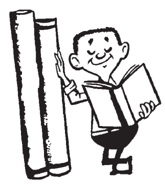
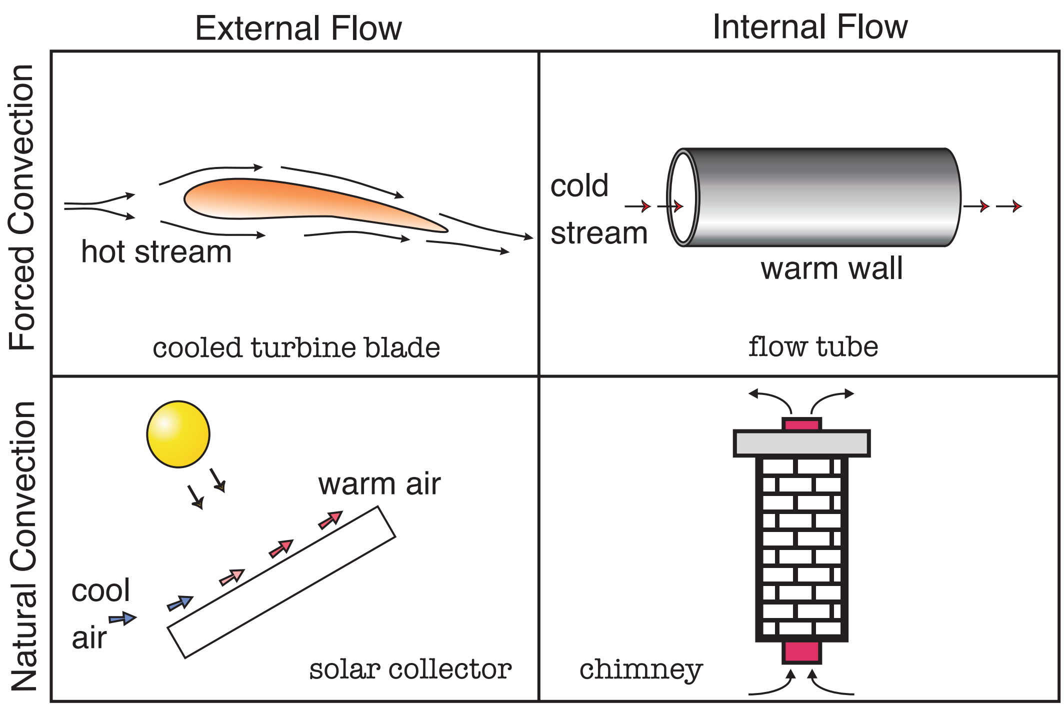
Convection Heat Transfer



Reading	**Problems**
19-1 $\longrightarrow$ 19-8	19-15, 19-24, 19-35, 19-47, 19-53, 19-69, 19-77
20-1 $\longrightarrow$ 20-6	20-21, 20-28, 20-44, 20-57, 20-79

Introduction



- in convective heat transfer, the bulk fluid motion of the fluid plays a major role in the overall energy transfer process. Therefore, knowledge of the velocity distribution near a solid surface is essential.
- the controlling equation for convection is *Newton's Law of Cooling*

$$\dot{Q}_{conv} = \frac{\Delta T}{R_{conv}} = hA(T_w - T_\infty) \qquad \Rightarrow \quad R_{conv} = \frac{1}{hA}$$

where

$$A = \text{total convective area, } m^2$$

h = heat transfer coefficient, $W/(m^2 \cdot K)$

T_w = surface temperature, $^\circ C$

T_∞ = fluid temperature, $^\circ C$

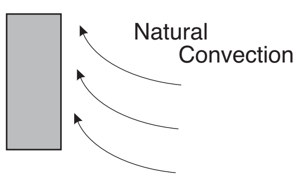
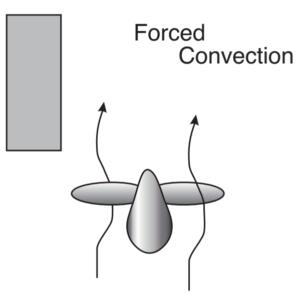
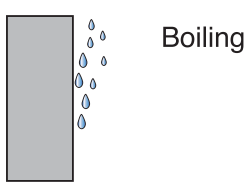
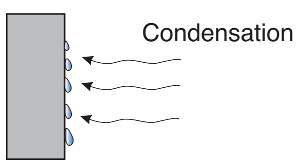
External Flow: the flow engulfs the body with which it interacts thermally

