ECE 309
Introduction to Thermodynamics and Heat Transfer

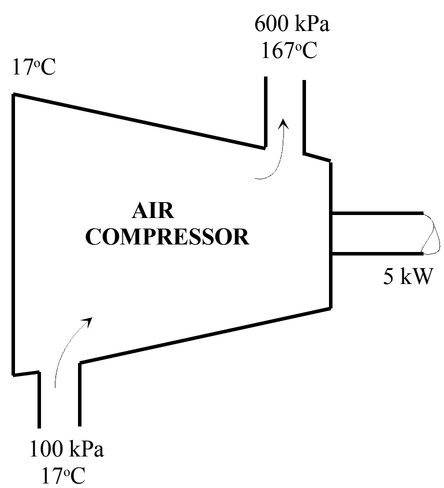
Tutorial # 6

Entropy

Problem 1

Air is compressed steadily by a 5-kW compressor from 100 kPa and 17^{o}C to 600 kPa and 167^{o}C at a rate of 1.6 kg/min. During this process, some heat transfer takes place between the compressor and the surrounding medium at 17^{o}C. Determine

(a) the rate of entropy change of air and
(b) the rate of entropy generation during this process.



Solution:

Step 1: Write the given data from the problem statement

State 1: P_1 = 100 kPa, T_1 = 17^{o}C
State 2: P_2 = 600 kPa, T_2 = 167^{o}C
Surrounding temperature: T_{surr} = 17^{o}C
Work input to the compressor: $\dot{W}$ =- 5 kW (negative sign indicates work is done on the system)
Mass flow rate: $\dot{m}_1 = \dot{m}_2 = \dot{m} = 1.6 kg/\min = 0.02666 kg/s$

Step 2: Write what we are asked to solve for:

(a) rate of entropy change of air: $\Delta\dot{S}_{air}$ = ?
(b) rate of entropy generation during the process: $\dot{S}_{gen}$ = ?

Step 3: State the assumption(s):

(1) At the specified conditions air can be assumed as an ideal gas
(2) Assuming steady state steady flow process (SSSF)
(3) Changes in kinetic and potential energies to be negligible

Step 4: Write the Energy balance (First Law of Thermodynamics) and Entropy Balance (Second Law of Thermodynamics) equations for the system shown in Figure P1

Energy Balance (First Law of Thermodynamics):

$$\dot{Q} - \dot{W} = \dot{m}(h_2 - h_1) \tag{1.1}$$

for ideal gas, we know that

$$dh = C_{p,av} dT \tag{1.2}$$

$$\dot{Q} = \dot{W} + \dot{m}C_{p,av}(T_2 - T_1) \tag{1.3}$$

Entropy Balance (Second Law of Thermodynamics):

$$\sum \frac{\dot{Q}}{T} + \dot{S}_{gen} = \dot{m}(s_2 - s_1) \tag{1.4}$$

Again for ideal gas, we know that

$$s_2 - s_1 = C_{p,av} \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \tag{1.5}$$

where R is gas constant

$$\sum \frac{\dot{Q}}{T} + \dot{S}_{gen} = \dot{m}\left(C_{p,av} \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right) \tag{1.6}$$

Step 5: Solve for the unknown quantities

From Eq. (1.3), calculate the heat transfer during the process

$$\dot{Q} = -5\ kW + (0.02666\ kg/s)(1.010\ kJ/(kg\cdot K))(440 - 290)K = -0.96101\ kW \tag{1.7}$$

Note: In the above calculation of heat transfer, $C_{p,av}$ is found from Table A-2b at T_{av}= 365 K

(a) Rate of entropy change ($\Delta\dot{S}_{air}$)

$$\Delta\dot{S}_{air} = \dot{m}(s_2 - s_1) = \dot{m}\left(C_{p,av} \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right) \tag{1.8}$$

where $s_2 - s_1$ is replaced by Eq. (1.5)

$$\Delta \dot{S}_{air} = (0.02666\ kg/s)\left((1.010\ kJ/kg\cdot K)\ln\frac{440\ K}{290\ K} - (0.287\ kJ/kg\cdot K)\ln\frac{600\ kPa}{100\ kPa}\right) \tag{1.9}$$

$$\boxed{\Delta \dot{S}_{air} = -0.00248\ kW/K}$$

(b) Rate of entropy generation during the process ($\dot{S}_{gen}$)

Using Eq. (1.6) and result (a), we can find the rate of entropy generation during the process

$$\dot{S}_{gen} = \Delta \dot{S}_{air} - \frac{\dot{Q}}{T_{surr}} = -0.00248\ kW/K - \frac{-0.96101\ kW}{290\ K} \tag{1.10}$$

$$\boxed{\dot{S}_{gen} = 0.00083\ kW/K}$$

Problem 2

Steam enters a turbine at 30 bars and 400°C with a velocity of 160 m/s. Saturated vapor exits at 100°C with a velocity of 100 m/s. At steady state, the turbine develops work equal to 540 kJ per kg of steam flowing through the turbine. Heat transfer between the turbine and its surroundings occurs at an average outer surface temperature of 500K Determine the rate at which entropy is produced within the turbine per kilogram of steam flowing, in KJ/kgK. Neglect the change in potential energy between inlet and exit.

