. The building is heated by electricity whose unit cost is $0.091/kWh. Determine the amount and cost of the heat "vented out" per month in winter.

San Francisco is at sea level, and thus we take the atmospheric pressure there 1 atm = 101.3 kPa. The density of indoor air at 1 atm and 22°C is

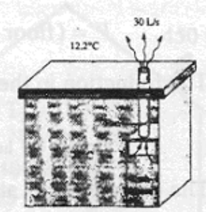


FIGURE P4-84

$$\rho_o = \frac{P_o}{RT_o} = \frac{(101.3\text{ kPa})}{(0.287\text{ kPa.m}^3/\text{kg.K})(22 + 273.15\text{ K})} = 1.20\text{ kg/m}^3$$

Then the mass flow rate of air vented out becomes

$$\dot{m}_{air} = \rho \dot{V}_{air} = (1.20\text{ kg/m}^3)(0.030\text{ m}^3/\text{s}) = 0.036\text{ kg/s}$$

Noting that the indoor air vented out at 22°C is replaced by infiltrating outdoor air at 12.2°C, this corresponds to energy loss at a rate of

$$\dot{Q}_{\text{loss,fan}} = \dot{m}_{\text{air}} C_p (T_{\text{indoors}} - T_{\text{outdoors}})$$
$$= (0.036\text{ kg/s})(1.005\text{ kJ/kg.°C})(22 - 12.2)°\text{C} = 0.0355\text{ kJ/s} = 0.0355\text{ kW}$$

Then the amount and cost of the heat "vented out" per month (1 month = 30×24 = 720 h) becomes

Energy loss = $\dot{Q}_{\text{loss,fan}} \Delta t$ = (0.0355 kW)(720 h / month) = **25.6 kWh / month**

Money loss = (Energy loss)(Unit cost of energy) = (25.6 kWh / month)($0.09 / kWh) = **$2.30 / month**