Internal Flow: the heat transfer surface surrounds and guides the convective stream

Forced Convection: flow is induced by an external source such as a pump, compressor, fan, etc.

Natural Convection: flow is induced by natural means without the assistance of an external mechanism. The flow is initiated by a change in the density of fluids incurred as a result of heating.

Mixed Convection: combined forced and natural convection

Process	h $[W/(m^2 \cdot K)]$
	• gases 3 - 20 • water 60 - 900
	• gases 30 - 300 • oils 60 - 1 800 • water 100 - 1 500
	• water 3 000 - 100 000
	• steam 3 000 - 100 000

Dimensionless Groups

Prandtl number: $Pr = \nu/\alpha$ where $0 < Pr < \infty$ ($Pr \to 0$ for liquid metals and $Pr \to \infty$ for viscous oils). A measure of ratio between the diffusion of momentum to the diffusion of heat.

Reynolds number: $Re = \rho U\mathcal{L}/\mu \equiv U\mathcal{L}/\nu$ (forced convection). A measure of the balance between the inertial forces and the viscous forces.

Peclet number: $Pe = U\mathcal{L}/\alpha \equiv RePr$

Grashof number: $Gr = g\beta(T_w - T_f)\mathcal{L}^3/\nu^2$ (natural convection)

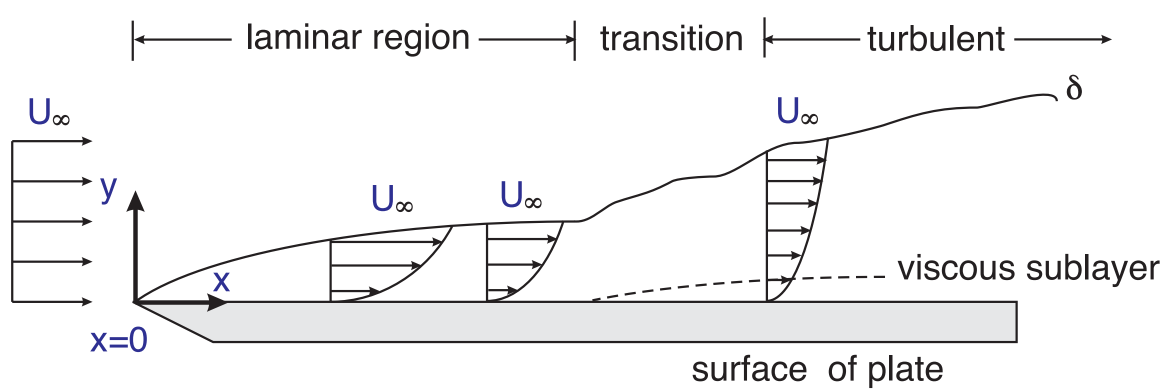
Rayleigh number: $Ra = g\beta(T_w - T_f)\mathcal{L}^3/(\alpha \cdot \nu) \equiv GrPr$

Nusselt number: $Nu = h\mathcal{L}/k_f$ This can be considered as the dimensionless heat transfer coefficient.

Stanton number: $St = h/(U\rho C_p) \equiv Nu/(RePr)$

Forced Convection

The simplest forced convection configuration to consider is the flow of mass and heat near a flat plate as shown below.