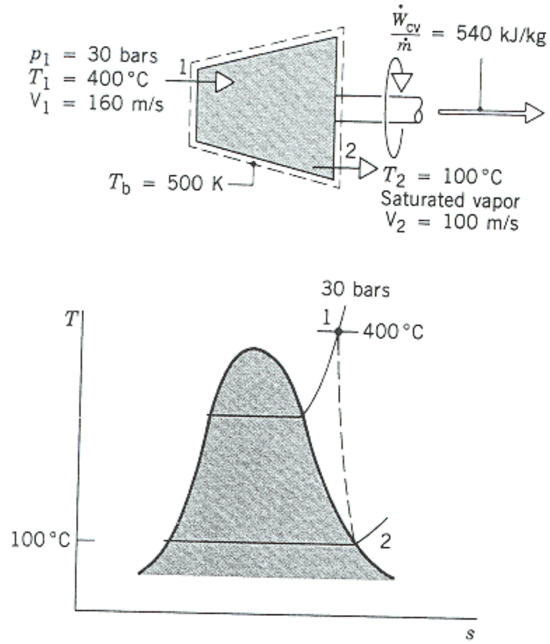


Figure P2

Solution:

Step 1: Write the given data from the problem statement

State 1: P_1 = 30 bars, T_1 = 400°C, V_1 = 160 m/s

State 2: Saturated vapor, T_2 = 100°C, V_2 = 100 m/s

Boundary temperature: T_b = 500 K

Work output from the turbine: W = 540 kJ/kg

Step 2: Write what we are asked to solve for:

Rate of entropy produced per kg of steam flowing within the turbine: S_{gen} = ?

Step 3: State the assumption(s):

(1) Assuming steady state steady flow process (SSSF)
(2) Change in potential energy between the inlet and outlet is negligible
(3) Heat transfer between the turbine and the surroundings occurs at a boundary temperature T_b

Step 4: Write the Energy balance (First Law of Thermodynamics) and Entropy Balance (Second Law of Thermodynamics) equations for the system shown in Figure P2

Energy Balance (First Law of Thermodynamics):

$$\dot{Q} - \dot{W} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \right) \tag{2.1}$$

$$\frac{\dot{Q}}{\dot{m}} = \frac{\dot{W}}{\dot{m}} + h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \tag{2.2}$$

$$Q = \frac{\dot{Q}}{\dot{m}} = W + h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \tag{2.3}$$

$$Q = 540\left(\frac{kJ}{kg}\right) + (2676.1 - 3230.9)\left(\frac{kJ}{kg}\right) + \left(\frac{100^2 - 160^2}{2}\right)\left(\frac{m^2}{s^2}\right)\left(\frac{1\,N}{1\frac{kg\,m}{s^2}}\right)\left(\frac{1\,kJ}{10^3\,N\,m}\right) \quad (2.4)$$

$$\boxed{Q = -22.6\,kJ/kg}$$

{Note: From Table A-4, @ T_1 = 400°C and P_1 = 30 bars, h_1 = 3230.9 kJ/kg and
s_1 = 6.9212 kJ/kg K
From Table A-2, @ T_2 = 100°C and x_2 = 1, h_2 = 2676.1 kJ/kg and
s_2 = 7.3549 kJ/kg K }

Entropy Balance (Second Law of Thermodynamics):

$$\frac{\dot{Q}}{T_b} + \dot{S}_{gen} = \dot{m}(s_2 - s_1) \quad (2.5)$$

$$\frac{\dot{S}_{gen}}{\dot{m}} = (s_2 - s_1) - \frac{\left(\frac{\dot{Q}}{\dot{m}}\right)}{T_b} \quad (2.6)$$

$$S_{gen} = (7.3549 - 6.9212)\left(\frac{kJ}{kg\,K}\right) - \frac{(-22.6)\left(\frac{kJ}{kg}\right)}{500\,K} \quad (2.7)$$

$$\boxed{S_{gen} = 0.4789\,kJ/kg\,K}$$