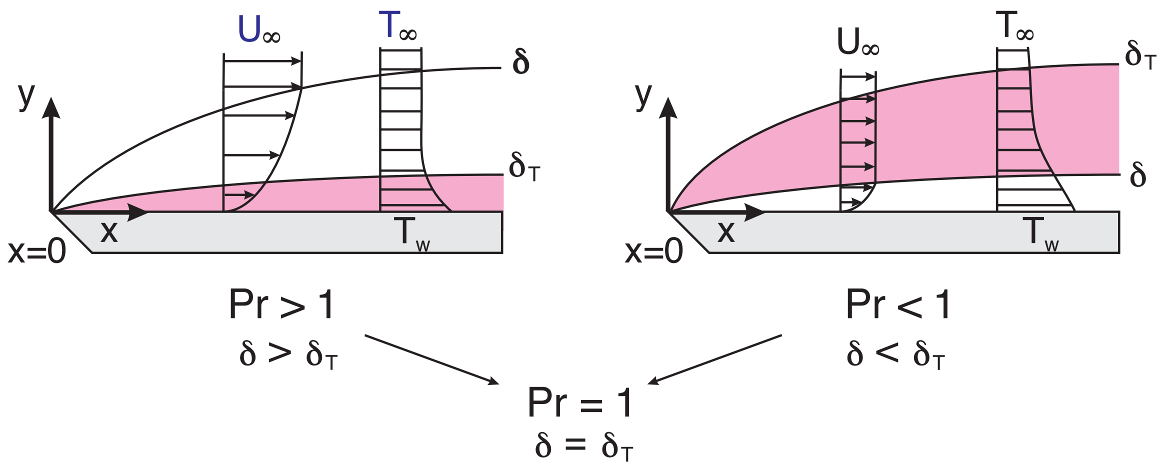
- as Reynolds number increases the flow has a tendency to become more chaotic resulting in disordered motion known as turbulent flow
  - transition from laminar to turbulent is called the critical Reynolds number, Re_{cr}

$$Re_{cr} = \frac{U_\infty x_{cr}}{\nu}$$

  - for flow over a flat plate $Re_{cr} \approx 500,000$

– for engineering calculations, the transition region is usually neglected, so that the transition from laminar to turbulent flow occurs at a critical location from the leading edge, x_{cr}

Boundary Layers



Velocity Boundary Layer

- the region of fluid flow over the plate where viscous effects dominate is called the *velocity* or *hydrodynamic* boundary layer
- the velocity of the fluid progressively increases away from the wall until we reach approximately $0.99\ U_\infty$ which is denoted as the δ, the *velocity boundary layer thickness*. Note: 99% is an arbitrarily selected value.

Thermal Boundary Layer

- the thermal boundary layer is arbitrarily selected as the locus of points where

$$\frac{T - T_w}{T_\infty - T_w} = 0.99$$

Heat Transfer Coefficient

The local heat transfer coefficient can be written as

$$h = \frac{-k_f \left(\dfrac{\partial T}{\partial y}\right)_{y=0}}{(T_w - T_\infty)} \equiv h(x) = h_x$$

The average heat transfer coefficient is determined using the mean value theorem such that

$$h_{av} = \frac{1}{L}\int_0^L h(x)\, dx$$

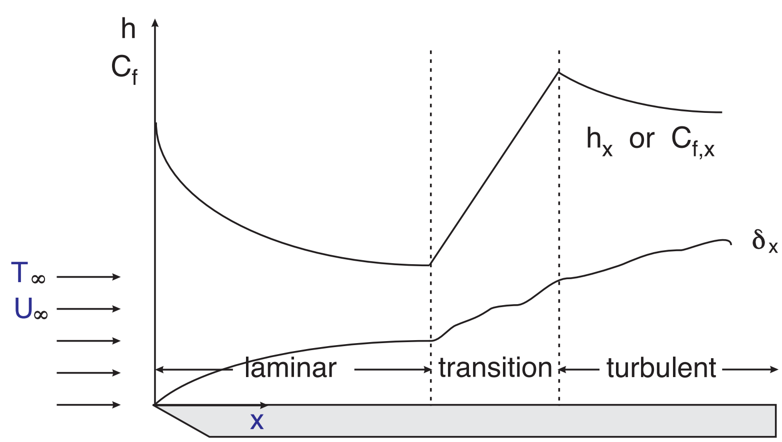
The Nusselt number is a measure of the dimensionless heat transfer coefficient given as

$$Nu = f(Re, Pr)$$

While the Nusselt number can be determine analytically through the conservations equations for mass, momentum and energy, it is beyond the scope of this course. Instead we will use *empirical correlations* based on experimental data where

$$Nu_x \quad = \quad C_2 \cdot Re^m \cdot Pr^n$$

Flow Over Plates



1. Laminar Boundary Layer Flow, Isothermal (UWT)

The local value of the Nusselt number is given as

$$\boxed{Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}} \quad \Rightarrow \text{local, laminar, UWT, } Pr \geq 0.6$$

where x is the distance from the leading edge of the plate.

$$\boxed{Nu_L = \frac{h_L L}{k_f} = 0.664\, \mathrm{Re}_L^{1/2}\, \mathrm{Pr}^{1/3}} \quad \Rightarrow \text{average, laminar, UWT, } Pr \geq 0.6$$

For low Prandtl numbers, i.e. liquid metals

$$\boxed{Nu_x = 0.565\, Re_x^{1/2}\, Pr^{1/2}} \quad \Rightarrow \text{local, laminar, UWT, } Pr \leq 0.6$$

2. Turbulent Boundary Layer Flow, Isothermal (UWT)

$$\boxed{Nu_x = 0.0296\, Re_x^{0.8}\, Pr^{1/3}} \quad \Rightarrow \begin{array}{l} \text{local, turbulent, UWT,} \\ 0.6 < Pr < 100, \quad Re_x > 500,000 \end{array}$$

$$\boxed{Nu_L = 0.037\, Re_L^{0.8}\, Pr^{1/3}} \quad \Rightarrow \begin{array}{l} \text{average, turbulent, UWT,} \\ 0.6 < Pr < 100, \quad Re_x > 500,000 \end{array}$$

3. Combined Laminar and Turbulent Boundary Layer Flow, Isothermal (UWT)

$$\boxed{Nu_L = \frac{h_L L}{k} = (0.037\, Re_L^{0.8} - 871)\, Pr^{1/3}} \quad \Rightarrow \begin{array}{l} \text{average, turbulent, UWT,} \\ 0.6 < Pr < 60, \quad Re_L > 500,000 \end{array}$$

4. Laminar Boundary Layer Flow, Isoflux (UWF)

$$\boxed{Nu_x = 0.453\, Re_x^{1/2}\, Pr^{1/3}} \quad \Rightarrow \text{local, laminar, UWF, } Pr \geq 0.6$$

5. Turbulent Boundary Layer Flow, Isoflux (UWF)

$$\boxed{Nu_x = 0.0308\, Re_x^{4/5}\, Pr^{1/3}} \quad \Rightarrow \text{local, turbulent, UWF, } Pr \geq 0.6$$

Flow Over Cylinders and Spheres

1. Boundary Layer Flow Over Circular Cylinders, Isothermal (UWT)

The Churchill-Berstein (1977) correlation for the average Nusselt number for long ($L/D > 100$) cylinders is

$$\boxed{Nu_D = S_D^* + f(Pr)\, Re_D^{1/2}\left[1 + \left(\frac{Re_D}{282000}\right)^{5/8}\right]^{4/5}} \Rightarrow \begin{array}{l}\text{average, UWT, } Re < 10^7 \\ 0 \leq Pr \leq \infty, \;\; Re \cdot Pr > 0.2\end{array}$$

where $S_D^* = 0.3$ is the diffusive term associated with $Re_D \rightarrow 0$ and the Prandtl number function is

$$f(Pr) = \frac{0.62\, Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}$$

All fluid properties are evaluated at $T_f = (T_w + T_\infty)/2$.

2. Boundary Layer Flow Over Non-Circular Cylinders, Isothermal (UWT)

The empirical formulations of Zhukauskas and Jakob are commonly used, where

$$\boxed{Nu_D \approx \frac{\overline{h}D}{k} = C\, Re_D^m\, Pr^{1/3}} \Rightarrow \text{see Table 19-2 for conditions}$$

3. Boundary Layer Flow Over a Sphere, Isothermal (UWT)

For flow over an isothermal sphere of diameter D

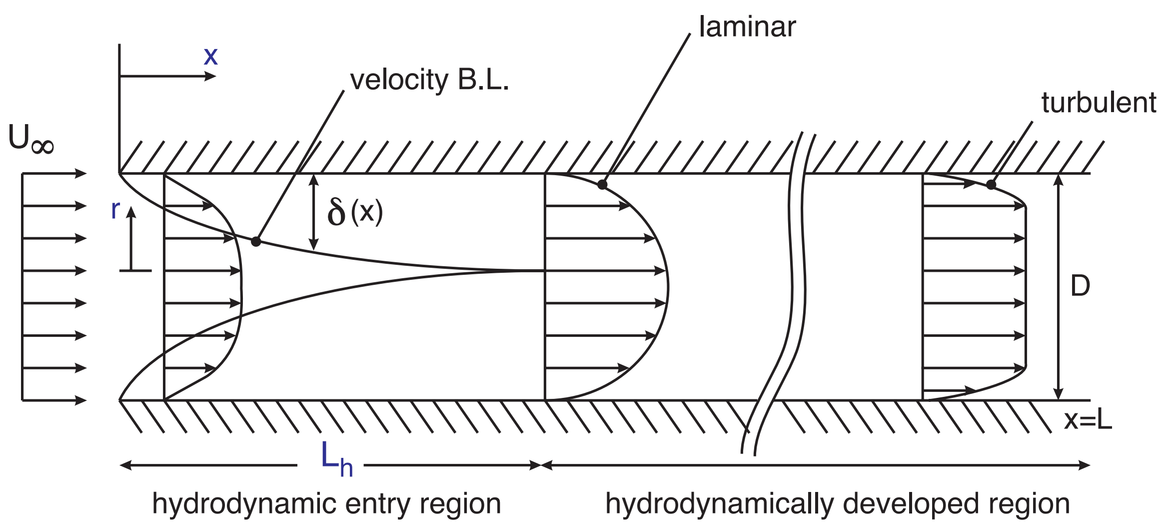
$$\boxed{Nu_D = S_D^* + \left[0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}\right] Pr^{0.4}\left(\frac{\mu_\infty}{\mu_w}\right)^{1/4}} \Rightarrow \begin{array}{l}\text{average, UWT,} \\ 0.7 \leq Pr \leq 380 \\ 3.5 < Re_D < 80,000\end{array}$$

where the diffusive term at $Re_D \rightarrow 0$ is $S_D^* = 2$

and the dynamic viscosity of the fluid in the bulk flow, μ_∞ is based on T_∞ and the dynamic viscosity of the fluid at the surface, μ_w, is based on T_w. All other properties are based on T_∞.

Internal Flow

Lets consider fluid flow in a duct bounded by a wall that is at a different temperature than the fluid. For simplicity we will examine a round tube of diameter D as shown below



The velocity profile across the tube changes from $U = 0$ at the wall to a maximum value along the center line. The average velocity, obtained by integrating this velocity profile, is called the *mean velocity* and is given as

$$U_m = \frac{1}{A_c}\int_{A_c} u\, dA = \frac{\dot{m}}{\rho_m A_c}$$

where the area of the tube is given as $A_c = \pi D^2/4$ and the fluid density, ρ_m is evaluated at T_m.

The Reynolds number is given as

$$Re_D = \frac{U_m D}{\nu}$$

For flow in a tube:

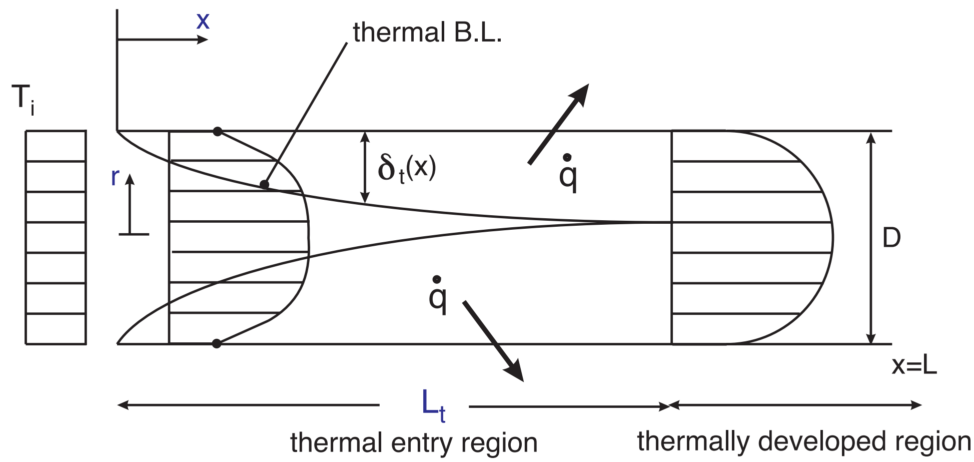
$Re_D < 2300$	laminar flow
$2300 < Re_D < 4000$	transition to turbulent flow
$Re_D > 4000$	turbulent flow

Hydrodynamic (Velocity) Boundary Layer

- the hydrodynamic boundary layer thickness can be approximated as

$$\delta(x) \approx 5x \left(\frac{U_m x}{\nu} \right)^{-1/2} = \frac{5x}{\sqrt{Re_x}}$$

Thermal Boundary Layer



- the thermal entry length can be approximated as

$$L_t \approx 0.05 Re_D Pr D \qquad \text{(laminar flow)}$$

- for turbulent flow $L_h \approx L_t \approx 10D$

1. Laminar Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For laminar flow where $Re_D \leq 2300$

$\boxed{Nu_D = 3.66}$ $\Rightarrow$ fully developed, laminar, UWT, $L > L_t$ & L_h

$\boxed{Nu_D = 4.36}$ $\Rightarrow$ fully developed, laminar, UWF, $L > L_t$ & L_h

$$\boxed{Nu_D = 1.86 \left(\frac{Re_D Pr D}{L}\right)^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.4}} \Rightarrow$$

developing laminar flow, UWT,
$Pr > 0.5$
$L < L_h$ or $L < L_t$

For non-circular tubes the hydraulic diameter, $D_h = 4A_c/P$ can be used in conjunction with

Table 10-4 to determine the Reynolds number and in turn the Nusselt number.

In all cases the fluid properties are evaluated at the mean fluid temperature given as

$$T_{mean} = \frac{1}{2}\left(T_{m,in} + T_{m,out}\right)$$

except for μ_w which is evaluated at the wall temperature, T_w.

2. Turbulent Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For turbulent flow where $Re_D \geq 2300$ the Dittus-Bouler equation (Eq. 19-79) can be used

$$\boxed{Nu_D = 0.023\, Re_D^{0.8}\, Pr^n} \quad \Rightarrow \quad \begin{array}{l} \text{turbulent flow, UWT or UWF,} \\ 0.7 \leq Pr \leq 160 \\ Re_D > 2,300 \\ n = 0.4 \;\; \text{heating} \\ n = 0.3 \;\; \text{cooling} \end{array}$$

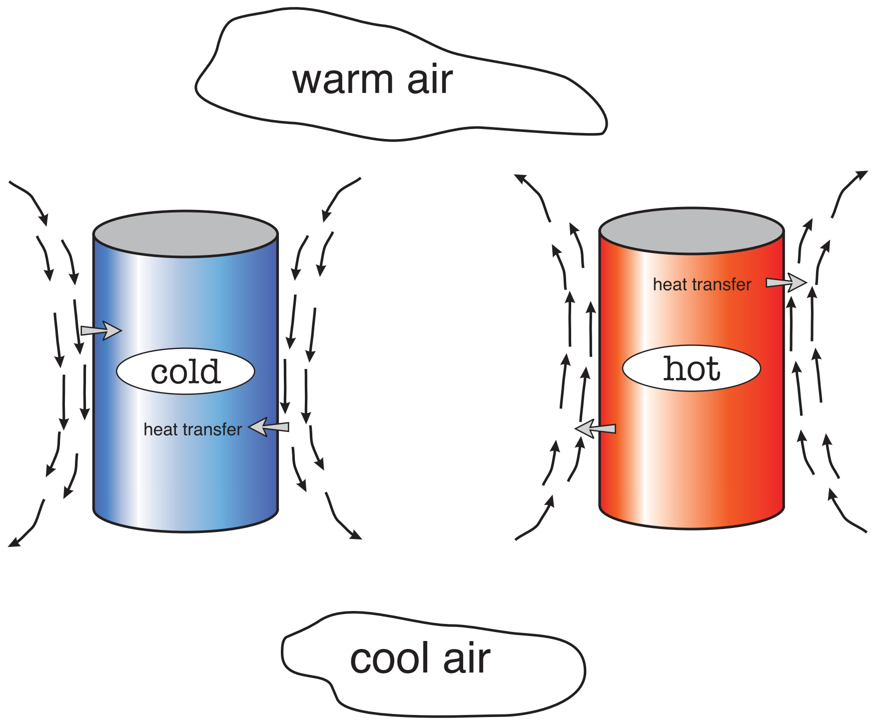
For non-circular tubes, again we can use the hydraulic diameter, $D_h = 4A_c/P$ to determine both the Reynolds and the Nusselt numbers.

In all cases the fluid properties are evaluated at the mean fluid temperature given as

$$T_{mean} = \frac{1}{2}\left(T_{m,in} + T_{m,out}\right)$$

Natural Convection

What Drives Natural Convection?



- a lighter fluid will flow upward and a cooler fluid will flow downward
- as the fluid sweeps the wall, heat transfer will occur in a similar manner to boundary layer flow however in this case the bulk fluid is stationary as opposed to moving at a constant velocity in the case of forced convection

We do not have a Reynolds number but we have an analogous dimensionless group called the *Grashof number*

$$Gr = \frac{\text{buouancy force}}{\text{viscous force}} = \frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\nu^2}$$

where

$$g = \text{gravitational acceleration, } m/s^2$$

$$\beta = \text{volumetric expansion coefficient, } \beta \equiv 1/T \ (T \text{ is ambient temp. in } K)$$

$$
\begin{aligned}
T_w &= \text{wall temperature, } K \\
T_\infty &= \text{ambient temperature, } K \\
\mathcal{L} &= \text{characteristic length, } m \\
\nu &= \text{kinematic viscosity, } m^2/s
\end{aligned}
$$

The volumetric expansion coefficient, β, is used to express the variation of density of the fluid with respect to temperature and is given as

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

Natural Convection Heat Transfer Correlations

The general form of the Nusselt number for natural convection is as follows:

$$Nu = f(Gr, Pr) \equiv C Ra^m Pr^n \qquad \text{where } \; Ra = Gr \cdot Pr$$

- C depends on geometry, orientation, type of flow, boundary conditions and choice of characteristic length.
- m depends on type of flow (laminar or turbulent)
- n depends on the type of fluid and type of flow

1. Laminar Flow Over a Vertical Plate, Isothermal (UWT)

The general form of the Nusselt number is given as

$$Nu_{\mathcal{L}} = \frac{h\mathcal{L}}{k_f} = C \left(\underbrace{\frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\nu^2}}_{\equiv Gr}\right)^{1/4} \left(\underbrace{\frac{\nu}{\alpha}}_{\equiv Pr}\right)^{1/4} = C \; \underbrace{Gr_{\mathcal{L}}^{1/4} \, Pr^{1/4}}_{Ra^{1/4}}$$

where

$$Ra_{\mathcal{L}} = Gr_{\mathcal{L}} Pr = \frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\alpha\nu}$$

2. Laminar Flow Over a Long Horizontal Circular Cylinder, Isothermal (UWT)

The general boundary layer correlation is

$$Nu_D = \frac{hD}{k_f} = C \left(\underbrace{\frac{g\beta(T_w - T_\infty)D^3}{\nu^2}}_{\equiv Gr} \right)^n \left(\underbrace{\frac{\nu}{\alpha}}_{\equiv Pr} \right)^n = C \underbrace{Gr_D^n \, Pr^n}_{Ra_D^n}$$

where

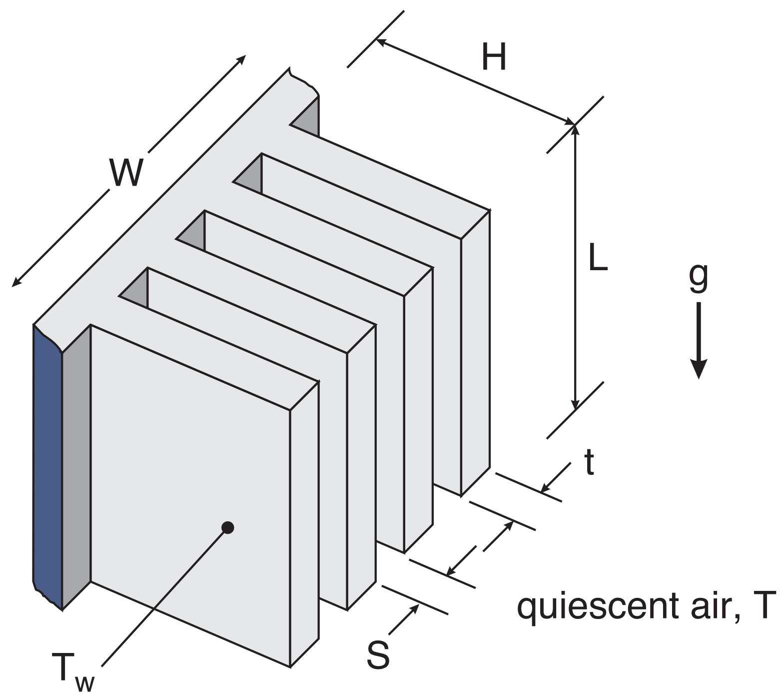
$$Ra_D = Gr_D Pr = \frac{g\beta(T_w - T_\infty)D^3}{\alpha\nu}$$

All fluid properties are evaluated at the film temperature, $T_f = (T_w + T_\infty)/2$.

Natural Convection From Plate Fin Heat Sinks

Plate fin heat sinks are often used in natural convection to increase the heat transfer surface area and in turn reduce the boundary layer resistance

$$R \downarrow = \frac{1}{hA \uparrow}$$



For a given baseplate area, $W \times L$, two factors must be considered in the selection of the number of fins

- more fins results in added surface area and reduced boundary layer resistance,

  $$R \downarrow = \frac{1}{hA \uparrow}$$

- more fins results in a decrease fin spacing, S and in turn a decrease in the heat transfer coefficient

  $$R \uparrow = \frac{1}{h \downarrow A}$$

A basic optimization of the fin spacing can be obtained as follows:

$$\dot{Q} = hA(T_w - T_\infty)$$

where the fins are assumed to be isothermal and the surface area is $2nHL$, with the area of the fin edges ignored.

For isothermal fins with $t < S$

$$S_{opt} = 2.714 \left(\frac{L}{Ra^{1/4}} \right)$$

with

$$Ra = \frac{g\beta(T_w - T_\infty L^3)}{\nu^2} Pr$$

The corresponding value of the heat transfer coefficient is

$$h = 1.31k/S_{opt}$$

All fluid properties are evaluated at the film temperature.